LUND SOCIAL SCIENCE STUDIES
4

PUBLISHED BY WRITERS CONNECTED WITH THE INSTITUTES OF SOCIAL SCIENCES,
LUND UNIVERSITY, SWEDEN

THE PROBLEM OF SUMMATION IN ECONOMIC SCIENCE

Contents: ix

THE PROBLEM OF SUMMATION IN ECONOMIC SCIENCE

A METHODOLOGICAL STUDY WITH
APPLICATIONS TO INTEREST,
MONEY AND CYCLES

By
GÖRAN NYBLÉN

LUND 1951
C. W. K. GLEERUP

PRINTED BY
SKÅNSKA LITOGRAFISKA AB
MALMÖ – SWEDEN
1951

TO THE MEMORY OF
MY PARENTS

EDITORIAL NOTE

This edition has been prepared due to the extreme scarcity of the original text. At this time (November 2007) there is no copy available for sale through the usual WEB booksellers. Even the library copies are disintegrating due to the decay of the paper. It is intended that this edition be freely available, in PDF form, on the WEB. Redistribution of the PDF is expressly permitted and, in fact, encouraged. Since the original publisher no longer exists and I could not locate a successor, I have assumed the copyright of this edition.

This edition was produced by, first, OCR converting a scanned copy of the text. The OCR was fairly good, but the equations had to be reset in TEX, and the usual number of incorrect OCR conversions required correction. My intention was to reproduce the text as exactly as possible so that nothing would be lost. I corrected obvious typographical errors. There were surprisingly few of these. There was only one point where there is an error (or possibly incomplete statement). This occurs on page 177 (this edition) following equation (17) where the variable g_i is used. This variable is not defined. This omission could be corrected by looking up the referenced paper by Marschak, but I have not pursued this. There are a few other places where the mathematical expression is, I think, not particularly clear. I have left these unchanged. In all cases, I am confident that the analysis and conclusions are correct.

There surely remain typographical errors which were introduced by the OCR and by my editing. I have recently corrected a number of minor typos and improved the layout in several cases. I would appreciate reports of any remaining errors. My email address is below.

This work is of fundamental importance to the development of economics. It is for this reason that I have made it more accessible. Had Nyblén lived, he would have had the opportunity to develop and explain his ideas, to the great benefit to economic science and society. As it is, the incompatibility of the present macro economic theory and econometrics with the *two summation theorems* described here makes this theory and statistical analysis meaningless, and of no valid use for economic planning, prediction, or regulation.

This publication is available in my library on: archive.org. This is an edited copy introducing some minor typographical changes mainly to make the text more accurate to the original. Minor changes to the 2014 text. However, the *Contents* and the *References* were edited to be more complete and accurate. This version as of June 2024 makes mainly a number of edits to more accurately show the usage of the $\gg$ symbol.

Michael D. Godfrey
June 2024

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ACKNOWLEDGMENTS

This study was initiated in the academic year 1948–49, when I was working at the Lund Institute of Economics; it was further developed during the year 1949–50 in the course of my stay in the United States on a scholarship, granted by the Swedish Social Science Research Council; it assumed its present form in the year 1950–51 at the Lund Institute of Economics; and it was finally submitted as a doctoral thesis at Lund University in September 1951.

Only the errors and opacities in the volume are completely my own. I cannot possibly express in adequate terms my debt to my teacher, Professor *Johan Åkerman,* Lund University. His guiding influence has been indispensable in bringing the analysis up to whatever level it has attained; his warm personal interest in my work has been a great encouragement. I should also like to acknowledge the interest shown me by the following scientists of the American universities which I visited: Professor *Wassily W. Leontief,* Harvard University, Professor *Jacob Marschak,* the University of Chicago, Professor *Oskar Morgenstern,* Princeton University, Professor *John von Neumann,* the Institute for Advanced Study, and Professor *Paul A. Samuelson,* Massachusetts Institute of Technology. Particularly I wish to thank Professor Morgenstern for most instructive and stimulating discussions.

I further extend my thanks to Mr. *C. Montagu Evans,* M. A. (Cantab.) for improving my English, and to Mr. *Wilhelm Trotzig* for seeing the volume through the press.

Lund, September 1951. *Göran Nyblén*

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INTRODUCTION

The *problem of summation* arising in economic science is due to the multi-economic-subject character of most social organizations. It is only absent in societies comprising no more than one economic will. Although seldom explicitly singled out as a distinct entity, the problem is touched upon in almost all economic theories and its treatment extends far back in the history of doctrines — as well-known examples we may mention the discussion of the comparability and transferability of individual utilities and the discussion of the additivity of supply curves of separate enterprises, both topics obviously only relevant to economies with many decision units.

Recently two important theorems in the field have been published. *The first summation theorem* is found in von Neumann's and Morgenstern's[1)] 1944 [1]. theory of games and states that economic values are in general non-additive, if by economic value is meant the amount which a group of economic subjects can secure for themselves through collaborating within a social economy. Considering two or more groups of economic subjects, the theorem signifies that the amount which the economic subjects in all groups involved can achieve through collaboration is in general greater than the sum of those amounts which the various (sub)groups can gain, if not cooperating and instead acting in isolation. To illustrate, we may provisionally think of the subgroups as local labor unions and of the inclusive supergroup as a national federation of local labor unions. *The second summation theorem* is due to Arrow[2)] and states that it is in general impossible to aggregate many individual value schemes to one social value scheme, if by value scheme is meant a complete ordering of the various alternatives among which an economic subject may choose, i.e. an ordering showing for each pair of two alternatives that one alternative is preferred to the other or that the, alternatives are indifferent, this under the condition that if alternative **A** is preferred to alternative **B** and **B** is preferred to alternative **C,** then **A** is always preferred to **C.** The feasibility of aggregating individual value schemes to a social value scheme is usually taken for granted in welfare economics, where it is believed that a social welfare function may

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[1)] John von Neumann and Oskar Morgenstern, ≫Theory of Games and Economic Behavior≫, Princeton 1944

[2)] Kenneth J. Arrow, ≫Social Choice and Individual Values≫, New York and London 1951 [2], *and* previously circulated unpublished material.

be based on individual preference scales. *In this thesis we mean by the problem of summation in economic science precisely these two theorems and their implications.* They can both be put under the following heading: *the economic problem of the relation between the parts and the whole where the parts are in the last instance the individual decision units and the whole is in the last instance a society or a number of coexisting societies.* In this wide formulation the economic problem of summation was early posed by Johan Åkerman.[3)] Obviously this problem is absent only in a society where the parts are identical to the whole, i.e. a society with no more than one economic subject.

Our task is to investigate certain further implications of the two summation theorems. The implications in which we are interested fall into three categories. The first category comprises the relations between the summation theorems themselves. We see immediately that these constitute a field of investigation because at first sight the theorems seem to be incompatible with each other: the first-mentioned uses economic values tied to sets of economic subjects, the second mentioned proves the general impossibility of aggregating individual value schemes into a value scheme attached to a group of economic subjects. The second category comprises the relations between the summation theorems and the traditional economic theory, notably the Walrasian constructions and the econometric systems. It is easily concluded that there are points to be clarified in this context. A Walrasian construction, for instance, claims to involve many economic subjects without any non-additivity of economic values or any conflict between various individual value schemes appearing. The third category comprises the relations between the summation theorems and certain fields of empirical economic research; here lies our main interest. The investigation of the first two categories of implications is a necessary preliminary to the study of the third; it may be labeled the methodology stage of our analysis. It is carried out in Chapters I–IV of our thesis. At the outset we discuss the econometric equation systems intended to deal with the kind of empirical problems in which we are chiefly interested. As the main result of the analysis in Chapters I–IV our distinction between the ≫production aspect≫ and the ≫distribution aspect≫ is established.

The analysis in Chapters V–VII starts from a realization of ≫the fundamental problem≫, i.e. the endeavor to let empirical economic research be guided by other kinds of theories than essentially Walrasian constructions, ≫whose empirical foundations be in limbo≫.[4)] Utilizing the

[3)] Johan Åkerman, ≫Ekonomisk kausalitet≫, Lund 1936 [3].

[4)] Oskar Morgenstern, ≫Professor Hicks on Value and Capital≫, The Journal of Political Economy, 1941, p. 393.

distinction between the production aspect and the distribution aspect, and certain consequences of the first summation theorem already established we discuss certain empirical problems of interest rates, inflation and business cycles, i.e. implications, belonging to the third category are established: this investigation may be labeled the application stage of our analysis. At the same time we shed some further light on the first two categories of implications by paying attention in the special contexts involved to the distinction between the production aspect and the distribution aspect, and, e.g. by briefly discussing the possible treatment of "the particular empirical problems within a Walrasian universe. In the last section of every chapter and in a concluding section of the thesis we give a detailed summary of our conclusions and results as regards the problem of summation in economic science.

It need hardly be added that our analysis is tentative and has but modest claims, intruding as it does into a field so far very little systematically explored. At several junctures the analysis has to cease after having pointed at the existence of important, unsolved problems. On the other hand we think that we make definite progress tn certain directions where the silence of economic science has been almost complete up to the present time. This is true, for instance, of the corroboration established between certain important results of game theory and certain well-known observations.

In this context it may be pointed out that the parts of game theory — consequences of the first summation theorem — which we utilize are fairly few: beyond the three main concepts of characteristic functions, imputations and solutions, we refer to the distinction between objective and discriminatory solutions, mainly to be found in the zero sum three-person game; also to the theory of excess to be found in the game discussion of composition and decomposition; and finally to the multiplicity of characteristic functions signifying zero-sum n-person games with $n \geq 4$. We try to explain and discuss all concepts and theorems utilized, both of game theory and of other economic theories, but must presuppose a certain knowledge of some of the constructions involved. Quotations are frequently made when this is found convenient; they are of two main types. The first type — chiefly appearing at the methodology stage of our thesis — serves in the comparison of various theories; the second type — appearing in some places at the application stage of our thesis — serves to convey observations of empirical phenomena, whether they be of a more directly observable quantitative or qualitative kind or of a more subtle kind like the ≫social norms≫ which we shall discuss.

It should be emphasized that the problem of summation is almost completely distinct from ≫the problem of aggregation≫ described in econometric literature, although there may be a point of tangency which will also be considered in this thesis. The latter problem is not chiefly concerned with the multi-unit-of-decision character of the society studied; it could as well be posed within a single-unit-of-decision economy because it deals with the reduction of a comprehensive economic equation system with finely subdivided variables into a system containing only a few relations between ≫aggregative≫ index-number-variables, easier to handle computationally.

We conclude this introduction with a technical note. 1) The literature references given are of two kinds. Works only more or less casually referred to are placed in footnotes. Works of importance for the main arguments of the chapter are placed in a list included at the end of the book; to which indications are made in footnotes. 2) Important concepts of certain theories and investigations discussed are, to avoid confusion with our own concepts, spelled with capital initial letters. 3) Square brackets [ ] in quotations enclose words which must necessarily be added to elucidate the meaning of the quotations, when these are taken from their original context. Three stops ... indicate as customarily that a section in the text quoted is excluded; one stop within a sentence indicates that we have dropped a symbol or a reference which is devoid of interest in the new context. 4) The numbering of the formulas refers to each chapter separately.

Chapter I

The Econometric Equation Systems

1) The triple character of econometric equation systems; the behavior equations

The economist observes and measures prices, quantities, interest rates wages and many other economic variables; he follows their courses in the country studied during time-spans of inflationary processes, of the ups and downs of business cycles or of war-planned social economies, thus composing time-series of the variables for a certain region and period. His aim is to understand and explain the movements of such sets of time-series; his criterion of success in the task is the power of prediction, even though it may be conditional in various respects. The time-series observations of economics, as the observations of all sciences, may give hints about fruitful problem statements but must also be complemented by theoretical hypotheses about the mechanism that produces the variables. The scientist regards these hypotheses in the light of the observations known to him, other than those directly used in their construction; that is, he studies the activities of the various units of economic and social decision which both in conflict and collaboration form a society, the results of whose activities are the observed variables. The economic scientist is not primarily a politician; he undertakes his investigations out of idle curiosity. He does not believe in his findings being utilized by a representative of a Utopian Common Good; he merely hopes his investigations will at least contribute towards man's understanding of his rôle in a changing world of struggle and peace, prosperity and chaos.

One very important way of looking upon a set of time-series for a given region and period is to consider the variables in question as connected by a system of several simultaneous relations, i.e. their time-paths as generated by an equation system. We do not dwell here upon the particular characteristics usually attributed to the system of relations, its being dynamic and stochastic, etc., but only stress the fundamental idea that economic variables are produced by a mechanism, which can be described as a system of simultaneous equations. The idea, explicitly originating in the works of Cournot and Walras — partly already in Quesnay's *Tableau* — and almost unconnected with the quantitative empirical study of economic variables, was brought into contact with econometric time-series analysis especially by

Tinbergen in the 1930's, above all through his *Statistical Testing of Business Cycle Theories* (1939),[1] following the somewhat earlier work by Frisch and others. Tinbergen consistently adheres to the idea that the economic variables considered in USA from 1919 to 1932, the time-paths of which the economist observes, are generated by an equation system connecting the variables. On the basis of certain economic plausibility conditions he puts forward a system of relations and he estimates from the observed time-series the numerical values of parameters in these relations by regression analysis, also trying to shun equations that seem to be in contradiction to the facts. In the econometric work following Tinbergen's pioneering and carried out mainly by research workers of the Cowles Commission, e.g. Klein's *Economic Fluctuations in the United States, 1921–1941* (1950),[2] the fundamental idea of economic variables as produced by a system of simultaneous relations is left completely untouched; only its complicated implications for statistical estimation methods of the parameters, etc. are much more thoroughly explored.

This state of affairs is completely in accordance with the views of many prominent contemporary economic theoreticians. Thus Samuelson writes on the topic in question: ≫It is not to be thought that the content of [equation] systems as described above must be restricted to the variables usually considered in price and value theory. On the contrary, one employs such constructions throughout the whole field of theoretical economics, including monetary and business cycle theory, international trade, etc. It goes without saying that the existence of such systems in no way hinges upon the employment of symbolic or mathematical methods. *In fact any sector of economic theory which cannot be cast into the mold of such a system must be regarded with suspicion as suffering from haziness*≪.[3] (Italicized here). Marschak speaks in the same vein: ≫ ... the multiequational nature of economics (whether micro- or macro-economics) ought to be more strongly emphasized. The student should learn to express in equations any 'theory of international trade,' or 'tax incidence,' or 'money and capital markets,' or any other hypothesis of his own making or of some writer's making; to

[1] Tinbergen [4] (Numbers in square brackets refer to the List of References at the end of the volume.)

[2] Klein [5].

[3] Samuelson [6], p. 9. Reproduced by permission of Harvard University Press.

show whether such a system is consistent and complete; and to discuss the effects of a given change in its parameters≫.[4)]

The functional form and the parameters of the relations included completely characterize an equation system; we may briefly refer to these entities as the ≫coefficients≫ of the system. In econometric literature the set of coefficients in an economic equation system is labeled the *structure* of the system. Assuming the fundamental idea correct and knowing the structure thus defined, e.g. obtained from observations through the application of certain regression methods, and valid for a definite region and period, we should have a complete explanation and understanding *of the course of events* there and then; this is the first characteristic of econometric equation systems. It means that in case no change in the coefficients — no structural change — has occurred, we should be able to predict for the next time-period the results of the activities of the many units of decision composing the society studied — if only within the limits set by the possible stochastic character of the relations and by the level of possible ≫exogenous≫ variables.[5)]

It would at first seem as if the task of the scientist be fulfilled at this point, the demands of scientific curiosity satiated. Nevertheless, the formal characteristics of the systems make it clear that further possibilities exist and these possibilities have also been explored in actual research. In Tinbergen's pioneering work referred to above the structure is not only determined, but also changes in structure are discussed from the viewpoint of their impact on a certain economic-political goal of the society in question; thus, e.g. the influence is calculated of a change in the coefficient ≫the marginal propensity to consume of earners of wages and salaries≫ on the periodicity of the dynamic equation system. We here encounter another characteristic of econometric equation systems besides that of describing and explaining the course of events, namely that of being an *instrument of decision* for society, or let us say the government. This second characteristic is elaborated and formalized in Marschak's article ≫Statistical Inference in Economics: an Introduction≫, in ≫Statistical Inference in Dynamic Economic Models≫ (1950).[6)] Policy may be of two kinds: ≫non-structural≫ and ≫structural≫. A non-structural policy fixes the values of variables,

[4)] John Marschak, ≫On Mathematics for Economists≫, *The Review of Economic Statistics,* 1947, p. 271. Reproduced by permission of Harvard University Press.

[5)] Variables, the time-paths of which are taken as given in the system. For a rigorous definition see Ed. Koopmans [7], pp. 7–8; cf. e.g. Koopmans [8], p. 169.

[6)] Ed. Koopmans[7], pp. 1–50.

controlled directly by the (government) policymaker, that is the values of certain exogenous variables of the system; a structural policy changes the structure, that is, changes some coefficient(s) of the system. Assuming that the objectives, ≫the gain≫, ≫the welfare≫, of society can be expressed as a function of all the variables of the system, and assuming the structure known, Marschak shows how the way of action, maximizing the gain, can in principle be found through a straightforward maximization procedure, taking into account both non-structural and structural policy and also adjustments of possible uncontrolled exogenous variables and unintended structural changes.[7)]

The second characteristic is heavily stressed in recent econometric literature. In an article on ≫Methodological Issues in Quantitative Economics≫ (1949), Koopmans, after having mentioned that ≫political economy has traditionally sought justification for its speculations in the search of a scientific basis for public policy in economic matters≫, even expresses the opinion that failure in the determination of the structure of econometric equation systems, barring every attempt to establish a government policy of the kind just referred to, may lead towards the necessity of accepting authoritarianism. He says, e.g., thinking of the statistical determination of ≫behavior equations≫, the dominating equation type in the econometric systems, and the resulting policy recommendations: ≫Now, if we do not learn how to introduce greater stability in the economy by the indirect inducements of money supply, tax schedules, and other general incentives or deterrents to individual action — either because such knowledge is impossible or because we do not seek it — political processes and social necessities will make us move more to the method of direct administrative prescription of individual behavior≫.[8)]

We note the double character hitherto indicated of econometric equation systems. Let us now begin to examine the various types of equations assumed to compose such an equation system in order better to be able to grasp the significance of the whole construction.

No doubt the *behavior equations* already mentioned, play the rôle most emphasized within the econometric equation systems. Familiar examples of such equations are demand relations, supply relations, consumption functions and investment functions; they are said to describe economic action, to describe ≫the behavior of individuals or collective units in their economic activity, their decisions to produce and consume≫,[9)] ≫a certain type of

[7)] Ed. Koopmans [7], pp. 8–19.
[8)] Koopmans[9], p. 89.
[9)] Haavelmo[10], p. 3.

economic decisions taken by a certain category of economic agents≫,[10] i.e. they represent ≫the joint response of groups of individuals or firms to a common economic environment≫.[11] Sometimes the crucial features of the relations of this type, the variables included, the signs of the coefficients, etc., are assumed ad hoc, but often attempts are made to derive the relations from supposedly more fundamental ones. This procedure is carried out by means of a principle of human economic or social behavior; in these cases it is almost always assumed that the unit of decision or the group of units concerned tries to maximize its utility or gain. Through a process of maximization the behavior equations are derived from the underlying ≫preference functions≫, ≫production functions≫ and ≫expectation functions≫ — we refer particularly to Hurwicz's article "Theory of the Firm and of Investment" (1946)[12] for a clear exposition and formalization of this important line of thought. The functional form and the parameters of the underlying relations, the preference, production and expectation functions, characterize a given decision unit (or possibly a group of decision units); they depend solely on its psychological, material, etc., equipment; they exist completely by themselves without reference to any other decision unit. The behavior equations being derived from the underlying functions, the coefficients of the behavior equations must in principle depend completely on the same individual psychological and other traits of the decision unit in question; changes in these components of the structure must be attributed to changes in the preference, production and expectation functions.

If such a process of derivation be applied, the resulting behavior is called ≫rational behavior≫. The process being applied or not the fact remains, that the type of econometric relations usually considered to be the most important one, gives *an account of the behavior of economic non-governmental decision units,* that is they describe quantitatively how economic objects, preferably in fulfillment of some goal, make decisions to adjust variables at their disposal to variables controlled by other agencies in the society; this is the third and last important characteristic of econometric equation systems which we wish to stress. Thus on the (rational) decisions by the individual economic subjects, depicted through the structural coefficients of the behavior equations, is superimposed the (rational) policy of society, expressed by changes of structural coefficients and fixation of certain exogenous variables, the set of structural coefficients of a system further describing the course of events in a given space and time. We may

[10] Ed. Koopmans[7], p. 54.
[11] Koopmans[11], p. 64.
[12] Hurwicz[12].

say that we have *a unified theory* with a *triple character.* We bear this statement in mind and reserve our comments on it until later in the analysis.

2) The econometric equation systems and the permanence of economic laws

Before continuing the examination of the various types of equations composing an econometric equation system, let us consider a group of problems already touched upon above when the notion of structural change was mentioned. As explicitly stated, e.g. in the quotation from Samuelson on p. 6, the fundamental idea of an equation system connecting the economic variables is usually believed to be of universal applicability, i.e. this theoretical construction is supposed to be general enough for application to every social economy at any time; the structure of an equation system is, however unique, e.g. the particular numerical values of the parameters, etc., in question are only valid for a specified social economy during a definite time-period. From country to country and from period to period the coefficients may change. To fix ideas we refer to a statement by Tinbergen, that the same system of relations can be used to analyze the economies of USA and of Russia during, let us say, the 1930's. The two social economies however show different numerical values of certain structural coefficients, notably the parameter called ≫the marginal propensity to invest≫.[1)] Further it may be recalled that the discussion under the heading ≫Keynes vs. the Classics≫ partly dealt with questions about the numerical values of certain crucial parameters in the equation system scrutinized. This is particularly pregnant in Klein's clear exposition of the controversy. What constitutes this revolution? Klein answers: ≫Suppose that we live in a Euclidean world in which there exists a perfectly competitive capitalist system. It shall now be argued that this Utopia will not automatically solve the unemployment problem under the conditions of Keynesian economics. However, full employment will always be insured in a classical situation in this world. Now it becomes possible to see the fundamental difference between Keynesian and classical economics ... The Keynesian Revolution rejected the classical theory of interest. It denied that the equation[2)]

$$S(r, Y_0) = I(r, Y_0)$$

need always have a positive solution for the interest rate, r, when Y_0 (income) is given at the *full-employment level.* When the saving process is

[1)] Jan Tinbergen, *Economische Bewegingsleer,* Amsterdam 1946, pp. 239-243.

[2)] S= savings; I = investment; r = interest rate; Y_0 = income at the full employment level. (The symbols are slightly changed by us).

analyzed, the slope of the savings function with respect to the interest rate might be negative or positive and will probably be small in absolute value. More recently, we have come to believe that the investment function is also interest-inelastic. It is more likely than not that there will be no positive value of r which satisfies this equation. *Perfect equilibrium of perfect competition is not in general compatible with the system of Keynesian economics* ... Incomes will have to adjust to that level, Y_1, at which savings out of that income will be equal to investment out of that income≫.[3)]

The difference between the classical thought-structure and the Keynesian one is condensed in Klein's analysis to a problem of the numerical values of the coefficients in the behavior equations named ≫the savings (consumption) and the investment functions≫ — the classical conclusions follow from certain values of the coefficients, the Keynesian conclusions from other values. It would seem appropriate, although maybe not explicitly stated by Klein, to interpret these findings in the following way: the same economic theory is applicable to the social economies of, for instance, the Anglo-Saxon world both in the time of the Classics around 1830 and in the time of Keynes around, 1930, but certain parameters of relations included in this theory show different values for the two time periods.

Tinbergen thus compares regionally the economies of two countries with widely different social organizations; Klein (slightly reinterpreted) compares temporally the economies of the same countries under different conditions of social organization — in both cases the dissimilarities are compressed to different numerical values of structural coefficients in behavior equations.

We are now going to add two points about the permanence of economic laws, the first being based on purely empirical considerations, the second being stressed by economists sometimes known as institutionalists. Haavelmo writes: ≫Every research worker in the field of economics has, probably, had the following experience: When we try to apply relations established by economic theory to actually observed series for the variables involved, we frequently find that the theoretical relations are 'unnecessarily complicated'; we can do well with fewer variables than assumed a priori. But we also know that when we try to make predictions by such simplified relations for new set of data, the relations often break down, i.e., there appears to be a *break in the structure* of the data. For the new set of data we might also find a simple relation, but a *different* one≫.[4)] Similar empirical

[3)] Klein[13], pp. 83–86. Reproduced by permission of The Macmillan Company.

[4)] Haavelmo[10], p. 26. Reproduced by permission of the Econometric Society.

experiences play a great rôle in the writings of the Swedish economist Johan Åkerman.[5] According to his opinion economics at the present stage ought to recognize that institutionally dissimilar time-periods, i.e. periods that differ with respect to the social groups dominating the society, to the power of the government, etc., show different economic relations or ≫laws≫ between the variables observed; no universal theory is possible at the ordinary level but certain ≫laws about the laws≫ may be discovered. Åkerman's strongly empirically orientated researches have similarities in the writings of Veblen and some of his American followers, who, e.g. according to Gruchy, the historian of doctrines, hold the following views: ≫For every different cultural region there should be a theory of economic enterprise which explains the organization of economic life in that particular cultural area≫.[6]

The econometric systems of simultaneous equations are supposed to be universally valid up to changes in structure. The question appears whether the type of element of change in the systems, i.e. the structural coefficients, is a sufficient one to grasp the changing economic reality. In other words: is the idea tenable that systems of simultaneous equations are universally applicable to social economies of all kinds? To discuss this very important problem we first focus attention on the other types of econometric relations besides behavior equations.

3) Further types of econometric relations

The types of econometric relations left to be considered are: ≫institutional equations≫, ≫technical equations≫, ≫balance identities≫ and ≫equilibrium conditions≫.[1] Usually these categories play a decidedly minor rôle in econometric literature — possibly with the exception of balance identities —, and are considered to be relatively problemless, as contrasted to the ample space given to the behavior equations and the emphasis put on their intricacies. In our opinion, however; the examination of these types of relations may provide an answer to the question of the universal applicability of econometric equation systems, a question which cannot be solved through the concentrated and isolated study of behavior equations. The discussion opened in this section will continue throughout Chapters I–IV of our thesis, particularly as regards the balance identities and the equilibrium conditions.

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[5] Johan Åkerman, *Ekonomisk kausalitet,* Lund 1936 [3] och Johan Åkerman [14], [15], [16].

[6] Gruchy [17], pp. 552–553.

[1] For lists of the various types of such relations, see e.g. Ed: Koopmans [7], pp. 54–55; Koopmans [18], p. 125; Marschak [19], p. 81; Klein [5].

Institutional equations are not frequently encountered in applied econometrics but examples of these relations are often listed: price controls, tax laws, bank reserve regulations; they are said to describe ≫behavior patterns set by law or rule≫.[2] Although their name may suggest that they should be of peculiar interest to the present investigation, their importance ought not to be overstressed; they give, however the first indications of the point towards which the whole discussion of econometric equation systems will tend.

As long as the equation system is viewed only as an instrument of decision for the government, no particular difficulties are connected with the institutional relations: they serve to modify the values of some structural coefficients, or may possibly restrict the variations of otherwise exogenous variables; in our opinion they could be viewed as government policy measures, actually carried out, and being either of the structural or the non-structural type according to Marschak's above-mentioned distinction.

If, however, that aspect is stressed which views the equation system as comprising several decision-units, the decision of each being described by a set of behavior equations, the problem arises as to whose decisions stand behind the institutional equations, or in Koopmans' words: ≫These [institutional] rules (tax schedules, reserve requirements, etc.) can to some extent be taken as given for the analysis of economic fluctuations. In a deeper analysis, these rules and the changes in them would need to be explained further from choices by individuals interacting, in various degrees of association with each other, through political processes≫.[3] It is not a priori clear how ≫economic choices≫ could be separated from ≫institutional choices≫, also the latter as exemplified above being certainly of importance for the amount of the gains of the individual decision units, nor is it clear how ≫institutional choices≫ could be described in the usual equation system setup, e.g. in terms of behavior equations derived from individual preference, production and expectation functions. If these types of decisions, however, are not explicitly included in the analysis, the set of structural coefficients cannot give a completely satisfactory understanding and explanation of the course of events, as possibly established values of the coefficients may be upset wholly unsystematically by new decisions in the category. As we do not consider institutional equations very important we leave the matter at this, and continue our analysis by commenting briefly on the technical equations.

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[2] Ed. Koopmans [7], p. 54

[3] Koopmans [9], p. 87. Reproduced by permission of Harvard University Press.

As we remarked in Section 1 above, the behavior equations are often derived through some maximizing principle from certain underlying functions attached to the individual decision unit, among them a production, or ≫technical≫ function, or a set of such functions. In this setup the technical equations are, we can say, institutionally localized to particular units within the society; furthermore they do *not* appear in the final system of equations, but the coefficients of the behavior equations included here depend of course on the shape of the technical restrictions, being partly derived from these. Thus it seems that a consistent use of the second viewpoint, society looked upon as composed by several decision units whose activities are described through behavior equations which in themselves depend on e.g. technical conditions, should exclude the presence of technical equations in the system of simultaneous equations supposed to generate the observed time-series. The matter is, however, changed if the first viewpoint be solely applied and the system of simultaneous equations regarded as a mechanism, put at the disposal of a central planning agency. In this case we have a situation similar to that facing an isolated individual decision unit, whose behavior is to be determined from the preference, production and expectation conditions; and the explicit introduction of technical equations seems obvious and necessary.

This being so, we find, although fairly weakly marked, that traces of the same dichotomy are present both in the application of the institutional equations and the technical equations: both types of relations are easily reconciled with the idea of a society consisting of only one decision unit but their inclusion in the system of simultaneous equations presents difficulties, if the many-decision-unit-character of society is stressed. We shall meet the same dichotomy in a much more pronounced way when analyzing the balance identities and the equilibrium conditions; we shall refer to these as belonging to the ≫link category≫, the name alluding to their rôle of knitting together the behavior equations. The analysis will be carried out mainly in the following three chapters; in this section we only note a few examples of the uncertainty found in the literature concerning the interpretation of the two relation types.

A most well-known relation encountered in economic analyzes is the statement that the sum of total consumption, C, and total investment, I, equals national income, Y; $C + I = Y$. The relation is usually said to be derived through application of the principles of economic accounting and it is viewed as a ≫balance identity≫ or a ≫balance equation≫.[4)]

[4)] See e.g. Paul A. Samuelson, "The Simple Mathematics of Income Determination", in *Income, Employment and Public Policy,* New York 1948, p. 134.

The characteristic and important thing for balance identities is that they hold by assumption; we may say that the equality between ≫supply≫ and ≫demand≫ is assumed by the theory. They are found throughout the literature on social accounting and similar topics, e.g. in Frisch's ecocirk-system.

Let us now in the usual manner complete the relation (1) $C + I = Y$ into a complete equation system by adding the two behavior equations (2) $C = C(Y)$ and (3) $I = I(Y)$. We now have a system of three variables in the relations. Concerning this system, unessentially modified, Marschak makes the following statement ≫[This system] has been repeatedly misunderstood in economic literature. One hears, for example, that because of [(1)], the sum of the propensities to consume and to invest is equal to unity: individuals can either consume or not consume the product, and the not consumed (saved) part is added to the nation's plant and inventories, i.e. is identical with investment. The investment so defined is *not* the variable quantity I that obeys the behavior relation [3]; instead it is only a particular value of the variable I, viz. the value that satisfies not only [(3)] but also [(1)]. This particular value is called equilibrium value because [(1)] is an equilibrium condition; [(1)] is actually a mere approximation of some dynamic statement such as

$$(Y_t - C_t - I_t) \to 0 \quad \text{as } t \to \infty \gg. \tag{4}$$

Marschak's[5] words explain very well what is meant by an equilibrium condition: as contrasted to a balance identity it does not hold by assumption, but instead it is satisfied through the working of some mechanism that the (complete) theory has to describe; we may say that the equalization of ≫supply≫ and ≫demand≫ has to be evidenced by the theory. Behind the question whether $Y = I + C$ is to be looked upon as a balance identity or an equilibrium condition we recognize without difficulty the old Keynesian savings-investment controversy. Although recommending the last-mentioned interpretation, Marschak, however, is not sure about the implications of equilibrium conditions. In another context he discusses a very similar setup: (5) $s = d$; (6) $d = d(p)$; (7) $s = s(p)$ [s = supply, d = demand, p = price], and concludes: ≫The economic basis of the various 'laws of supply and demand' is very weak. While [the behavior equations] the supply and demand curves [(6), (7)] have been satisfactorily derived from utility-maximizing behavior of households and firms, the adjustment

[5] Jacob Marschak, ≫20 Lectures on Income, Employment, and Price Level. Given at the University of Chicago, 1938≫, Cowles Commission for Research in Economics. The University of Chicago, Chicago 37, Illinois; mimeogr.; Lecture 13.

equations such as [(5)] have not been derived with any clarity. It is not even clear whose behavior is expressed by equation [(5)].≫ In what follows he considers among other things the relation (1) and concludes that it may presumably describe the behavior of producers.[6)]

In conclusion we would stress that these varying attitudes towards the significance of the equations in the link category do not preclude the circumstance that one or both types of relations are found in probably all econometric equation systems hitherto set up.

4) Summary

a) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS AND THE ECONOMETRIC SYSTEMS

The econometric systems show no signs of any particular difficulty inherent in the superposition of social decisions on individual decisions — in an essentially inclusive or unified theory they try to give both an instrument of decision for the government or the society and an account of the (utility-maximizing) behavior of the many individual units composing the society; moreover, no problem about the relations between the individual units — e.g. their cooperation in a distribution struggle — is posed, a feature which is closely tied to the purely economic — not political or sociological — character of the main ≫social groups≫ considered: consumers and investors. In order further to explore the rationale of this fact we single out for consideration two particular types of econometric equations — the balance identities and the equilibrium conditions — which form the connecting links between the various behavior equations, assumed to be derivable from assumptions on economic action of individual units or the government. Our discussion of these relations will later give as result that the balance identities embody the nucleus of a teleological element, which when further extended and stripped of its most restrictive features leads into systems, comprising only one value scheme — and hence being only able to take into account social decisions which are not superimposed on individual decisions (≫the pure production aspect≫); the equilibrium conditions on the other hand include a rudimentary theory of distribution between many economic subjects, which when further developed and stripped of its most restrictive features leads into systems which cannot easily comprise any value scheme, involving commodities and services, but view the aim of the economic subjects as consisting in the maximization of their respective

[6)] Jacob Marschak, ≫Introduction to Econometrics≫, Course given in March–May 1948, The University of Buffalo, Buffalo 14, New York; mimeogr.; pp. 27–28.

shares of the money income in the society, i.e. their relative income shares (≫the pure distribution aspect≫).

b) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS AND EMPIRICAL RESEARCH

The econometric systems try to convey in a unified theory both a social instrument of decision and an account of the (utility-maximizing) behavior of the many individual units composing the society — this construction also serves to explain the economic course of events in a society. We observe a special feature of the relation between the systems and empirical research — a feature which is correlated with the unproblematical superposition of social decisions on individual decisions within the systems and with the unproblematical character of the relations between the individual units or the groups of such units: the data or ≫the empty boxes≫ or the elements of change of the systems consist of structural coefficients, supposedly derivable from preference, technical and other functions characterizing single economic decision units. The aim of empirical research along traditional econometric lines is to find the numerical values of structural coefficients. These may vary between regions and periods, but particularly as regards variations between periods within the same region, no features of the systems make abrupt structural changes likely to occur.

Chapter II

The Production Aspect and the Distribution Aspect

1) The programming production analyzes

a) LEONTIEF'S SYSTEM

It is often pointed out[1)] that marked similarities exist between Keynesian econometric systems and the analysis presented in Leontief's *The Structure of the American Economy 1919–1929* (1940)[2)] and in later articles of his — the Leontiefian system being only much more general including as it does a large number of commodities and activities as contrasted to the aggregative income-consumption-investment-analysis, and hence also more difficult to handle computationally. The outlines of Leontief's system are easily given.[3)] The national economy is divided into n industries or ≫activities≫ (in the empirical applications published n is at most about 40, still implying a quite considerable amount of aggregation) with outputs $X_1, X_2, \ldots X_n$. For each activity $[I]$ is investigated, *first,* how its output $[i]$ is allocated to the other activities — thus out of X_1 the activity may itself retain x_{11}, activity no. 2 gets x_{12}, activity no. 3 gets x_{13}, etc.; — *second,* from what sources its inputs, i.e. its productive factors, come — thus activity no. 1 may retain x_{11} of its own product, it gets x_{21} from activity no. 2, x_{31} from activity no. 3, etc. The well-known input-output-matrix with this information for all activities gives us the following set of

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[1)] See e.g. Lloyd A. Metzler, "Three Lags in the Circular Flow of Income," in *Income, Employment and Public Policy,* New York 1948, pp. 14–15. Metzler speaks, however, in terms of monetary flows, while Leontief and most recent works in econometrics utilize real flows.

[2)] Leontief [20].

[3)] For a good exposition of Leontief's system and related topics see Alexandre Chabert, "Le système d'input-output de W. Leontief et l'analyse économique quantitative," *Économie Appliquée* 1950, pp. 173–205.

equations:

$$(1)\qquad \begin{aligned} X_1 &= x_{11} + x_{12} + x_{13} + \ldots + x_{1n} \\ X_2 &= x_{21} + x_{22} + x_{23} + \ldots + x_{2n} \\ &\vdots \\ X_n &= x_{n1} + x_{n2} + x_{n3} + \ldots + x_{nn} \end{aligned}$$

$$(x_{ij} : \text{ from I to J}).$$

or

$$X_i = \sum_{j=1}^{n} x_{ij} \qquad (i = 1, \ldots, n)$$

Now the hypothesis is introduced that the input-amounts which each activity requires from itself and the other activities are proportional to the output of this first activity — thus activity no. 1 with output X_1 requires the amount $a_{11}X_1$ from its own output, the input $a_{21}X_1$ from activity no. 2, the input $a_{31}X_1$ from activity no. 3, etc., while activity no. 2 with output X_2 requires the input $a_{12}X_2$ from activity no. 1, the input $a_{22}X_2$ from its own output, the input $a_{32}X_2$ from activity no. 3, etc. In all, this hypothesis gives us the following equations:

$$(2)\qquad \begin{aligned} x_{11} = a_{11}X_1; x_{21} = a_{21}X_1; \ldots; x_{n1} = a_{n1}X_1 \\ x_{12} = a_{12}X_2; x_{22} = a_{22}X_2; \ldots; x_{n2} = a_{n2}X_2 \\ \vdots \\ x_{1n} = a_{1n}X_n; x_{2n} = a_{2n}X_n; \ldots; x_{nn} = a_{nn}X_n \end{aligned}$$

or

$$x_{ji} = a_{ji}X_i \qquad \begin{bmatrix} i = 1, \ldots, n \\ j = 1, \ldots, n \end{bmatrix}$$

(1) and (2) may be summarized thus:

$$(3)\qquad X_i = \sum_{j=1}^{n} a_{ij}X_j \qquad (i = 1, \ldots, n).$$

When the system (3) is discussed and criticized attention is usually focused on the part of it which is represented by equations (2), the technical equations. Fulfilling our program we are first going to consider here the set of equations (1), which obviously are of the link category. Leontief states about these relations: they describe ≫balance — the external relationships

— between all the various sectors of the national economy in terms of the outputs (supply) and the inputs (demand) for each and every kind of goods and services. These supply- and demand-equations are simple in form and non-controversial in content≫.[4)]

In the ensuing discussion several participants commented on this statement; we quote Koopmans: ≫Supply- and demand-equations are usually conceived as relationships expressing by what quantity suppliers and consumers respond to a given price. The same term should not be used for the identity (or equilibrium condition) which says that, in the absence of changes in stocks of a commodity, the quantity supplied equals the quantity demanded≫[5)]

We agree with Koopmans in the terminological question, but from our point of view it is important to discuss the real character of equations (1), if they are balance identities or equilibrium conditions and what implications follow from either interpretation. We see immediately that every relation (1) expresses how the sum of the requirements of all activities for a particular commodity is put equal to the amount of the commodity produced, that is the rate at which commodity i flows from activity I is equal to the rate of input of i to all activities in the system. The system — as far as presented up to this point — does not give any indications at all about a possible ≫mechanism≫ or procedure, through which this equalization be achieved, and nothing contradicts the interpretation that the relations (1) [and the equality signs in the combined system of equations (3)] are valid by assumption, that is according to our definition are balance identities, representing the conservation of real flows. What does this interpretation imply?

The Leontiefian system is very ≫mechanical≫ in the sense that no fundamental economic decisions are explicitly tied to it, involving choices of ways of action or choices of sets of commodities or agreements between various decision units or similar things; there are at first sight no teleological assumptions included in it at all, involving the search of economic decision units for some objective, for profit or utility, since the technological necessities expressed by equations (2) and the balance identities determine everything. It may be questioned, however, if the balance identities do not embody the nucleus of precisely a teleological postulate: they embody no

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[4)] Wassily W. Leontief "Recent Developments in the Study of Interindustrial Relationships," *The American Economic Review,* Papers and Proceedings, 1949, p. 212.

[5)] Tjalling C. Koopmans, "Discussion of Leontief's paper," *The American Economic Review, Papers and Proceedings,* 1949, p. 234. The quotations are reproduced by permission of the American Economic Association.

doubt a full-utilization-of-resources assumption in assuming the equality of output and input, of ≫supply≫ and ≫demand≫. Who achieves this equality? If no, let us say, ≫market mechanism≫ with an equalization possibility can be added to the system, the idea that the equality is established by a central planning authority would appear to be present, and thus the proposition may even arise that the application of balance identities in a somewhat disguised manner implies the existence of a central planning authority or the ≫planning-system≫ character of the construction.

Before further investigating the possible addition of such a ≫mechanism≫ of equalization we would mention a ≫dynamic≫ extension of the Leontiefian system, which also allows teleological traits of a certain kind to appear, still vaguely, but in a somewhat stronger manner than above. This extension has been elaborated by Hawkins in his article "Some Conditions of Macroeconomic Stability" (1948),[6] and unpublished researches by Leontief move in the same direction. It takes into account the existence of stocks and the formation of new capital. Let us introduce n stocks or types of real capital, one for each activity; denote further by $y_{ij}(i = 1, \ldots, n; j = 1, \ldots, n)$ the type of real capital, briefly ≫the capital≫, from activity I that is stored in the productive processes of activity J — above, we denoted by x_{ij} the type of commodity from activity I that is employed in the productive processes of activity J — and assume that the physical amounts of various capitals stored in a certain activity are proportional to the output of this activity: $y_{ji} = b_{ji}X_i(i = 1, \ldots, n; j = 1, \ldots, n)$ — above, we assumed that the physical amounts of various commodities employed in a certain activity were proportional to the output of this activity [relations (2)]. The rate of change of a capital, — of increase, that is new investment or of decrease, that is disinvestment —, $\frac{dy_{ij}}{dt} = \dot{y}_{ij}$, is then proportional to the rate of change of output,

$$\frac{dX_i}{dt} = \dot{X}_i; \qquad \text{we get}$$

$$\dot{y}_{ji} = b_{ji}\dot{X}_i \qquad \begin{bmatrix} i = 1, \ldots, n \\ j = 1, \ldots, n \end{bmatrix}. \tag{4}$$

We now combine relations (4) with relations (2) and apply the full-utilization-of-resources assumption, i.e. introduce balance identities, stating how the sum of the requirements of all activities for the product of a particular activity, to be used partly as a commodity or a ≫consumed factor≫, partly as a capital or a ≫stored factor≫ in the productive processes, is put

[6] Hawkins [21].

equal to the amount of production of this activity. We first obtain balance identities of this kind:

$$X_i = \sum_{j=1}^{n} x_{ji} + \sum_{j=1}^{n} y_{ji} \quad (i = 1, \ldots, n) \tag{5}$$

and the final result is the following combined system of linear differential equations, replacing relations (3):

$$X_i = \sum_{j=1}^{n} (a_{ij} X_j + b_{ij} \dot{X}_j) \quad (i = 1, \ldots, n) \tag{6}$$

This means that the rate at which product i flows from activity I is equal to the rate of input — both as current factor of production and as stored factor of production — of i to all activities in the system.

It may still be asked: who achieves the equality between supply and demand, that is, the full utilization of resources? This viewpoint is here somewhat underlined by the fact that phenomena of growth appear and the full utilization of resources implies that the maximum rate of growth is established for the economy, and further stressed, if the maximum rate of growth be assumed to be established not for an indefinite stretch of time but for a definite time-span from $t = 0$ to $t = T$, i.e. for the ≫planning period≫ $0 \to T$.

The general solution of the system of linear differential equations (6) can be written

$$X_i = \sum_{j=1}^{n} C_j D_{ij} e^{\alpha_j t} \quad (i = 1, \ldots, n) \tag{7}$$

where the C_j are arbitrary constants, the D_{ij} determined by (6) and the α_j are the roots of the characteristic function of (6). It may now be shown that there is in fact at most one such [root] that has empirical meaning as a [steady] state in which the economy can function. The other solutions represent transient states meaningless if taken by themselves. The reason for this is that there is at most one particular solution for which the coefficients [a_{ij} and b_{ij}] are all of the same sign, and no meaning has been given to a negative rate of production≫.[7)] Let us denote this root by α_1. The corresponding solution or time-path of the variables reads

$$X_i = C_1 D_{i1} e^{\alpha_1 t} \quad (i = 1, \ldots, n) \tag{8}$$

[7)] Hawkins [21], p. 314.

and α_1 is the rate of growth of the whole economy [in the steady state]. Equation (6) may now be written:

$$X_i = \sum_{j=1}^{n} X_j(a_{ij} + b_{ij}\alpha_1) \quad (i = 1, \ldots, n) \tag{9}$$

We shall return to this setup, both in Chapter V and Chapter VII.

We spoke above about the possible supplementing of the input-output-systems by some ≫mechanism≫, e.g. a ≫market mechanism≫ to account for the equalization of supply and demand. The thing we have to do in order to investigate this possibility is obviously to consider the rôle of prices in the systems, a rôle which has not up to this point been mentioned at all. On carrying out this investigation we find that the prices do not embody an equalization procedure; on the contrary their definition is based on the equality-assumption. Thus the prices of the static Leontiefian system are given by the following relations:

$$P_i = \sum_{j=1}^{n} P_j a_{ji} \quad (i = 1, \ldots, n) \tag{10}$$

They state that the price of a unit output shall be equal to the sum of the values of the inputs used in the production of that unit. The prices of the dynamic system are similarly given by the relations

$$P_i = \sum_{j=1}^{n} P_j(a_{ji} + b_{ji}\alpha_1) \tag{11}$$

which state that the price of a unit output shall be equal to the sum of the values both of consumed factors and of stored factors used in the production of that unit. We note for later discussions three very interesting (partly overlapping) features of these prices.

First: it is not necessary to introduce them into the analysis; it may be convenient to do so but everything can be expressed, every manipulation with the systems can be carried out, in real terms. The prices are adequately named ≫fictitious prices≫ (Hawkins) or ≫shadow prices≫.

Second: There exists an one-to-one correspondence between real quantities and monetary prices in the systems, which we may symbolize thus:

$$\overrightarrow{X} \quad \longleftrightarrow \quad \overrightarrow{P} \tag{12}$$

where $\overrightarrow{X}$ is the vector of commodities[8] and P the vector of prices. We compare equations (3) and (9) respectively involving quantities with equations (10) and (11) respectively involving prices. One set of comparable equations is obtained from the other by transposing the matrix of technical coefficients $[a]$ or $[a+b]$, that is by interchanging its rows and columns; a vector of prices (quantities) expressed as explicit functions of the coefficients is obtained from a vector of quantities (prices) expressed as explicit functions of the coefficients through substituting a_{ji} and b_{ji} for a_{ij} and b_{ij} (a_{ij} and b_{ij} for a_{ji} and b_{ji}).

Third: the procedure just outlined, which leads from quantities (prices) to prices (quantities), leaves the basic teleological assumptions about full utilization of resources and maximal growth of the economy unaffected. We may say that the coefficients and the teleological assumptions constitute the very mechanism to be studied; the working of this mechanism may be observed either from the quantity or real side, or from the price or monetary side, and for all essential purposes it is immaterial from which side it is observed.

Having made plausible our interpretation of the relevant relations in the link category as balance identities, holding by assumption, and having made certain tentative remarks about the implications of this interpretation, we now shift our direct attention from these relations themselves to the second set of relations in the input-output-systems, the linear-proportional technical relations, so often discussed as being extremely oversimplified.

Leontief says about these relations: ≫In treating technical input coefficients as independent structural parameters, this approach assumes them to be independent of the prices of the respective cost factors and thus eliminates from this particular general equilibrium model the 'substitution effect' of the marginal productivity theory. This can be considered to be its fundamental weakness≫.[9] Opinions holding that the (static) input-output-system is a special case of the Walrasian general-equilibrium system because it substitutes linear-proportional production functions — absolute complementarity between factors assumed — for nonlinear production functions, follow the same direction; sometimes reservations are added for the peculiar rôle of prices in Leontief's analysis. In our opinion these statements require considerable qualification in the light of recent investigations.

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[8] The symbol $\overrightarrow{X}$ stands for the n-uplet $[X_1, X_2, \ldots, X_n]$.

[9] Wassily Leontief, "Econometrics" in *A Survey of Contemporary Economics,* Philadelphia 1948, p. 409. Reproduced by permission of The Blakiston Company.

b) THE PROGRAMMING SYSTEMS PROPER

The hypothesis that each productive process can only be carried out in one given fixed way seems to contradict elementary everyday observations, which demand that a choice between several ways of producing the same commodity be admitted. Examining Leontief's systems from these considerations we may in a first step regard the productive processes actually employed in it, i.e. the i sets of coefficients $[a_{i1}, \ldots, a_{in}]$ as chosen from a wider set of ≫all technically possible production processes≫. The question then arises: how is this choice made or how can the choice be conceived of as possible to make at all? The way out is in a second step to introduce into the system explicitly more activities than commodities, that is, to allow for the possibility of producing some or all of the commodities in more than one way. This second step was first taken by von Neumann in his paper "A Model of General Economic Equilibrium" (1936).[10] Such a system with more activities than commodities is in general undetermined. In order to get a determinate system some device must be introduced by which the choice between the various alternative production possibilities be made, that is the problem of best utilization of resources appears. In a third step this is achieved through the explicit statement of the goal or the objectives of the economy, or, we might say, its ≫preference scale≫ or ≫value scheme≫, and the attempted optimal fulfillment of this entity. The step was first taken by Dantzig and Wood in the articles on ≫Programming of Interdependent Activities≫ (1949)[11]; their Objective Function, which is to be maximized, serves the purpose. In this extended setup the teleological assumptions are quite definite; it is explicitly no longer only the question of full utilization of resources but also of best utilization of resources, and the attainment of this is relative to one single value scheme of the society.

The Leontiefian systems[12] may be viewed as giving an explanation of the course of events in a multi-unit-of-decision society, granted the simplifications that every unit has no more complicated decisions to make than adhering to certain fixed technical rules and that full employment of all resources is achieved by chance. In our opinion the interpretation does injustice to the systems. We first concentrate on the static variant (3). This

[10] von Neumann [22]; compare the comments in Kurt Singer, ≫Robot Economics≫, *Economic Record* 1949, pp. 48–73.

[11] Dantzig [23].

[12] From now on we put under this headline both the static variant or Leontief's system proper, and the dynamic variant, or Hawkins' system.

may be viewed as the result of a choice from a wider system, comprising more activities than commodities:

$$X_i = \sum_{j=1}^{n} a_{ij} X_j \qquad (i = 1, \ldots, m); n > m \tag{13}$$

The choice based on one single value scheme of society, retains n of the m activities at positive production levels ($X_j > 0$) and puts equal to zero the levels of $(m - n)$ superfluous activities ($X_j = 0$). According to this interpretation the smaller system (3), granted its static character, describes the course of events in a society with one common set of objectives as long as the set of objectives is left unchanged and the plan fulfilled, that is during one single planning period. The wider system consisting of (13) and the Objective Function does more than this; it depicts the whole process of decision whereby the plan is established. The smaller system is more tangible than the wider one, but can it be given a real meaning, if stripped from the greater context?

A programming system could be said to consist of two parts, the *real* part and the *virtual* part. The real part includes those activities among the possible ones that are actually singled out and operated at positive production levels. The virtual part includes firstly those activities which are considered in the decision process but which do not materialize, secondly the set of objectives, formalized in the Objective Function — the value scheme of society — which makes the choice between the alternatives possible. In our opinion a Leontiefian system should be looked upon as the real part of a programming system, depicting the actual fulfillment of a particular plan of society, i.e., describing the course of events in a centrally planned economy during one planning period — disregarding certain types of possible disturbances. If this interpretation be not adhered to, most serious problems arise in judging the implications of the extremely restrictive linearity assumptions of a Leontiefian system. If the interpretation be followed, however, the relation indicated between the programming analyzes proper and the Leontiefian systems throws new light on the significance of linear relations in economic theories.[13)] Admitting several alternative

[13)] On the predominance of linear relations in economic theories and the uncertain significance of this fact compare the following words by R. G. D. Allen: ≫Mathematical economics in the past has been dominated by the mathematical convenience of linear systems. It seems likely that linear assumptions are not adequate in the treatment of economic dynamics. If so, some other simplification is needed to prevent the mathematics from getting too formal. No one has yet hit upon the appropriate simplification. There is

ways of producing the same commodity — as does a general non-linear production function — a programming system proper does not rest on a restrictive linearity assumption; the real part of such a system shows, however, linear relations, which hold so long as no changes take place in the virtual part of the system — e.g. a change of the value scheme of society, or the appearance of new production alternatives, both types of phenomena being likely to become of actual importance at the beginning of a planning period.

The static assumption is not inherent in the programming analyzes. In this particular context we label ≫static≫ a system, which for all periods within the time-span considered shows the same values of the variables, that is of the quantities and the shadow prices. Systems (3) and (13) are static; (13) is written by Dantzig in the following form:

$$\sum_{j=0}^{n_t} \beta_{ij}^{(t)} X_j^{(t)} = 0 \qquad (i = 1, \ldots, m; \quad t = 1, \ldots, T) \tag{14}$$

The $\beta_{ij}^{(t)}$ correspond to the Leontiefian technical coefficients, and the balance identity expresses, as do (3) and (13), that ≫the net rate of flow of any commodity, considered over all activities during a period, vanishes≫[14] — the full-utilization-of-resources assumption. A ≫period≫ is the time from $(t-1)$ to t within the span from 1 to T. As long as the static assumption prevails all periods are alike. We label ≫dynamic≫ in this particular context a system which for all periods within the time-span considered does not show the same values of the variables. Hawkins' equations (6) are dynamic. In our discussion of them in this section we disregarded the transient components of their solution, but *the* steady-state solution alone took into account a certain dynamic phenomenon of growth, represented by the real rate of growth a_1; if $a_1 = 0$ we get the static Leontiefian system (3). Von Neumann's above-mentioned system is dynamic and we shall stress in Chapter V that it shows — just as Hawkins' steady-state solution — a maximum real rate of growth of the economy. The dynamics of both Hawkins' steady state and von Neumann's system is restricted in the sense that it presupposes the proportions between the various quantities remaining constant — there is only one coefficient of growth, valid for all sectors of the economy. Dantzig introduces dynamics in a way which embodies that of von Neumann but which goes further and also admits changes in the

much to be done by the mathematician as well as by the economist. ("The Mathematical Foundations of Economic Theory," *The Quarterly Journal of Economics,* 1949, p. 127).

[14] Dantzig [23]; p. 206.

proportions between the various quantities. To the balance identities (14) he adds the balance identities

$$\sum_{j=0}^{n_t} a_{ij}^{(t)} X_j^{(t)} = \sum_{j=0}^{n_t-1} \overline{a}_{ij}^{(t-1)} X_j^{(t-1)} \tag{15}$$

$$\left[\begin{array}{c} i = 1, \ldots, m; \quad t = 1, \ldots, T \\ \overline{a}_{ij}^{(0)} = 0 \end{array} \right]$$

A $\beta_{ij}^{(t)}$ represented a constant rate of flow during the period $(t-1)$ to t of commodity i to activity J per unit of its output; an $\alpha_{ij}^{(t)}$ is the discrete quantity added to the flow of commodity i to activity J per unit of its output at the point of time $(t-1)$; an $\overline{a}_{ij}^{(t)}$ is the discrete quantity subtracted from the flow of commodity i to activity J per unit of its output at the time t. Thus coefficients β by themselves depict only unchanged flows of the commodities, coefficients a and $\overline{a}$ stand for changes in the flows, i.e., for changes in the values of the quantity-variables. The relations (15) state ≫that the output (for each commodity) of all activities operating in the period [no.] $(t-1)$ must equal the input of all activities operating during the tth period≫.[15] Arguments of the familiar accelerator type and other problems of time-lags or stocks and capital equipment could be expressed in these relations. Together (14) and (15) express the full-utilization-of-resources assumption in the dynamic case.

Dantzig's relations (14) and (15) introduce more activities than commodities $(n > m)$ allowing for several ways of producing the same output. As the logical completion of the system the Objective Function permits a choice between the various possible alternatives or ≫feasible programs≫. It expresses the objectives of the economy in some way in terms of commodities, and ≫the optimum feasible program is that feasible program which maximizes a specified linear objective function:

$$\sum_{t=1}^{T} \sum_{j=1}^{n_t} \gamma_j^{(t)} X_j^{(t)} = \text{Max.} \tag{16}$$

where $\gamma_j^{(t)}$ are constants≫.[16] This is the best-utilization-of-resources assumption.

We note that the Objective Function, of course, takes into account the whole time span from 1 to T, the whole planning period. Dantzig underlines

[15] Dantzig [23], p. 206.
[16] Dantzig [23], p. 207.

that a wide variety of sets of objectives can be expressed through a linear Objective Function as (16); examples are given on pp. 203–204 of his article. Already the system (14), (15) and (16) leads to intricate mathematical and computational problems but in several respects Dantzig strives for a still greater conceptual generality in his handling of the three components: the flows of the various commodities to the activities [Dantzig's Postulates I–V, and VII–VIII; compare Leontief's equations (2)]; the full-utilization-of-resources assumption [Dantzig's Postulate VI; compare Leontief's equations (1)]; and the best-utilization-of-resources assumption (Dantzig's sections on ≫Linear Program≫, ≫The Objective Function≫ and ≫Linear Objective Function and the Mathematical Problem≫).[17)]

Let us restate briefly the main points of our preceding argument in this section. The Leontiefian system, often mentioned as an obvious extension of some aggregative Keynesian model, seems to be of a very mechanical kind in the sense that it does not explicitly embody any significant economic or social decisions, only the application of certain rigid technical rules. Can this system, as sometimes believed, be the nucleus of an explanation of the course of events in a social economy? In connection with its balance identities we found traits of a certain teleological assumption: the full-utilization-of-resources assumption. This assumption was retained and somewhat more strongly underlined in the dynamic version of the system discussed (Hawkins). Turning our attention to its technical relations we noted the common criticism against the linear-proportional equations, implying that they are only a very special case of the non-linear relations in the general Walrasian system, and sometimes even doubting their contact with any reality. We ourselves took up the position that its significance — even in its dynamic forms — for the explanation of the course of events in a multi-unit-of-decision society is highly doubtful, but not so its relevance for the explanation of the course of events in a centrally planned economy. We tried to make these conclusions plausible by pointing out the fact that the most obvious criticism of the linearity-assumptions could be discarded if the system be viewed as embedded in a wider system, the wider system containing more activities than products *and* an openly teleological assumption — one set of objectives permitting a choice between alternative productive processes. The smaller system is thus the result of the process of choice valid until a new set of objectives be established, we might say during one planning period. A wider system with these properties — besides having thoroughly dynamic features — is found in the programming analyzes, now being elaborated, by Dantzig and others.

[17)] For a still further extension see H. W. Kuhn and A. W. Tucker "Nonlinear Programming" (abstract), *Econometrica* 1951, pp. 50–51.

Until the topic be further discussed, we are going to maintain in the following pages that the programming analyzes — under which heading we now include Leontief's systems — are of significance at least to a centrally planned economy; and it is possible that they may be of still wider relevance.

c) THE MARGINAL PRODUCTIVITY THEORIES OF DISTRIBUTION

The programming analyzes deal with a problem of production — the fulfillment of one set of objectives of society under technical restrictions. Do they also treat some problem of distribution? It ought to be fairly clear that a theory which only takes into account *one* economic will, *one* economic decision unit, namely the maximizing agent behind the Objective Function,[18] can comprise no real treatment of a problem of distribution; such theories which assume perfect coordination of all interests in society could not be expected to have any place for a theory of distribution between *two* or more decision units. However obvious these statements may seem to be, the reader who thinks of the set of distribution theories, often curiously enough still considered as the distribution theories — the marginal productivity theories of distribution —, cannot help finding certain points of contact between these and the programming systems. This resemblance is no superficial one, but in our opinion the theories in question deserve, without any reservation, the label ≫completely spurious distribution theories≪. We are now going to discuss this matter, starting from our above considerations on the rôle of prices in the Leontiefian systems.

The prices in the Leontiefian systems are not necessarily to be included in the analysis, being shadow prices; they stand in an one-to-one correspondence to quantities; and they are based on the teleological assumption. Are these features retained through the transition to programming analyzes proper? The first condition that prices need not be taken into account still holds — Dantzig's whole analysis is fulfilled without the mentioning of prices, which also means that the theory of shadow prices in programming analyzes still remains to be worked out. There are, however, already strong indications that shadow prices might also be defined on the basis, of the more explicit teleological elements and depend, as do the quantities, both on the teleological elements as well as on the technical coefficients. In von Neumann's system quantities (≫intensities of production≪) and prices show ≫symmetry≪: ≫Another feature of our theory, so far without interpretation, is the remarkable duality (symmetry) of the monetary variables

[18] Cf. on this point p. 84 below.

(prices, interest factor .) and the technical variables (intensities of production, coefficient of expansion of the economy .).≫[19] As we know von Neumann introduces more activities than products but leaves the system indeterminate, adding no Objective Function. Beyond these remarks we do not consider it important for our purpose explicitly to compare von Neumann's teleological assumptions with those of the other pieces of research studied. Introducing more activities than commodities and making teleological assumptions, including those of full-utilization-of-resources, but only reaching part of the way towards best-utilization-of-resources, Koopmans elaborates a price-concept with the desired properties.[20] Samuelson, finally, in a yet unpublished memorandum gives some examples of shadow prices, based on complete teleological assumptions.[21]

Stigler gives the following excellent summary of the so-called marginal productivity theories of distribution: ≫The completion of the marginal productivity theory of distribution was achieved only with the development of the proof that if all productive agents are rewarded in accord with their marginal products, then the total product will be exhausted. This exhaustion-of-product problem is of course unique to the general marginal productivity theory. In this respect previous distribution theories fall into one of two categories. The residual theories form the first group. Distribution theories of this type always premised at least one residual share; in the classical system, rent was accorded this position in a first dichotomy, ≫profit≫ in a second. Clearly no problem of the exhaustion of the product can arise when there is a residual claimant. The second general category includes all distribution theories in which the exhaustion of the product by distributive shares is made an explicit assumption. In this class fall the doctrines based on fixed coefficients of production which were advanced by Walras and Wieser, and their more recent followers. Only the marginal productivity theory, consequently, has been confronted by the question, does this method exactly exhaust the total product:≫[22]

The researches mentioned in the quotation are centered mainly around a transformation or production function:

$$X_1 = f(X_2, X_3, X_4) \tag{17}$$

where X_1 is the quantity of the product produced, say ≫corn≫, and X_2, X_3, X_4 are the quantities of the factors employed, say ≫labor, capital and

[19] von Neumann [22], p. 1.

[20] Koopmans [24], *and* unpublished memoranda; cf. p. 85 below.

[21] Paul A. Samuelson, "Market Mechanisms and Maximization," 1949.

[22] George J. Stigler, *Production and Distribution Theories,* New York 1948, p. 320. Reproduced by permission of The Macmillan Company.

land≫. Let us for the sake of illustration study an isoquant, that is the relation between the quantities of two factors, the quantities of the product and the other factors being constant (17) could result in e.g. the following ≫general≫ shape of an isoquant:

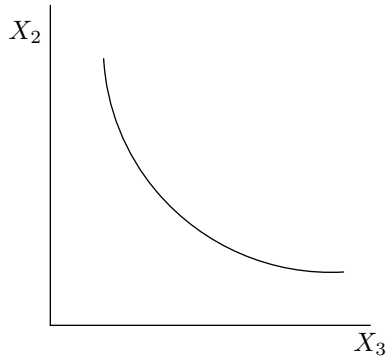

Fig. I

An isoquant in Leontief's original system is much more special:

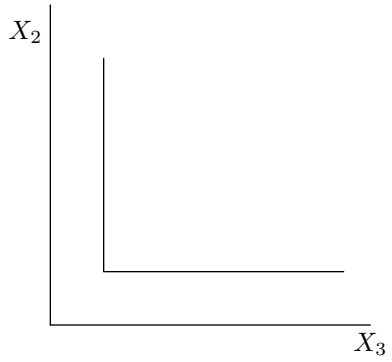

Fig. II

because of the assumption made which restricts factors to be combined only in *one fixed* proportion. In the programming analyzes an isoquant may approximate the ≫general≫ shape:

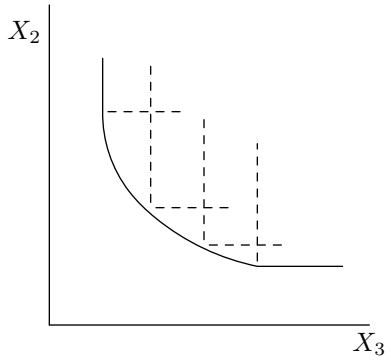

Fig. III

because they include many alternatively possible technical processes, admitting factors to be combined in *many fixed* proportions. The marginal productivity theory of distribution now requires the partial derivatives of (17) to be formed:

$$\frac{\delta X_1}{\delta X_2}, \quad \frac{\delta X_1}{\delta X_3}, \quad \frac{\delta X_1}{\delta X_4}$$

It then takes the first important step in postulating the transformation function to be homogeneous of the first order (≫if all factor-quantities are doubled, the product-quantity is doubled too≫). Because of this assumption the economically famous mathematical theorem by Euler gives:

$$X_1 \equiv X_2 \frac{\delta X_1}{\delta X_2} + X_3 \frac{\delta X_1}{\delta X_3} + X_4 \frac{\delta X_1}{\delta X_4} \tag{18}$$

The second important step consists in introducing the prices of the various quantities, say P, W, Z and J and putting the real ≫prices≫ equal to the partial derivatives:

$$\frac{\delta X_1}{\delta X_2} = \frac{W}{P}; \quad \frac{\delta X_1}{\delta X_3} = \frac{Z}{P}; \quad \frac{\delta X_1}{\delta X_4} = \frac{J}{P} \tag{19}$$

This reads: every productive factor is remunerated by the amount of the product that has been added by the last small input-increment of the factor in the productive process; the combination of (18) and (19) gives the conclusion:

$$PX_1 = WX_2 + ZX_3 + JX_4, \tag{20}$$

that is, the value of the final product is equal to ≫the sum of what goes to the laborers, of what goes to the capitalists and of what goes to the landlords≫ ; thus the factor shares exactly exhaust the social product and the problem of distribution is solved. We are now going to state why we consider this solution to be merely spurious.

The first step is eminently criticized by Samuelson, who asks why a production function *must* be homogeneous of the first order — this not being the case Euler's theorem does not apply and the whole theory with its total-exhaustion-of-product justification falls. ≫It is a scientifically meaningless assertion that doubling all factors *must*[23] double product.≫[24]

No doubt this criticism is very valid but the real weakness of the marginal productivity theory of distribution does not seem to lie solely in the homogeneity assumption; it is rather connected with the second step. Suppose the prices are considered as exogenous, determined by some forces outside the setup (17)–(21). Relations (19) then give the familiar maximization conditions for profits to be maximized under restriction of the production function, and prices being given. The conditions state that the marginal rate of substitution between product and factor is equal to the corresponding price ratio at the point of production where profits are at their maximum. This is the corner-stone of the traditional theory of the firm, but the approach does not depend on any homogeneity assumption and it does not say anything at all about what determines prices: it only allows derivation of supply and demand equations, showing the amounts of product supplied and the amounts of factors demanded, both as functions of the prices involved. We have here a marginal productivity theory of production of a firm but no theory of a social organization. It is in this context that the marginal productivity theory has played its rôle of historical importance.

Obviously a non-spurious theory of distribution cannot consider prices as given. In (19) there are as many new equations as new variables to be determined (price ratios) ; hence the inclusion of relations (19) with prices not exogenous into a marginal productivity setup does not in general change the determinateness of the system. If the quantities to be produced and employed are not determined by other parts of the system, the relations (19) cannot, under the condition stated, help to determine them; a basic indeterminacy thus appears. If the quantities be determined by other parts of the system the relations (19) fix the values of some sort of (superfluous) shadow prices. What is required for the quantities to be determined by other parts of the system? The experience from the linear programming

[23] Italicized here.

[24] Samuelson [6], p. 84.

analyzes strongly indicates that in all interesting cases — that is in all cases except the case of absolute complementarity of factors (Fig. II) — this is not possible without the establishment of some Objective Function, some value scheme of society. A glance at production function (17) suggests that in this case a program could possibly be formulated, where, e.g. production, is brought to a certain level, the amount used of one factor is minimized, the amount used of another factor is fixed at some level, etc. In our opinion the spuriousness of the marginal productivity constructions as theories of distribution has its prime origin precisely in the fact that the setups with prices not given logically require the statement of one set of objectives of society and hence must treat society as a unified organization with no diversity of interests and with no problem of distribution whatsoever; the prices consistent with this type of analysis are of a shadow character, and have no connection with any process of distribution between various economic subjects or groups of economic subjects. The circumstance that society is viewed as a unified organization is, however, frequently disguised in various ways, and not until the construction of the programming analyzes, foreshadowed by certain welfare analyzes and social accounting studies, has it become quite explicitly and strictly formulated.[25)]

Thus we have pointed out that an extension of the Leontiefian balance-identity system leads logically and directly into analyzes, which are adequately denominated production problems: they require the explicit statement of one set of objectives of the economy studied and they say nothing about any problem of distribution between various decision units, taking only one economic will into consideration.

2) The Walrasian systems

a) MOSAK'S SYSTEM

We begin by a terminological note. Our main interest does not lie in finding ≫what Walras really meant≪. In order to avoid history-of-doctrines disputes we are going to use expressions like ≫the Walrasian systems≪ only as a common name for two explicitly stated pieces of research, one being ≫Mosak's system≪ as presented in Mosak's *General Equilibrium Theory in International Trade* (1944),[1)] — which does not mainly deal with international trade but is a highly merited clarification of Hicks' more involved extension (1939) of certain earlier lines of thought — the other being

[25)] For a specific example of the basic indeterminacy of marginal productivity theories of distribution in the absence of a value scheme of society compare Chapter V, Section 2: a.

[1)] Mosak [25].

≫Marschak's system≫ as presented by Marschak in the articles "Money and the Theory of Assets" (1938)[2] and "The Rationale of the Demand for Money and Money Illusion" (1950),[3] which, belonging to the same category of thought, in certain respects goes further than does Mosak's analysis.

Mosak's system[4] contains two groups of relations. The first group shows a certain similarity to the Leontiefian set (1):

$$\begin{aligned} s_{11} + s_{12} + s_{13} + \ldots + s_{1m} &= d_{11} + d_{12} + d_{13} + \ldots + d_{1l} \\ s_{21} + s_{22} + s_{23} + \ldots + s_{2m} &= d_{21} + d_{22} + d_{23} + \ldots + d_{2l} \\ &\vdots \\ s_{n1} + s_{n2} + s_{n3} + \ldots + s_{nm} &= d_{n1} + d_{n2} + d_{n3} + \ldots + d_{nl} \end{aligned} \tag{21}$$

or

$$\sum_{j=1}^{m} s_{ij} = \sum_{k=1}^{l} d_{ik} \qquad (i = 1, 2, \ldots, n)$$

or

$$S_i = D_i \qquad (i = 1, 2, \ldots, n)$$

The first index denotes commodity, the second individual. Each equation states that the sum of the supply of one single commodity emanating from m suppliers equals the sum of the demand for this commodity emanating from l demanders. The second group of equations are behavior equations stating, for each individual, the dependence of his supply and demand of each commodity on the $(n-1)$ relative prices:

$$\begin{aligned} s_{11} &= s_{11}(p_1, p_2, \ldots, p_{n-1}) \\ &\vdots \\ d_{nl} &= d_{nl}(p_1, p_2, \ldots, p_{n-1}) \end{aligned} \tag{22}$$

Combining (21) and (22) we get[5]

$$S_i(p_1, p_2, \ldots, p_{n-1}) = D_i(p_1, p_2, \ldots, p_{n-1}) \quad (i = 1, 2, \ldots, n) \tag{23}$$

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[2] Marschak [26].

[3] Marschak [27].

[4] Mosak [25], pp. 1–56; 94–109 — we disregard here those parts of the book dealing with more special problems of international trade, and with ≫inter-temporal-equilibrium theory≫.

[5] We have here only $(n-1)$ independent equations, as one of the equations follows when the remaining $(n-1)$ equations are satisfied. Cf. Mosak [25], p. 36.

The behavior equations, the supply and the demand relations, one set for each decision unit, are derived under an assumption of rational behavior, i.e. they are obtained through a maximization procedure from the basic preference and technical functions of the single decision units. We now first put the question: how does this Walrasian setup attack the problem of production, the problem of utilization of resources, within the social economy studied? An answer to this problem will give us the possibility of a direct comparison with the programming analyzes.

No doubt, the following fact must be placed in the center of such an answer: the extreme compartmentalization of society assumed, or in other words the quasi-isolated treatment of the various decision units. The productive processes take part, so to speak, behind the surface of the behavior equations. Each producer has the choice between various ways of producing his commodities, as expressed by the technical equations — the production-possibility functions — attached to the decision unit in question. These relations may be of varying shape. In the ordinary approach — the approach of Mosak — a general non-linear technical equation is postulated, connecting the outputs and the inputs of the productive establishment; it would with all probability be possible to describe similar technical conditions in terms of a linear-programming system attached to the producing unit. As was the case with the extended input-output analyzes applied to a social economy, the choice between the various production possibilities, now regarded from the viewpoint of a single enterprise, requires a statement of objectives. One more thing, however, is needed: knowledge of the prices prevailing for commodities and factors involved in the process. These two requirements are inseparable from each other.

The statement of objectives is made in two steps, one basic and one auxiliary. For each final demander — this is the basic step — a preference function is postulated, expressing the objectives of this unit as regards consumption of the various commodities. Prices being considered as given, the set of demand equations of the decision unit are derived through maximization from this preference function. The demand equations for each commodity are summed over the various units, and the sums constitute part of the demand member of the relation (21). Each producer — this is the auxiliary step — is assumed to have as objectives in his production activity the maximization of monetary profits. With this assumption and considering the prices as given, the set of demand equations and supply equations for factors are derived from the technical equations of the unit. The demand equations for each commodity = factor are summed over the various units; the sums are superimposed on those emanating from the final demanders, constituting together the demand member of the relations (21); the supply equations for each factor are summed over the various units, and the sums

form the supply member of the relations (21). Through adjustment of prices, the demand member for each commodity is equalized to the supply member for, the same commodity; the equality is not assumed but instead the theory has to show how the equalization is achieved. Relations (21) are according to our definition clearly equilibrium conditions, but the nature of the equilibrating process can only become explicitly outlined if dynamic versions of the relation-type be utilized.

Thus consumer's tastes — the various sets of final objectives of society — exercise through the price mechanism their guiding influence on the choice between the various production possibilities, i.e. on the amounts of the different factors employed in each process and the amounts of goods produced. The strategic point in the construction is constituted by the equilibrium conditions. Whose behavior do they describe? In treating the equilibrium conditions we cannot continue to speak only of production: they depict in their adjustment of prices a distribution or imputation process, and according to our opinion they cannot be assumed to describe the behavior of any unit involved in economic activity, striving for profit or utility, but ought to be regarded as intended to describe an institutional arrangement of a very special character.

Why may the equilibrium conditions be said to depict a distribution or imputation process? Obviously, the behavior equations themselves do not determine the prices of the various commodities and factors; only through their summing for each commodity or factor, and the equalization of corresponding demand and supply-totals — i.e. the formation of equilibrium conditions — are the prices determined, upon which it depends, given tastes and technological possibilities, how far consumers may reach in satisfying their needs and how big profits producers may obtain.

Why may the equilibrium conditions be said to describe an institutional arrangement? Already Walras localized the relations to a market with an auctioneer — compare his ≫prix crié≪. This circumstance is especially brought out if Samuelson's dynamic version of the relation type be used:[6)]

$$\frac{dp_i}{dt} = f_i(D_i - S_i); \quad f_i > 0 \qquad (i = 1, 2, \ldots, n) \tag{24}$$

i.e., if at a given price demand exceeds supply, then the price will rise and if at a given price supply exceeds demand then the price will fall — such constructions are no doubt made with the functioning of a market in mind and are most easily associated to markets with auctioneers or more

[6)] Samuelson [6], pp. 263 and 270.

complicated arrangements serving the same purpose, e.g. corn exchanges. It is true that Walras, and particularly his follower Cassel speak in the same context about scarcity, ≫rareté≫, ≫knapphet≫; thus Cassel attaches explicitly ≫the principle of scarcity≫ to the relations and talks about the ≫task≫ of price in restoring equilibrium between demand and supply. In these and similar expressions the institutional localization of the equilibrium relations to auction-markets is glossed over, as is the case with many authors who consider the relations as self-evident.

We now make a side-remark about the dynamic character of relations (24). They describe explicitly the equalization process indicated by the static and rudimentary equilibrium conditions (21) and (23). How is this distinction between statics and dynamics related to that of the programming analyzes of our previous section? A dynamic setup can, it would appear, use the same mathematical tools (differential equations, difference equations) in both cases, but its economic meaning differs from one case to the other. In the programming analyzes statics denotes constant flows of all commodities through all the subintervals of one planning period; dynamics introduces phenomena of growth and other changes in the flows of commodities between the various subintervals of one planning period; the equality between supply and demand holds both in statics and in dynamics. In the Mosakian system statics denotes a state where the equalization is achieved; the equality between supply and demand only holds in statics, and dynamics describes the path towards the equilibrium. We shall return to these two distinctions between statics and dynamics in Chapter VII.

Why may the institutional arrangement — the market mechanism — be said to be of a very special kind? This circumstance partly lies in the auction-character of the market already referred to; above all, however it lies in the fact, that each decision unit is assumed to be absolutely on a par with the other units, without any particular possibility of its own, and without any possibility through collaboration with others, to influence the course of events and push the prices in desired directions, i.e., change the distribution in its favor. The distribution process described is *automatic,* because no particular agreements of any kind between the decision units are needed for it to function, and it is *ultra-harmonic,* because no conflicts of interest are present and no unit has more influence on the price-determination than any other. In other words: the data, the preference functions and the technical equations, or the structural coefficients of the behavior equations derived from these, being given, the ≫market mechanism≫ is assumed to effect a fixed distribution, no other factors intervening in this determination because of the quasi-isolation of the decision units. The relations between the various units is of a rudimentary limit-case-character, and the whole construction more or less explicitly and

justifiably builds on the assumption that the number of actively involved decision units tends towards infinity. The world of Walras as pictured by (21)–(24) consists of a number of essentially isolated decision units which meet on the markets, working under special conditions.

Thus it would seem to be an urgent task to develop a more general theory of distribution than that embodied in the Walrasian equilibrium conditions. In Chapter I, Section 1, we referred to recent developments in the derivation of behavior equations, i.e. at the last instance supply and demand equations. Very many modifications and refinements may be made, and have indeed been made, to ≫the theory of the firm≫ and ≫the theory of the household≫ or rather ≫the theory of the isolated economic unit≫. Firstly, however, the following general rule is valid: the further the introduction of refinements is pressed, the remoter lingers the explicit establishment of a system of simultaneous equations and with it the mere chance of developing a theory of distribution, as such a theory could not possibly be devised solely from a study of one single decision unit and its preference, technical and expectation functions without the simultaneous consideration of the other units taking part in the distributive process. Secondly, in the cases where refinements in behavior equations are really combined with explicit attempts to put up a whole system of equations comprising many decision units, the tendency seems to be to retain the simple Walrasian equilibrium conditions essentially unchanged.[7)]

b) MARSCHAK'S SYSTEM

A notable exception from both these categories seems to be constituted by Marschak's treatment of ≫market functions≫ or ≫revenue functions≫ in his above-mentioned articles.[8)] The main point of relevance to our discussion in Marschak's analysis is that the individual economic subject should not regard the prices as independent of his own actions. An assumption of the kind indicated promises to lead towards a more general and realistic theory of distribution than that presented by the quasi-isolated decision units facing the impersonal Mosakian market mechanisms.

How does Marschak introduce and develop his new element of theory within the system of simultaneous equations explicitly stated? Essentially he postulates that for each decision unit the price to be set depends explicitly on the amounts of the commodities sold and bought in the transaction concerned, i.e. market functions or revenue functions being introduced, which, in a way characteristic for each decision unit, give every price as a

7) Cf. e.g. Klein [5] .

8) Marschak [26] and Marschak [27] .

function of commodity quantities transacted — similar constructions are met in the literature on imperfect competition, but there no attention is usually paid to the possibilities of incorporation into a system of simultaneous equations.

It is to be observed firstly that these functions can hardly be classified among the equation types listed in Chapter I: behavior equations, institutional equations, etc.; secondly; they distinguish themselves from, the type, of functions, preference functions and others, from which behavior equations are often derived, by their forming for all decision units a set of functions not independent of each other and thus not attributable only to single decision units studied in isolation. We comment on this last circumstance.

The Walrasian system (21)–(24) is determined in the sense that it contains as many independent relations as variables. The addition of revenue functions would seem to amount to the establishment of, an overdetermined system with more relations than variables. In, his later paper Marschak says on, this topic: *The [revenue functions] do not form an independent set.* Their forms and parameters are restricted by the condition that the market be cleared. This is simply another way of saying that, in the market, one man's strength is another man's weakness. For [the number of participants] < 5, the Theory of Games has begun to discuss these restrictions. We need not try here to develop this discussion.[9)] In other words: the revenue functions are valid only under the modification that equilibrium conditions stating the equality of total demand and total supply be valid at the same time, this requirement leading to restrictions on the structure of the price-determining market functions, i.e. on the shape of the distribution in the society studied; these restrictions are said to be further analyzed in the theory of games for small numbers of decision units.

The most interesting thing in Marschak's extended Walrasian system is precisely the observation that the various market functions cannot be assumed to be independent of each other but must be treated as a whole, attention being paid simultaneously to all economic units. This conforms to a common-sense-observation that any not completely spurious theory of distribution must contain elements that are not only attributable to single decision units like preference functions and production functions — the equilibrium conditions of the Mosakian system also obey this condition — but unfortunately no hints are put forward as to how the theory of distribution indicated may be developed, except the reference given.

[9)] Marschak [27], p. 93.

Notwithstanding this difficulty, hitherto insufficiently explored, the distribution process of Marschak's system is still *automatic,* because no particular agreements between the decision units are tied to it, the only decisions in the system being those of individual-utility-maximizing (or profit-maximizing) choice between sets of ≫assets≫; it is not ultra-harmonic, because all decision units do not have the same influence on the prices, but it may still be adequately called *harmonic,* because the data, including the revenue functions, being fixed and the various choices made, everything is assumed to be settled — no set of alternative schemes of distribution results, each alternative favoring particular group-interests. If there be some conflict of interest and ensuing potentialities for a struggle of distribution, this must lie completely outside the system; inside the system the given data are postulated to fix the scheme of distribution as a result of the choice-activities of the decision units. This feature will be brought out clearer in the comparisons which follow in the next section.

c) THE WALRASIAN SYNTHESIS BETWEEN A PRODUCTION ASPECT AND A DISTRIBUTION ASPECT

The programming analyzes only dealt with the production aspect *Man vs. Nature.* The goals, the objectives of society — formalized in Dantzig's Objective Function — were stated on one side, and the technical restrictions conditioning the fulfillment of these objectives were introduced on the other side — formalized in Dantzig's flow functions. The Walrasian systems dealt at the same time both with a production aspect *Man vs. Nature* and with a distribution aspect *Man vs. Man.* We have emphasized at length particularly the Mosakian systems rudimentary treatment of the last-mentioned problem. At a first glance it seems, however, as though the production aspect were treated in a much more general manner in the Walrasian systems than in the programming analyzes, because the former do not assume the existence of only *one* set of objectives, *one* Objective Function, *one* preference scale in the social organization studied but a set of preference scales, *tastes,* one for each decision unit. How is then a conflict between these varying value schemes avoided? We now re-stress that the prices in the Walrasian systems are necessary links in their solution of the production problem, it being through them that the tastes of the final consumers in an automatic way partly govern the production, i.e., the choice between the feasible technical possibilities. The prices are thus by no means of a shadow character, but they are formed in the extremely special automatic and (ultra)harmonic distribution process. Hence the solution of the production problem and the solution of the distribution problem are interdependent and the generality of the whole construction cannot exceed that of the distribution process.

What happens to the production aspect when the distribution aspect is generalized? — we have stressed the obvious need for such a generalization and remarked that the Marschakian system takes a step in this direction, although not a very elaborate step. Also in the Marschakian system the apparent harmony between the various value schemes is maintained — each decision unit has a preference scale of its own but no conflicts between the various sets of objectives appear in the system. Otherwise we should expect a priori an analysis that really tried overcoming the Walrasian quasi-isolation of the decision units to encounter difficulties in this respect, when for instance problems of collaboration in the distribution struggle between individuals with different preference scales presented themselves. The main line of the analysis now leads us directly to ≫The Theory of Games and Economic Behavior≪.

Thus the Walrasian systems treat both a problem of production and a problem of distribution, i.e., they give a particular synthesis between a production aspect and a distribution aspect. The whole construction builds on an extreme compartmentalization of the society imagined that is most easily disguised through a direct discussion of the very special distribution mechanism included, on the functioning of which also the solution of the production problem depends; but it is inherent too in the fact that a multitude of different value schemes guide in harmony with each other the production through the medium of prices and the auxiliary profit-maximization objectives, given certain technical restrictions.

3) The game distribution theory

a) THE ASSUMPTION OF TRANSFERABLE UTILITY

The theory of games[1)2)] makes the assumption about the free transferability of utility between the various decision units in a social eco-

[1)] von Neumann and Morgenstern [1].

[2)] Obviously the exposition that follows is not intended to be completely self-contained. It is aimed firstly to give an outline of some basic concepts of the theory of games which will be to some degree utilized in Chapters V–VII, secondly to give a certain comparison with the economic theories already discussed. Among the many reviews of ≫The Theory of Games and Economic Behavior≪ we refer particularly to the article by Abraham Wald in the *Review of Economic Statistics* 1947, pp. 47–52, containing a good description of the theory, and to the article by Georges Th. Guilbaud, "La théorie des jeux," *Économie Appliquée* 1949, pp. 275–319, containing some interesting comments on the theory. For a clear and compact treatment of some of the basic concepts to be introduced below see also J.

nomy:[3] ≫We shall ... assume that the aim of all participants in the economic system, consumers as well as entrepreneurs, is money, or equivalently a single monetary commodity. This is supposed to be unrestrictedly divisible and substitutable, freely transferable and identical even in the quantitative sense, with whatever 'satisfaction' or 'utility' is desired by each participant≫.[4] The authors acknowledge the assumption to be a serious limitation of the analysis and some commentators even declare the bulk of the book to be without significance because of the crucial transferability postulated.[5])

The analyzes discussed above also made assumptions about utility, assumptions that were embodied in the preference functions or the Objective Functions. The important thing with these constructions is that they are uniquely attached to a single decision unit. They build on the hypothesis that the economic subject concerned is able to make a ≫complete ordering≫[6] of the alternatives — e.g. bundles of commodities — among which he may choose; that is he can for any two alternatives decide which one he prefers or state that he is indifferent between them; further, if he at the same time prefers alternative **A** to alternative **B** and **B** to alternative **C,** this implies that he prefers **A** to **C** (transitivity) — synonymous expressions are that the subject has an ≫ordinal≫ utility or that utility ≫is

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C. C. McKinsey, "Isomorphism of Games, and Strategic Equivalence" in *Contributions to the Theory of Games,* edited by H. W. Kuhn and A. W. Tucker, Princeton, New Jersey, 1950, pp. 117–130. (In this book is also found a bibliography of game literature, pp. 193–201). On the relations between economics and the theory of games compare Oskar Morgenstern, "Oligopoly, Monopolistic Competition, and the Theory of Games," *The American Economic Review* 1948, Papers and Proceedings, pp. 10–18, and Oskar Morgenstern, "Economics and the Theory of Games," *Kyklos* 1949, pp. 294–308.

[3] In speaking about ≫the theory of games≫, ≫games≫, etc. we refer throughout our volume, to Zero or Constant-sum n-person Games with $n > 2$, unless otherwise explicitly stated. The remaining categories of games, the Zero-sum two-person Games and the General, Non-zero-sum Games will only be considered at special places below, p. 88 and pp. 86, 88–91 resp. Compare, however, footnote 27 on p. 89.

[4] von Neumann and Morgenstern [1], p. 8.

[5]) Cf. e.g. Giovanni Demaria, "Su una nuova logica economica," *Giornale Degli Economisti e Annali di Economia,* 1947, pp. 661–671.

[6] ≫Complete ordering≫ according to the terminology of von Neumann and Morgenstern [1]; ≫weak ordering≫ or ≫ordering≫ according to Arrow [2].

measurable up to any monotone transformation≫. Sometimes a stronger hypothesis is utilized, making utility ≫cardinal≫ or ≫measurable up to any linear transformation≫, that is a number determined up to addition of any constant and multiplication by a (positive) constant. Thus even in the last-mentioned and strongest case there is nothing to fix a unit of utility (and a zero of utility). That this precludes inter-individual comparisons and transfers of utility is obvious and has long been known — compare the following drastic statement by Wicksell: ≫As far as I know no marginal utility theoretician is in doubt that *real* — not only imagined — measurements of utility [with inter-individual comparisons] by necessity require a unit of measurement established in advance; it is further obvious that this unit of measurement can only consist in a utility, moreover in a concrete utility, emerging under quite determined conditions — let us e.g. say the utility that a middle-aged farm-hand in Brandenburg derives from his high boots≫.[7)]

This problem of transferability of utility does not arise in the programming researches because only one economic subject is taken into account by them[8)] ; the Mosakian and the Marschakian systems no doubt comprise many decision units, but we have already stressed that no conflicts occur between the various individual value schemes included, a circumstance which we put in connection with the quasi-isolation of the decision-units underlying the Walrasian theories of distribution — and hence also the theories of production. The theory of games removes this quasi-isolation — we shall have repeated opportunities to return to this fact below.

Let us now mention another assumption made in the theory of games, the assumption of the constant-sum character (including the zero-sum case) of the games. This postulate is not maintained throughout the book — we are going to return to the topic in our next chapter — but for the time being we disregard the other possibility analyzed. The assumption implies that the participants in a social economy have a given sum to divide between themselves, be this sum of their shares zero or not; nothing can be made to increase the sum as such; the only thing that matters are the relative shares of the units, not the absolute amounts. If we compare with the traditional constructions we also find in this respect outstanding differences. The programming systems obviously imply that the ≫utility ≫of the single unit considered is not fixed and given but something to be maximized under the technical restrictions. The Mosakian and Marschakian systems can of course say nothing about any sum of utilities and its eventual

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[7)] Knut Wicksell , "Zur Verteidigung der Grenznutzenlehre" *Zeitschrift für die gesamte Staatswissenschaften* 1900, p. 579.

[8)] This statement will be further substantiated on pp. 80–84 below.

maximization but they no doubt assume that the utility of every participant increases as a result of the production and the ensuing exchange. However, if we look at the theories solely from the distribution aspect, the only thing that matters is the relative prices, not the absolute amounts.

Thus we have, on one side, individual utilities to be maximized and a single decision unit or many quasi-isolated decision units, and, on the other, transferable utilities adding up over society to a constant sum and many decision units in contact, cooperation and conflict with each other — which setup to choose? The quasi-isolation is an evil thing — almost all criticisms of Walrasian economics from Veblen to von Neumann and Morgenstern seem to share this view; the transferability of utility combined or not with the constant-sum assumption is another evil thing — almost all commentators on utility theory are of this opinion. We may characterize the situation (as it appears at first) as a paradox. This paradox lies more precisely in the fact that a construction which stands out directly as a generalization of the Walrasian theories of distribution seems to lead to an extreme specialization of the Walrasian theories of production, i.e., the generalizations of the part of the Walrasian systems which contains explicitly the atomistic assumptions on additivity, etc., — the equilibrium conditions and possibly the revenue functions — imply the acceptance of a device which completely blots out most traits of the Walrasian theory of production with its individual taste — expressions for non-transferable, non-constant-sum utilities — which through the price-medium influence the production.

We are going to try to handle this paradox in three steps. *First,* we point out that the ≫Theory of Games≪ — in the chapters to which we refer[9] — may be interpreted only to deal with a distribution problem. The only thing that matters in the analysis is the relative shares of the participants and we may forget about utility and interpret the whole setup in terms of money, which is freely transferable, substitutable and comparable between individuals, i.e. possesses the measurability-characteristics that even cardinal individual utility is lacking — of course the authors of the ≫Theory of Games≪ mention the possibility of the money interpretation too. *Second,* we repeat that the programming analyzes only pay attention to a production problem, the solution of which does not depend upon any rudimentary distribution considerations as in the Walrasian cases. We now contrast with each other, on one side, the theory of games distribution analyzes, on the other, the programming production analyzes and (re)state our interpretation of their basic assumptions. The first-mentioned theories do not assume any goal of society at all but it postulates that each decision

[9] Cf. footnote 3 on p. 45 above.

unit strives to secure for itself as big a money-income as possible, i.e. to make its position as good as possible relative to the positions of the other units; the last-mentioned theories assume the existence of one common goal of society that society strives to fulfill. The first case excludes that *all* economic subjects in society have something in common because of the fact that only the relative shares matter — it assumes an absolute diversity of interests, but it takes most explicitly into account the fact that various decision units may collaborate to better their relative positions; the second case amounts to all economic subjects in society having strictly parallel interests — it assumes absolute unity of interests.

In the first case we have *Man vs. Man,* in the second case *Man vs. Nature;* in the first case we have the pure distribution aspect, in the second case the pure production aspect. If for a moment we recollect some most elementary empirical experiences we find that under certain circumstances the production aspect dominates the life of a social economy: during wars and periods of preparation for wars, e.g. the conflicts of interest between the citizens that at other times result in a struggle of distribution are traditionally pushed aside and a united attempt is made to reach a common goal, and in the same direction lie the conditions in a communistic Utopia, in a centrally planned economy; under other circumstances the production aspect is remarkably absent: during economic crises, in the depth of many a depression, during certain types of inflation very little of a common goal of society is discernible but a conflict of interest is quite evident to the observer, although the traditional economics and the political speeches sometimes try to obliterate it. In our *third* step we ask: is a synthesis between the two aspects possible? We observe again that the Walrasian systems try to give such a synthesis, of course not between the pure production aspect as presented by the programming analyzes and the pure distribution aspect as presented by the theory of games but between the two aspects modified and specialized in respects with which we have already dealt.

b) THE MAIN CONCEPTS OF THE THEORY OF GAMES

α) *Inessential and Essential Games*

We are not going to treat the question about the eventual synthesis between the production aspect and the distribution aspect any further here but we shall return to it in our next chapter. Before doing this we must investigate more closely the main lines of the game distribution theory, about which we have only made introductory remarks. The three principal

concepts of the theory are the Characteristic Function,[10] the Imputation, and the Solution. It should be stressed that all interpretations and all comparisons with other theories are completely our own, unless otherwise explicitly stated.

We characterized above the traditional theories of distribution, the Mosakian theory and the Marschakian theory, as follows. They depict a distribution process which is *automatic* because no agreement of any kind whatsoever between the decision units is necessary for the scheme of distribution to be put into effect, and which is *harmonic* because all real conflicts of interest are assumed away — the decision units *may* be of unequal importance and have varying influence on the prices (Marschak and some theories of imperfect competition), but their influence is completely fixed and given in advance. No measures must be taken by the privileged units to preserve their position, no measures can be taken by the weak units to change the state of society — except through actions which fall completely outside the social theory in question and which possibly could cause unsystematic changes in its structure. The two characteristics are formally expressed through the fact that as soon as, firstly, the data are given, that is the forms and coefficients of the preference functions and the production functions, and the equilibrium conditions in Mosak's system and the forms and coefficients of the preference functions, the transformation functions and the market functions, and the equilibrium conditions in Marschak's system, and secondly, the decision units have decided to fulfill their objectives, be it the primary objectives of utility maximization or the auxiliary objectives of profit maximization, then among other things the prices and hence the scheme of distribution would be fixed.

One special case in the theory of games resembles closely these traditional distribution theories: the Inessential Games. Let there be n decision units or participants in a social organization, denote the set of all units by I, and any subset of units by S.[11] Let further $v((i))(i = 1, 2, \ldots, n)$ indicate

[10] In our thesis the word ≫game≪ is used synonymously with the word ≫Characteristic Function≪.

[11] A few set-theoretical notions will be utilized below. A *set* is an arbitrary collection of objects, the *elements* of the set in question. Let i and j be elements and S, T, and I be sets. The expression $i, j \in S$ means that the elements i and j are contained within, belong to, the set S; similarly $S \subset I$ implies that the set S is contained within the set I; S is a subset of I and I is a superset of S. The symbol Θ denotes a set which contains no elements at all, the *empty* set. The *sum* of two sets S, T is the set of all elements of S together with all elements of T, to be denoted by $S \cup T$. The *product* or *intersection* of two sets S, T is the set of all common elements of S and of

what decision unit no. i can secure for itself in the distribution process acting alone, that is without any kind of agreement or cooperation with the other economic subjects,[12)] and let $v(S)$ indicate what the group of decision units S can obtain in the distribution-process through agreement with each other,[12)] the total sum to be distributed being equal to $v(I) = s =$ a constant.

Already here we see why the construction meets difficulties as regards ≫transferability of utility≪. If only the $v((i))$ were to be defined it would appear likely that no problems of the transferability kind would arise, and that it would be sufficient for the expressions to be numbers lacking the transferability characteristic, lacking a common unit — they might, e.g. be numbers determined up to any linear transformation. If the Mosakian and the Marschakian systems admit a unique determination of the unknowns — the prices and the quantities — then for a given setup of the data it would be evident to what individual satisfaction level the mechanism could push the various decision units, and the whole thing could at the game level of analysis be expressed by n ordinal or cardinal numbers $v((k))$. If the data changed, the numbers $v((k))$ would take different values.

If, however, e.g. $v((1,2,3))$ — what decision units 1,2 and 3 can obtain by collaboration — is to be formed as a number, this seems to necessitate the existence of a common unit for the three economic subjects to count in, a common unit as possessed by everyday money or Utopian transferable utility. Otherwise the expressions $v(S)$ would be no numbers but some sort of many-dimensional vectors. In the Inessential case the only numbers that finally matter are the $v((i))$, but it is assumed that all the $v(S)$ may be formed, and thus everything should be counted in money. The inessentiality is expressed through the equality

$$(C') \qquad v(S) + v(T) = v(S \cup T), \quad \text{if } S \cap T = \Theta \text{ and for any } S, T \in I$$

and hence, e.g.

$$v((1)) + v((2,3)) = v((1,2,3)).$$

The additive formula (C') expresses the fact that nothing can be gained in the distribution process through agreements between the participants;

T, to be denoted by $S \cap T$. If two sets S, T have no elements in common, then their product is empty: $S \cap T = \Theta$. The *difference* of two sets I, S is the set of all those elements of I which do not belong to S, to be denoted by $I - S$. $I - S$ and S are then *complements* in I. Compare further von Neumann and Morgenstern [1], pp. 60–67.

[12)] Cf. p. 88 below.

what goes to a certain set of units, say $S \cup T$, is always equal to the sum of what goes to two subsets acting independently, say T and S, which are non-overlapping, that is have no members in common — $S \cap T = \Theta$ — and which completely exhaust the bigger set. The formula gives as a corollary

$$\sum v((i)) = v(I) = s, \tag{25}$$

that is the sum of the amounts accruing to the individual participants acting in isolation from each other is exactly equal to the total amount which is to be distributed. Hence there is no surplus

$$\Big[v(I) - \sum v((i))\Big]$$

the distribution of which must be settled through agreements. This brings out clearly the *automatic* character of the distribution process pictured. In order further to illustrate the *harmonic* character of the distribution process we introduce the concept of Imputation, that is a scheme of distribution to be denoted by $[a_1, a_2, \ldots, a_n]$ or shorter $\vec{a}$, the components of the vector being the shares of the various participants — this step is in fact superfluous in the rudimentary special case discussed but will serve as a link to the more complicated cases. For the Inessential Games we simply get

$$a_1 = v((1)); \quad a_2 = V((2)); \ldots; \quad a_n = V((n)), \tag{26}$$

that is, a unique Imputation corresponds to a given set of the $v((i))$; if these were changed, the Imputation would change too. Thus there is only one possible Imputation to be established on the background of an Inessential set of numbers $v((i))$; no conflicts of interest are effectively present and the distribution process is harmonic.

Formula (C') is only a special case of the general expression

$$v(S) + v(T) \leq v(S \cup T), \text{ if } S \cap T = \Theta \text{ and for any } S, T \in I \tag{C}$$

— this is the first summation theorem, referred to in the Introduction. In the case of Essential Games the equality sign does not hold for any S, T, thus

$$v(S) + v(T) < v(S \cup T), \text{ if } S \cap T = \Theta \text{ and for some } S, T \in I \tag{C''}$$

The inequality (C'') expresses the fact that ≫the whole is greater than the sum of its parts≫, a sentence often used in less precise contexts. At the

point where it is introduced the game distribution theory departs from traditional economics in a constructive direction. The progress made is due to this non-additivity characteristic and so are also the difficulties that the development of the construction meets. It might be said to be in accord with empirical experiences of the kind that a trade-union may secure higher wages to the member-laborers than these can obtain acting independently, but more generally it implies that the scheme of distribution in a society is influenced by the activities of many decision units, the distribution process being non-automatic, as explicit agreements are required to fix a particular distribution, and non-harmonious, as conflicts of interest are a main feature of most societies, which admit several possible alternative distributions, favoring in a varying degree different economic units. This stands out clearer if a corollary from (C'') be derived:

$$\sum v((i)) < v(I) = s \tag{27}$$

stating that for Essential Games the total amount to be distributed is greater than the sum of the amounts going to each participant if acting in isolation from the rest of the community. Hence a surplus $[v(I) - \sum v((i))]$ arises, the distribution of which must be settled through agreements. The formula also implies that several alternative distributions are possible, each corresponding to a particular power constellation in society; this will be further substantiated below. Obviously much cannot be judged from the mere inspection of the formula; it is clearly a straight-forward generalization of the additive setup, a construction which resembles closely the traditional analysis with the evident special-case status; its further implications have to be explored.

β) The Imputation and the Essential Characteristic Function

Before proceeding to a closer scrutiny of formula (C'') we introduce the formal definition of an Imputation — this was provisionally made in our discussion of the Inessential case above. An Imputation, expressing a possible scheme of distribution among the members of a society, is a set of n numbers $[a_1, a_2, \ldots, a_n] = \vec{a}$ with the following two properties[13)]

$$\begin{aligned} &(a) \quad a_i \geq v(i) \qquad \text{for } i = 1, 2, \ldots, n \\ &(b) \quad \sum_{i=1}^{n} a_i = s \,[= v(I)]. \end{aligned} \tag{28}$$

[13)] von Neumann and Morgenstern [1], pp. 263 and 350.

The first property states that the share which goes to a participant must not be less than what he can obtain alone[14] — in the Inessential case the equality sign is valid for *all* participants. The second property states the condition that the sum of all shares add up to a given amount.

One such Imputation is established at a point of time in a society — how is it determined and how does the non-automatic and non-harmonic character of its determination appear? The principal concepts to be discussed are firstly that of Characteristic Function, secondly that of Solution.

Von Neumann and Morgenstern ≫translate≫ the notion of Characteristic Function by ≫rules of the game≫ or ≫physical background≫, i.e. ≫the physical laws which give the factual background of the economic activities under consideration≫[15),16)] No doubt the idea that the economic activities of the various decision units in a society take place against a certain background, against certain data, is a necessary one, as is also the idea that this background, these data, may change. Probably such ideas are to be found in every piece of economic analysis, although they are expressed in widely differing ways and with varying emphasis on the two constituent points. In order to stress in what direction the real difficulties seem to lie, we mention again the set of coefficients or flow functions which constitute the ≫physical background≫ of the programming systems and which express clearly technological facts; to these must be added coefficients or functions which describe the set of objectives of the economic organization concerned, i.e. its preferences. Certainly these data may be of a very complicated nature from the computational point of view — this is granted by the difficulties that already meet the computation of discrete linear programs — but the thing that is by necessity avoided in these setups are data describing the chances and possibilities that the various decision units of a society have in the process of distribution. The theory of games indicates with emphasis that the expression of data of this kind meets difficulties of quite another quality than the above-mentioned computational ones; we might say conceptual difficulties. The Mosakian and Marschakian systems comprise many economic subjects. As we know the former analysis is chiefly concerned with a production aspect, having the same types of data as the programming systems and treating the distribution aspect in the

[14] Thus a certain minimum is fixed, below which an income-share cannot be pressed. The existence of such a minimum is not alien to economics — compare for instance Ricardo's ≫iron law of wages≫ and Cassel's discussion of the connection between the average life-length of man and the minimum interest rate.

[15] von Neumann and Morgenstern [1], p. 32.

[16] Cf. below, particularly Chapter VII, Section 4.

most rudimentary way; the latter analysis takes a step towards indication of the conceptual difficulties referred to — it remarks that the market functions tied to the various economic subjects cannot be considered as an *independent* set because they are restricted by the equilibrium conditions and this is said to express the fact that in the market one man's strength is another man's weakness.[17] The implication of the system, however, which is assumed to be valid, namely the unique determination of the unknowns, among them the prices, places its distribution theory very close to the Inessential Games, which depict a society where the chances and possibilities of the decision units in the process of distribution are unique and fixed; the expression of data does not encounter any particular conceptual difficulties in the Inessential case. The conceptual difficulties in this respect of the Essential Games are due entirely to the fundamental non-additivity-characteristic, which in this context may be said to imply that the economic activity of one decision unit influences the result of the activities of other units.

We now give the formal definition of a Characteristic Function. A Characteristic Function is a numerical set-function, v, defined for all subsets S of I, I having the elements $(1, 2, \ldots, n)$, and $v(I)$ being equal to a constant. It satisfies the following conditions:[17,18]

$$
\begin{aligned}
&(A) \quad v(\Theta) = 0 \\
&(B) \quad v(S) + v(I - S) = v(I) \\
&(C) \quad v(S) + v(T) \leq v(S \cup T) \quad \text{if } S \cap T = \Theta.
\end{aligned}
\tag{29}
$$

A $v(S)$ denotes the amount which the set of decision units S can expect to be able to achieve in the distribution process, if they form an agreement and collaborate in an intelligent manner. The first condition trivially states that an empty set of players gets the money amount 0; the second condition expresses also somewhat trivially, that the amount which S may secure and the amount accruing to the complement of S, add up to the total amount available. The third and fundamental condition and its corollary have already been mentioned above.

We concentrate on the Essential case where the equality sign in (C) does not always hold. Thus for Essential Games all the $v(S)$ are of final importance, not only the $v((i))$; already this indicates the *non-automatic* character of the distribution process, as $v(S)$ is the money result of a possible agreement. A set of $v(S)$ forming a Characteristic Function gives a catalog of the agreements that may be formed between all conceivable sets

[17] Cf. footnote 11 on p. 49 above.

[18] von Neumann and Morgenstern [1], pp. 241 and 349.

of decision units involved, each agreement being characterized solely by the maximum money amount which it can give to the collaborating members. Observe that we speak only of the objective possibilities that the units are facing, not of the state of things that is really put into effect, i.e. of the agreements that are actually carried out. It is at first sight obvious that many different catalogs of possibilities, i.e. many different Characteristic Functions, may exist, each Characteristic Function being a set of numbers $v(S)$, all S in I taken into account. The problem then appears, how to classify the different cataloges, how to enumerate all different Characteristic Functions?

The first step on this way is to form classes of Characteristic Functions, the differences among the members within each class being insignificant with regard to the further concepts of theory to be established. This is achieved through the procedure of Reduction, whereby classes of Strategically Equivalent Games are formed, each class being denoted a Reduced Game. The Reduction procedure applied consists in putting all the $v((i))$ of a Characteristic Function equal to each other (and to a common value)

$$(29:D) \qquad v(1)) = v((2)) = \ldots = v(n)) \ [= -\gamma].$$

Thus all Characteristic Functions differing *only* with respect to the values $v((i))$ form one class of Strategically Equivalent Games and constitute one particular Reduced Game.

The procedure may be justified as follows. Consider two Games or two Characteristic Functions which only differ in the respect that a given subject no. i in one game *always* gets a fixed amount a_i^0 more than the corresponding subject no. i in the other game; thus

$$v'(S) = v(S) + \sum_{i \in S} a_i^0$$

where $v'(S)$ is the Characteristic Function of the first game and $v(S)$ the Characteristic Function of the second game. It is fairly clear conceptually that these two Characteristic Functions do not differ from each other in any fundamental aspect — the position of every unit in each possible agreement is only shifted by a fixed amount — and it is easy to show that the addition of the fixed amounts a_i^0 are of no importance for the further concepts of the theory, i.e. ≫the strategic possibilities, the inducements and possibilities to form coalitions, etc., are entirely unaffected≪.[19] We may remark that in the Inessential case only differences of the type $v'((i)) = v((i)) + a_i^0$ could occur, corresponding closely to the structural changes of traditional

[19] von Neumann and Morgenstern [1], p. 246.

economics. We now have at our disposal n coefficients a_i^0. If we impose the restriction $\sum_{i=1}^{n} a_i^0 = 0$, in order to get the same total amount of shares in both games, $v'(I) = v(I) = s$, then $(n-1)$ additional restrictions may be chosen without changing the setup in any fundamental respect. The $(n-1)$ relations $(29:D)$ are the additional restrictions chosen. Obviously after this reduction rule has been applied only one Inessential Characteristic Function exists.

A great variety of Reduced Essential Characteristic Functions however remain, the number of different cases increasing steeply with the number of decision units, n, in the society imagined, and the second step on the way of enumerating all feasible Characteristic Functions gives rise to considerable trouble. Only $n = 3$ is an exception with a unique Reduced Characteristic Function; for $n = 4$ already a family of Reduced Characteristic Functions with 3 indefinite parameters exists, that is a cube may be chosen, each point in which represents a Reduced Characteristic Function. This cube has only been partly explored up to the present time. The complexities grow rapidly with the number of participants; the ten-person Games filling a 501-dimensional continuum and the set of n-person Games having $2^{n-1} - n - 1$ coordinates. The tools for handling such setups, are still very incompletely elaborated.

A remarkable exception from the tendency of enormous increase in complication, when the number of players increases, is found in the so-called Symmetric Games, the n-person Symmetric (Essential) Game being characterized by $(\frac{n}{2} - 2)$ parameters when n is even, and by $(\frac{n+1}{2} - 2)$ parameters when n is odd.

It may be remarked, however, partly as a third step on the way to classifying the different Characteristic Functions, that the number of active economic decision units in a society need not be large. Often the decisive economic questions seem to be settled through negotiations between relatively few, say, 4 to 10, fairly unified and coherent economico-political organizations or between big social groups, whose internal structure is so fixed that they may be considered to form only one decision unit; that is their splitting up in various subgroups is no actual potentiality. We want to go somewhat further into this important question — in fact this seems to be the proper place to discuss the setting of the so-called ≫aggregation problem≪.

The aggregation problem is described by Marschak as follows: ≫The equations of [an economic] model must refer to individual agents in specified markets (as consumers or manufacturers of certain goods or as workers, bankers, landlords; etc.). However, to reduce the model to a manageable size, variables referring to single individuals in finely subdivided markets

must be grouped into aggregates. Suppose the value of some variable relevant to a practical decision is calculated on the basis of such an aggregative model. This value will contain an error inasmuch as it will deviate from the corresponding value calculated on the basis of a true, detailed model, with separate equations for each commodity and individual. Optimum aggregation should combine highest manageability (e.g. shortest computations) with smallest error. This aggregation problem (which includes that of index numbers) has not been solved, not even formulated in detail, nor will it be studied in this volume.≫[20] Researches along the line indicated have been carried out by Dresch, May, Arrow, and others. The important thing to realize is that this aggregation problem ≫to shrinken an articulate, comprehensive and from the computation point of view unmanageable equation system into an inarticulate, small and manageable system≫ — is described solely from the point of view of a Walrasian construction. In the still rather imaginary process of shrinkage both the number of explicitly introduced variables, i.e. commodities and prices, and the number of explicitly introduced decision units are reduced — and aggregates result such as ≫total consumption of the farm population≫ —, but the distinction between reduction of the number of variables and reduction of the number of decision units is in this context not important, nor even clear-cut. Referring to the above-mentioned threefold character of the econometric equation systems it may be remarked that the aggregation problem is closely tied to the rôle of the systems as an instrument of decision for society; in this connection it may be indicated thus: to give a simplified, aggregative description of reality, i.e. of the (rational) behavior of the many decision units of society, in order to be able to arrive at practical policy measures.

A related aggregation problem is being considered within the Leontiefian system, e.g. by Morgenstern. In this special case of the programming analyzes there is of course no question of reducing the number of decision units; the problem posed is instead to investigate the consequences of grouping all activities of a social economy into let us say 20 or 30 or 150 categories as is done in an input-output matrix and the treating of each category as a homogeneous technical unit letting, e.g. its aggregative input of a particular commodity stand in a certain relation to its aggregative output. The research on this topic is still in its infancy. The more it is developed towards the programming analyzes proper, probably the clearer will be its aim, namely, to give a simplified aggregative, description of the technological possibilities of (a centrally planned) society in order to be able to arrive at practical policy measures. Heuristic hints as to the setting

[20] Ed. Koopmans [7], p. 7. Reproduced by permission of Cowles Commission for Research in Economics.

and solution of such an aggregation problem could perhaps be found in studying the procedures actually employed by some planned economy.

Thus, there appear two related but yet distinct types of aggregation problems, one within the Walrasian constructions and one within the programming analyzes, the first-mentioned being concerned both with the reduction of the number of variables and the number of decision units with the avowed principal aim of being capable of supplying policy measures for society — however remote this possibility may seem for a Walrasian construction from other considerations —, the second-mentioned dealing understandably only with the reduction of the number of variables with the aim of being capable of supplying policy measures for a planned society.

We are now going to contrast these investigations with the conditions of the theory of games, notably with the very considerable increase of complexity, which occurs as the number of participants — the number of decision units — grows, and we shall ask the question whether in this context some aggregation problem can be posed within the theory.

Because of the assumption of transferable utility or, as we put it, the monetary assumption, no particular problem of reducing the number of variables appears, as the only relevant variables are the money shares of the various decision units out of the given total amount; the decisive problem is whether a reduction of the number of decision units of a social economy could be achieved in some way. This problem is touched upon in connection with the Symmetric five-person Game in the theory of games. A comparison is made between a set of Symmetric five-person Games (Γ) and the corresponding four-person games (Γ') which are obtained through merging players nos. 4 and 5 into one indissoluble unit, a ≫composite player≫. There turns out in some cases ≫the interesting situation that even when 4 and 5 happen to cooperate in Γ, their joint position is dislocated by the possibility of their separation≫,[21] that is ≫already our first discussion of the three-person game disclosed that the division of proceeds within a coalition is determined by the possibilities of each partner in case of separation. But this situation, which we now visualize, is different. In our present Γ it can happen that even the total share of player 4 plus player 5 is influenced by this 'virtual' fact. A qualitative idea of such a possibility is best obtained by considering this: When a preliminary coalition is bargaining with prospective further allies, their bargaining position will be different if their coalition is known to be indissoluble (in Γ') than when the opposite is known to be a possibility (in Γ)≫.[22] Thus the merger of players

[21] von Neumann and Morgenstern [1], p. 338.

[22] von Neumann and Morgenstern [1], p. 338, footnote 2. The quotations are reproduced by permission of Princeton University Press.

4 and 5 into one unit takes the game from the more complex five-person category to the less complex four-person category, but unfortunately the resulting four-person game with a composite player does not give us only insufficient information about the original five-person game — which should be the case if the possible amounts accruing to a coalition of 4 and 5 in the five-person games always coincided with the possible amounts going to the composite player in the four-person game, this game of course supplying no information about the division of shares within the composite unit; the resulting four-person game gives us in general even misleading information about the original five-person game as there is in general no coincidence of the kind indicated. This circumstance seems to preclude all attempts of aggregation.

We believe, however, that the empirical reality suggests an important aggregation problem precisely in this context. The theory of games supposes ≫complete information≫ on the part of all decision units about the physical background upon which they operate, that is about the ≫rules of the game≫, formalized in the Characteristic Functions. Disregarding possible other sources of complication as the number of participants in a game increases, it seems reasonable to suppose that the assumption about complete information grows stronger approximately in pace with the increase of the number of parameters, characterizing an Essential Game, that is, except for Symmetric Games, extremely more rapidly than the number of participants augments. To put it in another way: the only decisive change that could occur to the objective possibilities of a Reduced three-person Game would be the shifting of it between the Essential category with its unique three-person member and the Inessential category with its obviously unique three-person member; in addition to this type of change many more decisive variations can occur to a four-person game, although every Essential four-person Game can be characterized by the relatively small number of three parameters, the same number of parameters being sufficient to characterize a Symmetric ten-person Game; but an Essential n-person game demands as we know $2^{n-l} - n - 1$ parameters. It is now suggested that the empirical way out of these intricacies is to form composite players, that is indissoluble mergers of many decision units, thereby decreasing quite considerably the number of active units operating in the various socio-economic fields. This does not mean that the distribution process within the composite units constitutes no problem. It is quite conceivable that the members of a composite unit, although unitedly meeting outside units, between themselves may ≫play a separate game≫ about the division

of the total amount accruing to the composite unit in the game on the higher level.[23)]

We are going to exemplify these considerations. The traditional distribution theory more or less developed as the ≫marginal productivity theory of distribution≫ uses the famous four-fold division: ≫laborers, entrepreneurs, savers and landlords≫, with their respective shares: ≫wages, profits, interest and rent≫. We think that there is a great descriptive realism in stressing these groups as the four participants in the distribution process on the highest level within a country. The categories mentioned each comprise very many decision units. During the last two decades the course of events has shown in a pronounced way that the individual decision units within a national economy often combine into strong organizations such as one or two labor unions, one national association of manufacturers, one farmers' association. Are these organizations to be viewed as more or less casual coalitions of independently acting decision units or as indissoluble mergers of decision units, that is ≫composite players≫? Taking into account the extreme increase in complication as the number of participants of a game grows, this question, posed to the empirical reality, seems to be a very important one, deserving careful study.

If we try to give a tentative answer to it, utilizing only the mot trivial observations, the result seems at first to be somewhat ambiguous. Experience from, e.g. Sweden, suggests that the decisive socio-economic distribution problems are settled in the last instance through negotiations under the auspices of the government between the national labor union, the national association of manufacturers, the farmers' association and sometimes one or two other interest groups; the basic issues are of course not settled through negotiations between all economic subjects in the country, having the potentialities to form groups at will; the possibility that, e.g. part of the farmers join part of the manufacturers, negotiating as one group with a union between the laborers and the rest of the farmers, as another group and the rest of the manufacturers as a third group seems further very remote and of a distinctly more unreal character than the possibility that the farmers during a certain period of time cooperate most closely with the laborers and form some sort of a casual coalition against the manufacturers, or the possibility that the farmers cooperate most closely with the manufacturers and form some sort of a casual coalition against the laborers. Thus considerations of this kind indicate that the big groups of the type mentioned ought really to be looked upon as essentially indissoluble units, representing fixed interests and acting as composite players.

[23)] Compare in this context François Perroux, "Les macro-décisions," *Économie Appliquée* 1949, e.g. p. 329.

On the other hand many observations stress that it is impossible to treat, e.g. ≫the manufacturers≫, as a unified organization within a country. Already within an enterprise there may be divergence of interest between, e.g. the owners = shareholders, the directors and the chief officials; the enterprises within the same branch of industry no doubt form distinct units which have problems to settle both in conflict and cooperation; various branch-organizations may operate as fairly strong associations in dealing with each other; often many branch-organizations seem to combine into higher units which after some steps at last form the national association of manufacturers. Thus considerations of this kind indicate that it is not sufficient to look upon the big groups as indissoluble units.

To reconcile these observations, pointing at first sight in two opposite directions, we enunciate the idea of a hierarchy of socio-economic distribution problems. On the highest (national) level of the hierarchy possibly three-four-five big interest groups like the laborers, the manufacturers, the farmers and the savers, participate in the distribution process; within all or some big groups there are sub-groups with questions to settle among themselves; each subgroup may be further differentiated; on a low level the individual enterprises within a limited branch may be active; possibly on a still lower level there may be divergent interests to be reconciled within an enterprise. We are of course fully aware that the assumption of such a hierarchy of distribution problems is an idealization of reality which ought to be explored empirically in a careful way, but we think that it has enough immediate plausibility to be used as a working hypothesis. The adoption of the hierarchy assumption, that is the assumption of the existence of composite economic subjects, acting as indissoluble units on a certain level, but comprising divergent interests which are developed on a lower level, would perhaps decrease the importance of games with very many participants. We stress the fact that decisive changes in the hierarchy grouping obviously may take place over time, e.g. the segregation of a new social group on the highest national level besides labor, enterprise, agriculture and savings, and we believe such variations to be important changes of the objective possibilities of the situations.

We wish to stress one further point in this context. In speaking about the four big groups on the highest national level we alluded particularly to experience from the last two decades concerning the formation of strong organizations; but we also made use of the fact that the traditional distribution theory, more or less developed as ≫the marginal productivity theory of distribution≫, had the same four groups, and we did not mention that one of the four big groups, the savers, that is the people dependent on the rate of interest as a source of income, are hardly organized in any country. We now express our opinion that the existence of composite

players is not necessarily dependent on the explicit presence of strong organizations. The ≫farm interests≫ may be sponsored against the interests of other big groups in many ways even if there be no national association of farmers, e.g. through the acting of a few influential representatives of an energetic Department of Agriculture; and the ≫saving interests≫ may be essentially disregarded both when there is and when there is not a national association of savers. Thus we believe the primary thing to be the existence of an ≫interest connection≫ between certain decision units; dependent to some extent on ≫irrational≫ socio-psychological factors, and the somewhat secondary thing to be the recognition of this interest connection in the form of an explicit organization, embracing the whole group. The development towards stronger organizations on the highest national level during the last two decades need, therefore, not imply that no distribution problem existed earlier between chiefly the same big groups, that is on the same level; it may be due to other factors which we shall try to touch upon below.

In surveying the three aggregation problems, that of the programming analyzes, that of the theory of games and that of the Walrasian constructions — the first-mentioned being yet only very incompletely handled in the literature, the second-mentioned being the problem of composite players briefly mentioned in the theory of games, the third-mentioned being fairly extensively discussed in the literature but ≫not solved, nor even formulated in detail≫ — we find how the first and the second problems seem to arise naturally out of the considerations of the economic units involved; the planning unit of the programming analyzes must ask how to manage the enormous number of various economic activities and commodities, the difficulties being due to the inexperience of practically handling very large numbers of variables even in the natural sciences and to the absence of perfectly ideal statistical and mathematical tools although it may try to utilize the recent electronic computers as Dantzig proposes; the decision units in a game situation undoubtedly also face a very difficult problem of keeping trace of the objective possibilities of the situation and its changes, as the number of potentially independent participants increases, the difficulties being clearly indicated by the progressive increase in the number of parameters necessary to characterize a given game, as the number of participants grows, and a tendency towards reduction of the number of potentially independent units in the situation is likely to be developed. The third problem, however, is not so clear-cut in its very setting. It is not formulated in such a way that the basic units of the Walrasian systems, the individual consumers, the entrepreneurs, etc. face any aggregation problem in their economic activities; and if only these two aspects of the systems be stressed, namely the description of the course of events and the account of the (rational) behavior of a large number of decision units, then the

whole problem seems to remain somewhere in the air. If a planning unit be superimposed on the whole construction, we approach the programming analyzes, but we shall see later some very grave difficulties which meet such a superposition.

When speaking in what follows about an economic or social group we mean a unit which is on a certain level essentially indissoluble — a composite player; thus the word *group* will be used from the point of view of game theory synonymously with expressions like player, participant, decision unit, unit, or subject. When hinting at a more or less casual coalition of players we will use the word *set,* the elements of which may be groups, decision units, economic subjects, etc.[24)]

γ) *The Solution*

Having discussed the physical background or the objective possibilities — as formalized in the Characteristic Function — against which the economic activities of the decision units in a society take place, the next question to be dealt with is the character of these activities. Are they, for instance, uniquely determined by the background? The economic activities of the programming, analyzes and of the Mosakian and the Marschakian systems could be called ≫choice-activities≫. In the first-mentioned case the economic subject is concerned by selecting among the various possible technical processes in such a way that his objectives be satisfied to the highest possible degree; in the last-mentioned cases the economic subjects have as their primary task the selection of such bundles of goods or assets that will maximize their individual utilities. As soon as the data are fixed, the best way to act is in principle given in all cases.

In the Inessential Games based on the Characteristic-Function-construction the activities of the economic subjects may be called ≫negotiation-activities≫, but the negotiation-possibilities are of such a rudimentary kind that nothing is changed for anybody through the negotiations; as soon as the data are fixed the distribution of income is given, that is the economic activities of one participant or a set of participants cannot influence the economic conditions of the other participants or sets of participants.

In the Essential Games the activities of the economic subjects are too of the negotiation type. There exists, however, a true kind of interdependence. What one participant, or set of participants does may influence the economic conditions of other participants or sets of participants and no unique ≫rational≫ way of determining the relative income-shares can be

[24)] About the hierarchy of distribution problems see further Chapter VII, Section 4: b: δ.

found when the data are fixed. How then is some order created out of this indeterminacy? An established agreement on how to distribute the income is necessary between some or all of the participants for the creation of stable conditions. Many different agreements may be realized, some favoring certain units in the society, some favoring other units. How is the agreement found that is really established in a society at a certain point of time?

We now turn to the concept of Solution. The concept is translated in the ≫Theory of Games≫ by ≫established order of society≫ = ≫accepted standard of behavior≫. The idea in this context is that the agreement actually put up must obey a certain principle, a certain mode of thinking then prevailing in the society, i.e. it must belong to the set of all agreements obeying this principle. Different principles of this kind or modes of thinking must possess the same kind of stability properties, but otherwise they may show very dissimilar traits. We quote the following passages from the ≫Theory of Games≫: ≫The single imputation is an often used and well understood concept of economic theory; while the sets of imputations [i.e. solutions] to which we have been led are rather unfamiliar ones. It is therefore desirable to correlate them with something which has a well established place in our thinking concerning social phenomena.

Indeed, it appears that the sets of imputations (**V**) which we are considering correspond to the 'standard of behavior' connected with a social organization. Let us examine this assertion more closely.

Let the physical basis of a social economy be given, — or, to take a broader view of the matter, of a society. According to all tradition and experience human beings have a characteristic way of adjusting themselves to such a background. This consists of not setting up one rigid system of apportionment, i.e. of imputation, but rather a variety of alternatives, which will probably all express some general principles but nevertheless differ among themselves in many particular respects. This system of imputations describes the 'established order of society' or 'accepted standard of behavior.'

Obviously no random grouping of imputations will do as such a 'standard of behavior': it will have to satisfy certain conditions which characterize it as a possible order of things. This concept of possibility must clearly provide for conditions of stability.≪[25)]

≫We have already seen that our concept of a solution of a game corresponds precisely to that of a 'standard of behavior' of everyday parlance.

[25)] von Neumann and Morgenstern [1], pp. 40–41.

Our conditions express just the kind of 'inner stability' which is expected of workable standards of behavior≫.[26),27)]

The formal definition of a Solution follows, including the stability criteria referred to. As a preliminary step the notions of Effectivity and of Domination must be introduced. The task before us may be said to consist in knitting together the concepts of Imputation and Characteristic Function, the aim of the negotiation activities being to establish an Imputation on the background given by a Characteristic Function. Already the definition of an Imputation involved certain elements of a Characteristic Function in stating that $a_i \geq v((i))$, that is, the share of an Imputation going to a participant must always be greater than or equal to the amount which the participant can obtain alone according to the objective possibilities of the situation. Obviously this is a very faint restriction on an Imputation outside the Inessential Games.

A set of units S is called Effective for the Imputation $\vec{\alpha}$ if

$$\sum_{i \in S} \alpha_i \leq v(S)$$

i.e. if the amount of the Imputation which goes to the set is not greater than the amount which the members have the possibility to secure for themselves through agreement and collaboration.

An Imputation $\vec{\alpha}$ Dominates another Imputation $\vec{\beta}$, if there exists a set S possessing the following three properties:[28)]

$$\begin{aligned} &a) \quad S \text{ is not empty} \\ &b) \quad S \text{ is Effective for } \vec{\alpha} \\ &c) \quad \alpha_i > \beta_i \quad \text{for all } i \in S \end{aligned} \tag{30}$$

i.e. if there exists a (non-empty) set of decision units which are able to enforce the acceptance of the Imputation $\vec{\alpha}$ and which all get higher shares in this Imputation than in another Imputation $\vec{\beta}$.

If in a society only one Imputation could be established, obviously this would have to possess certain optimum characteristics; in the technical terminology just introduced it should Dominate all other Imputations. If the social forces were such as to admit more than one Imputation, say a set **V** of Imputations, the elements of this set could not Dominate each other;

[26)] von Neumann and Morgenstern [1], p. 265. The quotations are reproduced by permission of Princeton University Press.

[27)] Cf. below, particularly Chapter V, Section 4.

besides, they should as a whole possess certain optimum characteristics in comparison with the other Imputations. Out of these considerations we get in a fairly straightforward manner the definition of the set **V** of acceptable Imputations; such a set is called a Solution. A set **V** of Imputations is a Solution, if it possesses the following properties:

(1) No $\vec{\beta}$ in **V** is Dominated by an $\vec{\alpha}$ in **V**

(2) Every $\vec{\beta}$ not in **V** is Dominated by some $\vec{\alpha}$ in **V.**

(1) and (2) are together identical to the single condition:

(1 + 2) The elements of **V** are precisely those Imputations which are Undominated by any element of **V.**[28)]

The first condition expresses a kind of inner consistency: no Imputation belonging to a Solution may Dominate another Imputation within the Solution, i.e. no Imputation in a Solution can be Effectively preferred by a certain group of units to another Imputation in the Solution. The second condition expresses a kind of outer stability: every possible Imputation outside a particular Solution must be ruled out by some Imputation belonging to the Solution, i.e. there always exists some Imputation in a Solution which is Effectively preferred by a certain set of units to a given Imputation not belonging to the Solution.

In case a Solution consisted only of one Imputation the first condition would be redundant and the second condition would give us an Imputation obeying the properties ordinarily attached to an optimum. Such an one-element Solution would denote that in the society imagined no real conflicts of interest were present — this unique Imputation being established, anybody and everybody would have reached the best position admitted by the objective possibilities they were facing. In the Inessential case the unique Reduced Characteristic Function admits only a unique Solution containing a unique Imputation — there the concept of Solution is completely superfluous, as it reduces to an one-element set of Imputations, but the straightforward application of the definition of a Solution yields this special result. In the Essential case a Solution never contains only one Imputation. This is simply the explicit consequence of the circumstance, that an Essential Characteristic Function depicts a society where a conflict of interest is present; the distribution process, envisaged is *non-harmonic;* no state of society can be established where all participants at the same time reach their best possible position; a certain set of participants are always in some sense — which may be a very complicated and subtle one — dominating, and establish an Imputation in their favor. Thus which

[28)] von Neumann and Morgenstern [1]; p. 264.

Imputation out of a Solution is really put into effect is a question of power; the theory does not indicate which particular set of economic subjects out of the possible ones that win — a word that ought to be conceived as having a large variety of nuances — the distribution struggle. An attempt to include the actual power-constellation in the data — compare Marschak's system — amounts in our opinion very closely to assuming away the problem that was to be attacked, the problem of distribution in a society where the decision units are not quasi-isolated from each other.

The distribution process described by the Essential Games is *non-automatic* and *non-harmonic.* It has a further major quality: it is in a certain sense *habitual.* The Essential Characteristic Functions hitherto fully investigated — among them of course the simplest ones of all — have admitted not *one* Solution but several Solutions, each as we know containing by necessity many Imputations. Which of the possible Solutions to be adopted is no question of power. It follows from the second property of a Solution, that of the outer stability, that everything a set of economic subjects may effectively claim can be found within each Solution: every Imputation not in the Solution is Dominated by *some* Imputation within the Solution. Probably it is a very common experience from the study of social and economic development that what is relevant and what can be judged is not if a principle is good or bad but only if it is consistently applied. A Solution is a habitual principle that gives some order to the conflicts of interest and the resulting struggle of distribution in a society. A set of solutions gives a classification of various types of distribution-agreements that may be put into effect, each member of the set embodying a particular type of agreements, or in other words a particular principle that the agreements have to obey. On one side a principle of this kind has sufficient stability characteristics not to be effectively questioned as such by group interests; on the other it is by no means unique as other principles may be substituted for it which show quite different traits without being preferable from any point of view; what matters is that the principle adopted be strictly adhered to.

We find that the task of knitting together the concepts of Imputation and Essential Characteristic Function cannot be carried out quite successfully; the condition that every acceptable Imputation on the background of a given Characteristic Function belongs to a Solution has been established, but this condition does not fix a unique Imputation in the Essential case.

δ) The game distribution theory and the permanence of economic laws

Thus the game distribution theory shows an extreme diversity and changeability in the schemes of distribution that may be put up in a society. Even if a Characteristic Function be empirically determined out of the bewildering amount of possible ones when the number of active economic subjects is large, various possible Solutions may probably exist on this background; even if the prevailing Solution be empirically determined, this does not imply that one and only one Imputation is valid; on the contrary it always allows for several possibilities. From still another point of view we may say: Suppose that a given Imputation, belonging to a given Solution, established on a given Characteristic Function, exists in a society. A change in this Characteristic Function may occur which shifts it into another Characteristic Function; as a result the Solution may change, although it is not in general uniquely determined what Solution will be established; within the new Solution a certain Imputation is put forward, although it is not given which of the possible ones this will be. Thus the three basic concepts are all elements of change of the theory, except for the Reduced Characteristic Function in the three-person case which is unique; the theory claims to be of universal validity up to the enormous variability of these elements of change, these ≫empty boxes≫.

This may be contrasted with the extreme fixity and unchangeability of traditional economics and of the similar Inessential games. We have pointed out how traditional economics is claimed to be of universal validity up to the structure, i.e., the coefficients of the structural relations, which may vary from case to case, and we have stressed how approximately this element of change is taken care of by the game Reduction procedure of the Characteristic Functions, and is shown to be insignificant at the game level of analysis.

The game distribution theory may be naturally looked upon as a formal generalization of the traditional distribution theory, via the Inessential case leading to the basic formula (C'').[29)] This generalization, however, immediately brought forward difficulties which were glossed over in the traditional harmony construction, the difficulties connected with transferable utilities and with conflicting value schemes. In our next chapter we shall study these difficulties in greater detail. We conclude this chapter by a few remarks.

[29)] p. 51 above.

We differ somewhat from the authors of ≫The Theory of Games≫ in stressing the pure distribution character of the part of the theory considered — Zero-sum or Constant-sum n-person Games with $n > 2$, postponing until later the mentioning of the Zero-sum two-person Games and the General Non-constant-sum, Games. In this respect we follow to a certain degree commentators who have criticized the utility-treatment in the work and asked why the whole thing could not be interpreted solely in terms of money. We differ from many commentators in the fact that we consider the analysis of the theory of games to be the general one and the traditional analysis to be the very special one and not vice versa. It is often held that the theory of games deals with exceptional, empirically rare cases of ≫oligopoly≫, while the traditional analysis deals with the ordinary and empirically common case of ≫free competition≫; we do not share these views, nor the opinion that traditional ≫dynamics≫ is a more general setup than game-theoretical ≫statics≫ — we refer here to what has been said above on fixity *vs.* diversity and on traditional statics *vs.* dynamics. We further differ from many commentators in not solely paying attention to the Zero-sum two-person Game and basing our remarks about the whole work on this particular setup.

c) THE THEORY OF GAMES AND OTHER TYPES OF NON-WALRASIAN ECONOMICS

Johan Åkerman wrote in 1938: ≫Is it an exaggeration to say that the mathematical school of economics is still today contained within the theoretical universe, as published by Leon Walras in 1874 under the title: *Élements d'économie politique pure*? Is it really possible that this school considers this construction to be a final expression of the mathematico-economic central theory, despite the vigorous activity of various authors in the field during almost three quarters of a century?≫[30] To this may be added that also many non-mathematical economists both in 1938 and today look upon the Walrasian universe as the *final* economic theory. Both mathematical and non-mathematical economists often make the reservation, however, that the original Walrasian or Paretian universe must be supplemented by considerations of imperfect competition and dynamics. Mosak (1944) represents the acceptance without conditions of the Walrasian construction, rejecting the application of theories of imperfect competition and dynamics ≫on the level of generalization employed in general-equilibrium analysis.≫[31] Marschak (1938 and 1950) represents the acceptance with

[30] Johan Åkerman, *Das Problem der Sozialökonomischen Synthese,* Lund 1938, p. 105.

[31] Mosak [25], p. 3.

reservations of the Walrasian construction, reservations that in this case have not remained mere unfulfilled propositions.[31a)] Von Neumann and Morgenstern (l944) have presented the first systematic — but still only incompletely elaborated — alternative to the Walrasian system expressed in the same mathematical language, although far more rigorously. It must not be forgotten, however, that a critical attitude towards the Walrasian central theory, linked with more or less systematic alternatives to it, has been presented during the course of the science of economics. We mention in chronological order the following four (groups of) contributors: The Austrian school, Carl Menger, von Wieser, Böhm-Bawerk (1871-); Veblen and his followers, e.g. Commons, discussed, e.g. in Gruchy's history of doctrines (1898-); Johan Åkerman (1936-); and François Perroux (1948-). No doubt there are points of contact between all these types of researches and the theory of games. Some relations will be discussed below in special contexts, but we do not propose to enter on a comparison at the most general level. We want, however, to make the following brief remarks.

Possibly an investigation of the connection between the Austrians and the theory of games should be concentrated on the concepts of Zurechnung and Imputation;[32)] for a splendid criticism in the Austrian tradition of Walrasian economics, see particularly Morgenstern's review of Hicks' *Value and Capital.*[33)]

Veblen and the institutionalist school[34)] hold the basic methodological principle that ≫the whole is greater than the sum of its parts≫, a principle which in a well-defined context is also expressed in the fundamental Characteristic Function inequality of the Essential Games, the first summation theorem. In the theory of games this inequality implies a great variety of elements of change of the theory: a multitude of different Reduced Essential Characteristic Functions, each with probably always many Solutions, each containing a set of Imputations; in contrast the fundamental Characteristic Function equality of the Inessential Games implies a unique Reduced Characteristic Function with one Solution, containing one Imputation. In the institutionalist writings the basic methodological principle leads similarly to an emphasis on the diversity of economic theory constructions as contrasted to the traditional uniqueness. The elements of change most

[31a)] Marschak [26] and [27].

[32)] Cf. von Neumann and Morgenstern [1], pp. 435–443.

[33)] Oskar Morgenstern, "Professor Hicks on Value and Capital," *The Jour nal of Political Economy* 1941, pp. 361–393. Compare too his review of Heinrich von Stackelberg , "Grundlagen der theoretischen Volkswirtschafts lehre," *The Jour-nal of Political Economy* 1950, pp. 267–268.

[34)] Cf. Gruchy [17].

generally employed are the — undoubtedly vague — concepts of ≫social field≫ and ≫norm of conduct≫. Economic life shows very many qualitatively different economico-cultural spheres or ≫social fields≫, each having essentially its own economic laws without much room for far-reaching generalizations. Each field conditions the activities of the various decision units or groups of decision units and has the characteristic that the result of the activity of one unit is dependent on the activities of other units. Therefore certain social ≫norms of conduct≫ are necessary in order to create temporarily stable conditions. However, the social field is apt to undergo frequent changes which also alter the social norms of conduct.[35)] Below we shall compare this concept of ≫norm of conduct≫ with the concept of Solution or accepted standard of behavior.

Johan Åkerman[36)] shares in much the methodological basis of institutionalism,[37)] and strongly emphasizes the diversity of economic theory as contrasted with the traditional uniqueness, particularly in the form that different, successive periods of time show their own economic laws or relations, with often sharp and sudden reversions from one period to the following one. The element of change elaborated is the ≫structure≫ (and ≫group≫) concept which is radically different from the econometric ≫structural coefficients≫ because it is, in its various quantitatively expressed forms, intended to characterize a whole, a social organization as such and is not, as the latter structure, tied to the parts of society that are studied, characterizing the psychology of the ≫group of consumers≫, etc. We shall have repeated opportunity below to refer to Åkerman's concepts of ≫structure≫, ≫structural change≫, and ≫structural limits≫.

When contrasting the elements of change of game theory with the elements of change employed by Johan Åkerman and also by the institutionalists it should be stressed that the three-fold division of the former is lacking in the latter, which are more directly based on empirical observations — yet Johan Åkerman in some places explicitly distinguishes between one technical and one institutional or power aspect of his concept of structure,[38)] and the institutionalists have the distinction between the background of a social field and the norms of conduct established on this background. It also seems likely that the trichotomy of game elements of change may be empirically somewhat obliterated by the circumstance that all three categories, the objective possibilities, the mode of thinking and

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35) Cf. Gruchy [17].

36) JohanÅkerman [14], [15], [16].

37) This point is also stressed by Erik Dahmén in a review of Gruchy [17], *Ekonomisk Tidskrift* 1948, p. 253.

38) Johan Åkerman [16], p. 2.

the actual power constellation, may sometimes be decisively changed at the same point of time in a society.[39)]

Perroux[40)] in a number of articles has strongly underlined the irrelevance of Walrasian and Paretian economics as an explanation of the reality of a multi-unit-of-decision economy. In such a society conflicts of interest, economic struggle and explicit economic agreements — alien to a Walrasian construction — are present, which require consideration of constraint and power, especially as exercised by one ≫dominating≫ decision unit or country against a ≫dominated≫ unit or country; and of ≫macro-decisions≫, taken not by a single individual acting in isolation but by groups of individuals, e.g. labor unions, which face the exterior problem of adjustment to groups on the same level, having different economic interests, and the interior problem of reconciling the economic actions of their constituent subgroups.

The difference in scientific procedure between scientists accepting the Walrasian constructions, at

least as the systematic starting point, and scientists rejecting these constructions, must be strongly emphasized. What the former category is apt to consider as the whole economic universe, the intricate details of which should be elaborated, is looked upon by the latter category at most as some very special cases of a general economic and social theory. It ought to be added, however, that genuine points of contact exist, e.g. between the theory of games and traditionalism, of which the programming analyzes constitute an example; these may be interpreted as Zero-sum two-person Games,[41)] and are besides in many ways connected with Walrasian constructions.

[39)] Certain points of contact between the theory of games and the writings of Johan Åkerman and the institutionalists, as well as some other ideas developed further in this thesis, were expounded in our article in Swedish: Göran Nyblén, "Atomistisk och holistisk ekonomisk teori," *Ekonomisk Tidskrift* 1949, pp. 258–282; English review summary in *The Economic Journal* 1950, pp. 429–430: "Atomistic and Holistic Economic Theory."

[40)] See e.g. François Perroux, "Esquisse d'une théorie de l'économie dominante," *Économie Appliquée* 1948, pp. 243–300, and François Perroux, "Les macro-décisions," *Économie Appliquée* 1949, pp. 321–354.

[41)] Cf. p. 89 below.

4) Summary

a) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS AND OTHER ECONOMIC THEORIES

Chapter I posed the task of investigating the balance identities and the equilibrium conditions of the econometric systems.

In this chapter the *balance identities* are first studied in Leontief's static system, closely resembling an ordinary econometric setup with balance identities but adding more variables and more linear relations; the balance identities in Leontief's static system imply a full-utilization-of-resources assumption. Hawkins' dynamic system adds to Leontief's constant-rate-flows system elements of capital formation and growth but retains the linearity of the relations; its balance identities imply both full utilization of resources and maximum possible growth of the economy. Dantzig's system abandons the linearity of the relations — in the sense of admitting more than one fixed way of producing the same commodity — and as a consequence of this step it must introduce — beyond the full-utilization-of-resources assumption — a best-utilization-of-resources assumption; formalized in the Objective Function, i.e. the value scheme of society, which is to be maximized. Hence this line of logical development of theories starting from the econometric balance identities seems to end at systems, comprising only one value scheme and being concerned with a process of production of various commodities and services guided by this value scheme.

The *equilibrium conditions* are first studied in the Walras-Mosakian system. This system embodies many individual value schemes, influencing the productive activities of the society through the pricing process which is formalized in the equilibrium conditions — most explicitly in their dynamic versions — and which makes supply equal to demand; no conflicts between the various value schemes occur. The last feature is, however, correlated with the fact that the pricing or distribution process depicted by the equilibrium conditions is of a very special nature. It builds on the assumption that no pricing problem has to be solved through explicit agreements between the decision units in the society, and hence that there cannot be any question of conflicting interests, each trying to establish an agreement to its favor out of a number of feasible alternatives; moreover, all decision units are supposed to have an equal and negligible direct influence on the formation of prices. The prices are determined through the working of the market mechanism; and the pricing or distribution process thus outlined is — due to reasons just stated — automatic and ultra-harmonic. The Walras-Marschakian system also includes many individual value schemes between which no contradictions appear. This feature is still correlated

with the existence of equilibrium conditions, but these are superimposed on revenue functions, describing in a way characteristic to each individual how the prices he can charge are dependent on the amounts of commodities involved in his transactions. The pricing process outlined does not imply that all decision units have an equal direct influence on the formation of prices, but it develops without any agreements being formed between the units; hence it is still automatic and also harmonic, because no alternative price systems are left to become a matter of dispute between various group-interests. Thus the equilibrium conditions, also when supplemented by market functions, depict a very special pricing or distribution process; the coexistence of the many value schemes in the Walrasian constructions is correlated with this automatic and harmonic character of the pricing process.

The Inessential Games also describe a pricing or distribution process in which all agreements are superfluous and no conflict of interest occurs, as there are no alternatives left for a struggle between various group-interests — the Imputation is unique. In these games economic values are additive, i.e. the income achievable by every possible set of units arises additively from the amounts which the units can acquire for themselves, acting without any agreements. The additivity characteristic can also be expressed thus: there is no part of the national income, the distribution of which can and must be settled through agreements between the members of society; all relative income shares are automatically and harmoniously fixed. Especially, if expressed in this form, the close similarity between the additivity characteristic of the Inessential Games and the basic features of the pricing and distribution process tied to the equilibrium conditions and related concepts is quite evident.

The additive Inessential Games, however, only constitute a rudimentary special case of all Games; their complement in the set of all (Zero or Constant-sum) Games is formed by the (Zero or Constant-sum) Essential Games. These formalize a distribution process which is neither automatic, nor harmonic — explicit agreements between the decision units are needed to establish a particular set of prices and incomes, and many feasible alternative forms of the agreements, each favoring particular group-interests, always exist. The Essential Games build on the first summation theorem, saying that economic values are non-additive, i.e. the income achievable by a set of economic subjects is in general greater than the sum of the amounts accruing to the various subsets of units, forming the set — or to the various individual units, composing the set — if these smaller groups act in isolation without any agreements between each other. The non-additivity characteristic can also be expressed thus: there is always one part of the national income the distribution of which can and must necessarily be settled

through agreements between the members of society; the distribution of the national income can never be completely settled in an automatic and harmonious way. Especially, if expressed in this form, the rôle of the Essential Games as a generalization of the pricing and distribution process tied to the equilibrium conditions and related concepts and to the Inessential Games is quite evident.

This generalization of the traditional, rudimentary distribution process causes, however, immediate trouble as regards the value schemes of the decision units. It must assume that economic values can be attached to sets of economic subjects in such a way that the values achievable through cooperation among every conceivable set of units can be expressed as numbers which have all the measurability and transferability characteristics of everyday money — or of Utopian transferable utility. We have adopted the interpretation in terms of money, which in connection with the constant-sum-assumption implies that an economic unit in a society depicted by an Essential Game strives to maximize its share of the money national income, i.e. its relative income share. Hence this line of logical development of theories starting from the econometric equilibrium conditions seems to end at systems comprising no value scheme at all but counting only in money, or, more adequately expressed, being concerned with a process of distribution of money incomes.

The first line of logical development of theories starts from the econometric balance identities and ends at the programming systems, dealing with a process of production of various commodities and services guided by a value scheme of society. We term this type of theories ≫the pure production aspect≫. The second line of logical development of theories starts from the econometric equilibrium conditions and ends at the (Zero or Constant-sum) Essential Games, dealing with a process of distribution of money incomes. We term this type of theories ≫the pure distribution aspect≫. The Walrasian constructions comprise both a production aspect and a distribution aspect, which are completely synthesized. The many coexisting value schemes guide the production of commodities and services — the Walrasian production aspect — through the pricing and distribution process of the equilibrium conditions and related constructions — the Walrasian distribution aspect. Thus a sufficient condition for the harmonious coexistence of many individual value schemes in a society is the empirical validity of the peculiar pricing and distribution process described by the equilibrium conditions. This process is, however, of an obviously rudimentary character, the empirical significance of which is often doubted, even without the process being viewed as a special case of a wider theoretical framework — as we have tried.

If the doubtful pricing process be completely removed from a Walrasian construction the equilibrium conditions turn into balance identities — then supply becomes identical to demand as no equalization mechanism is left. In this way the production aspect is obtained in a pure form. The resulting system is the Leontiefian one, if severe linearity restrictions be imposed; abandoning these restrictions the programming analyzes appear, containing only one value scheme, and thus seemingly presupposing absolute unity of interests in society. If the doubtful Walrasian pricing process be generalized, i.e. if the additivity element be removed, implying that no agreements are needed for any distribution problem to be settled, the Essential Games appear, containing no value schemes but describing the aim of the economic units as consisting in maximization of their respective relative income shares. This is the distribution aspect in its pure form, seemingly presupposing absolute diversity of interests in society.

The question then naturally arises: is a synthesis possible between the production aspect and the distribution aspect, which does not have to resort to the Walrasian pricing procedure, almost completely blotting out all significant distribution problems and thereby making possible the harmonious coexistence of many value schemes? Certain prospects of such a synthesis would exist, if it were possible to supplement the pure production aspect by building up its social value scheme from many individual value schemes, thus introducing into the picture a certain amount of diversity of interests; or, if it were possible to supplement the pure distribution aspect by some considerations of total utility and social productivity which is to be maximized before the distribution process starts, thereby rendering crucial not only the relative income shares but also the total amount to be distributed.

b) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS AND EMPIRICAL RESEARCH

The systems belonging to the pure production aspect and to the rudimentary Walrasian synthesis between a production aspect and a distribution aspect comprise the same type of elements of change as the econometric systems, namely, structural coefficients. Of special interest in this connection is the fact that the real part of a programming analysis — the plan actually carried out or the methods of production actually employed — could be depicted by a system of linear relations without a very serious loss of generality, if the dependence of this system on the virtual part of this analysis — the methods of production which are feasible but not employed due to the character of the value scheme of society, and this every value scheme itself — be recognized. Thus the system of linear relations should be expected to be valid only during one planning period of the society, i.e.

until the value scheme is reconsidered and/or new methods of production are taken into account. We have interpreted Leontief's system to be such a real part of a programming analysis.

The pure production aspect depicts no distribution or pricing process at all — apart from the shadow prices which have no independent rôle. The Walrasian systems comprise a rudimentary distribution aspect: as soon as this is generalized into the Essential Games the elements of change utilized alter their character in a profound manner. The Essential Games still comprise an element of change similar to the structural coefficients of the Walrasian systems: variations between Strategically Equivalent Games in nowise influence the capacity of the various units to form agreements, enter into coalitions, etc; as we should expect, all Inessential Games are Strategically Equivalent because no agreements concerning the distribution process can be formed in such a game precisely as is the case in a Walrasian system.

This so insignificant element of change from the point of view of a distribution theory being ruled out through the reduction procedure, the following elements of change or ≫empty boxes≫ remain in the Essential Games: Characteristic Functions, Imputations and Solutions. The Characteristic Functions represent the objective possibilities of the decision units in the distribution process, including, for instance, factors of technology; the Imputations represent a power factor, as it is a question of the power of various group interests which Imputation is established within a Solution; the Solutions represent a habitual factor, as it embodies a principle, which a set of alternative, feasible Imputations has to obey. Of great importance to the whole construction is the number of active units involved in a distribution process, the complexity of analysis growing rapidly with the number of such units simultaneously present. This should be compared with the tendency of individual economic subjects to compose in the short run indissoluble higher units, sometimes in the form of strong organizations; sometimes in the form of a looser ≫interest-connection≫, when a group of economic subjects is represented by an agency without this agency being backed by explicit organizations. An indication of this tendency to form such indissoluble higher units in the distribution process is the persistent utilization in distribution analyzes of the four-fold division: laborers, farmers, entrepreneurs and rentiers. It is, however, plausible that such a superunit, — e.g. a social group —, taking part in a distribution struggle with other units on the same level, may have its own distribution problems to settle, i.e. a distribution process concerning the relative position of the various subunits within the superunit, notwithstanding the exterior relations to other superunits. This is the essence of our suggestion concerning

the hierarchy of distribution problems, which, if relevant, would somewhat diminish the importance of games with very many participants.

The elements of change of the structural coefficient type are tied to Walrasian constructions. Outside these constructions the programming analyzes within the pure production aspect show the same elements of change, but the distinction between the real and the virtual parts of these analyzes is an additional feature which should be observed; and the game analyzes within the pure distribution aspect show — apart from insignificant variations between Strategically Equivalent Games — completely different elements of change to be determined by empirical research. The universal validity of the structural coefficients as elements of change must therefore be held in considerable doubt. This point is stressed if a comparison be made between the elements of change of game theory and certain concepts utilized by non-Walrasian economic schools. Some of these lines of thought, e.g. represented by the institutionalists and by Johan Åkerman, employ in their theoretical reasoning and empirical research elements of change which can hardly be summed up in the traditional structural coefficients, as they above all characterize a society as such and not individual economic subjects viewed in isolation from each other. Examples of such elements of change are the institutionalist norms of conduct, bearing certain similarities to the Solutions or accepted standards of behavior — the habitual element in the game distribution theory — and Johan Åkerman's structure (and group) concept, being differentiated in one technical and one power component. Only the gradual establishment of a contact between the game elements of change and the empirical reality can, of course, ultimately settle their significance.

Chapter III

Is a Synthesis Possible Between the Production Aspect and the Distribution Aspect?

1) Some attempts towards a synthesis

a) ARROW'S THEOREM

We have spoken about the production aspect and the distribution aspect in two different contexts. *First,* we made the distinction in our analysis of the Mosakian and the Marschakian systems. The production aspect showed how the various consumer preferences guided the production, and the distribution aspect showed how the relative prices of the various commodities and services were determined; moreover, the process of the formation of prices — the distribution process — was judged to be of an obviously very rudimentary kind. At the same time the two aspects were interdependent, the production problem depending for its solution on the existence of the prices: thus the generality of the whole construction could not exceed that of the evidently weak link. *Second,* we contrasted with each other what can be called the pure production aspect and the pure distribution aspect, that is the production problem approached without the introduction of other than (unnecessary) shadow prices and the distribution problem treated without the inclusion of any ordinal or cardinal utility at all. In the first-mentioned case a theoretical construction was found implying *absolute unity of interests* in the society imagined, and counting in utility; i.e., it depicted the economic activities in the society as aiming at the fulfillment of one single set of objectives. As the prototype of such constructions stands the linear programming field; there the fulfillment of the single set of objectives consists in maximization of the Objective Function. Another more familiar but much vaguer, and partly more obscure example of this sort of theoretical construction is the ≫welf are analyzes≫ with their maximization of a ≫social welfare function≫. In the second-mentioned case a theoretical construction was found implying *absolute diversity of interests* in the society imagined, and counting in money; i.e., it depicted the economic activities of a decision unit in a society as aiming at maximization of its own money income, or, as we prefer to put it, of its relative share of the money national income, and thus no net-gain could be achieved through the collaboration of *all* decision units. As the prototype of

such constructions stands the Zero or Constant-sum Games, but we have pointed out that kindred ideas are in various respects found throughout non-Walrasian economics.

Under the denomination of absolute diversity of interests we may distinguish two sub-cases, firstly the insignificant case of harmony of interests, represented by the Inessential Games, where nothing can be gained for any unit through agreements and collaboration with other members of the society, and secondly the dominating case of conflict of interests, represented by the Essential Games.

Fulfilling our intention expressed in the first section of the previous chapter we shall now discuss attempts to bridge over the gulf between the pure production aspect and the pure distribution aspect. If such a synthesis can be found the result will be a system, expressing the same kind of unity as the Walrasian systems, but lying on quite another level, freed from the limitations of the systems, which are particularly obvious when these are viewed from the distribution aspect. If the prospects for such a synthesis are judged to be bad, a very interesting situation arises deserving of further comment. The first point in the discussion will be a consideration of the crucially important theorem by Arrow on the aggregation of value schemes — this is the second summation theorem, referred to in the Introduction. The second point will be a brief account of the related problem on Efficiency Points as defined by Koopmans. Finally, the third point will be an inquiry into the field of the General Games, which abandon the constant-sum assumption and thus take a decisive step towards adding certain productivity elements to the pure distribution aspect.

In the modern ≫welfare-analyzes≫ it is usually concluded that utility is interindividually non-comparable, but individual value schemes or preference scales may be harmonized through the establishment of a ≫welfare function≫. This concept is due to Abram Bergson.[1] It may be applied in cases where there is no question of harmonizing various individual value schemes; there it is in principle, as we have already indicated, similar to the Objective Function of the programming analyzes. We are here only concerned with its first-mentioned use, e.g. as expounded by Samuelson in his chapter on welfare economics in *Foundations of Economic Analysis.*[2] We quote :≫... we take as starting point for our discussion a function of all the economic magnitudes of a system which is supposed to characterize some ethical belief ... We only require that the belief be such as

[1] Abram Bergson (Burk), "A Reformulation of Certain Aspects of Welfare Economics," *The Quarterly Journal of Economics* 1937–38, pp. 310–334.

[2] Samuelson [6], pp. 219 ff.

to admit of an unequivocal answer as to whether one configuration of the economic system is 'better' or 'worse' than any other or 'indifferent,' and that these relationships are transitive; i.e., A better than B, B better than C, implies A better than C, etc. The function need only be ordinally defined, and it may or may not be convenient to work with (any) one cardinal index or indicator... The subject could end with these banalities were it not for the fact that numerous individuals find it of interest to specialize the form of [the function], the nature of the variables, and the nature of the constraints... A more extreme assumption, which stems from the individualist philosophy of modern Western Civilization, states that individuals' preferences are to 'count'≫.[3)]

Let this welfare function or social (complete) ordering[4)] be denoted by W. Samuelson particularizes as follows the point in which we are especially interested:

$$W = F(U^1, U^2, \ldots, U^i, \ldots) \tag{1}$$

U^i being the individual ordinal utility function of decision unit no i.[5)] In words: the social ordering of relevant alternatives, W, is built up from the many different individual orderings of alternatives, $U^1, U^2, \ldots, U^i, \ldots$, that is the social value scheme, consistent in the sense of being a (complete) ordering, amounts to an aggregation of the individual value schemes, which are consistent in the same sense. This aggregation may assign different importance to the various individuals, etc., but it must obey the ordering character both of the social value scheme and of the individual value schemes.

If such a construction, such a consistent aggregation of value schemes, such a bridge from *many* individual values to *one* social value could be made, a way would emerge for letting the pure production aspect move from an absolute unity of interests towards a certain diversity of interests, opening up the hope for a final synthesis with the distribution aspect at the other extreme. The theorem proved by Arrow in his *Social Choice and Individual Values* (1951)[6)] shows that the construction is in general impossible.[7)] We give a short account of this theorem. Let $x, y, z, \ldots$ denote

[3)] Samuelson [6] pp. 221–223. Reproduced by permission of Harvard University Press.

[4)] On this term cf. p. 45, footnote 6.

[5)] Samuelson [6], p. 228; cf. p. 244.

[6)] Arrow [2].

[7)] Compare in this context Johan Åkerman [14], I, pp. 248–249, II, p. 598 and François Perroux, "Les macro-décisions," *Économie Appliquée* 1949, pp. 321–354.

the various alternatives or ≫social states≫[8] to be considered. Let every individual be able to make an ordering of the alternatives and denote his preference scale by R_i; denote further the social preference scale by R, expressing also an ordering of the alternatives. We interpret xR_iy and xRy to mean ≫x is preferred to or indifferent to y≫ by individual i and by society, respectively. The ordering character of the preferences implies:

I. For all x and y, either $xR_{(i)}y$ or $yR_{(i)}x$

II. For all x, y and z, if $xR_{(i)}y$ and $yR_{(i)}z$, then $xR_{(i)}z$[9]

We quote the following definition on forming a social preference scale from individual preference scales: ≫By a *social welfare function* will be meant a process or rule which, for each set of individual orderings $R_1, \ldots,$ R_n for alternative social states (one ordering for each individual), states a corresponding social ordering of alternative social states, R.≫[10]

Arrow's theorem proves that *no* such process or rule exists, granted that the following five quite plausible conditions are fulfilled. 1) Certain restrictions may be imposed on the individual orderings. It may be assumed that each individual orders the various ≫social states≫ solely according to the commodity bundles which these give just to him — the ≫individualistic assumptions≫,[11] dominating in traditional analysis. This restriction is fairly weak. An obviously very strong restriction is on the other hand the postulate that all individuals have an exactly similar ordering of the ≫social states≫. In that case, of course, *any* individual value scheme may be used as a social value scheme. Examples are also to be found in the literature of individual value schemes which, without coinciding, are so similar to each other that consistent social schemes may be based on them.[12] It has not been established exactly where the limit extends between the general case admitting no aggregation of value schemes and the special cases. with so pronounced a similarity between the individual value schemes that an, aggregation is possible. The condition that for each individual at least the ordering of three among the alternatives is completely without restrictions, suffices, however, for excluding the special cases of similar individual value schemes.[13] 2) If an alternative is raised to a better position or remains in

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[8] Cf. pp. 83–84 below.

[9] Arrow [2], p. 13.

[10] Arrow [2], p. 23.

[11] Arrow [2], pp. 61–69.

[12] Duncan Black, "The Decisions of a Committee Using a Special Majority," *Econometrica* 1948, pp. 245–261; and other writings. Cf. Arrow [2], pp. 74–80.

[13] Arrow [2], p. 24

the same place in all individual value schemes and no other changes take place, this alternative must not obtain an inferior position within a social value scheme, based on the individual orderings. (≫Positive Association of Social and Individual Values≫).[14] 3) Let the relevant alternatives form a certain class of alternatives. The social valuation of these alternatives must not be influenced by possible alternatives lying outside the class in question. (≫The Independence of Irrelevant Alternatives≫).[15] 4) The social ordering must not be such as never to prescribe for some pair of two alternatives that one is better than the other, notwithstanding the structure of the individual preferences. (≫The Condition of Citizens' Sovereignty≫).[16] 5) The social welfare function must not always, for each pair of two alternatives, follow one particular individual value scheme in deciding which one is to be preferred. (≫The Condition of Non-dictatorship≫)[17]

The most interesting condition, opening up a field for further exploration, seems to be the first one, which still admits a certain possibility for aggregation of individual value schemes, sufficiently similar to each other — in a sense which has so far not been established.

We have earlier met with ranking of different alternatives of choice both in the Objective Function of the programming systems and in the preference functions of the Walrasian constructions. It was pointed out that the objectives of an economy were naturally expressed in some way in terms of goods and that the linear Objective Function was claimed to be capable of embodying a great variety of different possibilities of formalizing a value scheme of a planned society. Every preference function in a Walrasian system gives an ordering of different bundles of commodities to be consumed (or assets to be possessed) by one individual decision unit, and the crucial point in the constructions is the market mechanism, the distribution process, through which these tastes of the various decision units are supposed in harmony with each other to influence the productive activities. In Arrow's considerations it may very well be the case that the only relevant thing to each individual, the only basis for each R_i, is the bundle of goods and services that accrues just to him in the various alternatives or social states (the ≫individualistic assumptions≫). This wording seems to fit the Walrasian systems too — in them the only basis for a preference ranking is certainly the bundle of goods and services accruing just to the decision unit itself. What is then the difference between the Walrasian constructions with harmony between the various value schemes and Arrow's investigations,

[14] Arrow [2], p. 26.
[15] Arrow [2], p. 27.
[16] Arrow [2], p. 29.
[17] Arrow [2], p. 30.

which give as result the theorem showing the impossibility of harmonizing various individual value schemes to a consistent value scheme of society? The sharp difference lies in the fact that the Walrasian systems presuppose all issues between two or more decision units to be settled solely in the market place through the functioning of the particular atomistic market mechanism. Arrow's, theorem is not limited to an imaginary society where such mechanisms take care of all economic and social intercourse between the decision units; on the contrary, it applies in full generality to every type of collaboration and social decision making; moreover, it requires as the end product of the aggregation a consistent social value scheme as defined above to form the basis of ≫social rationality≫, and no such social ordering of the alternatives emerges from the Walrasian systems. The symbolism in Arrow's work does, however, admit of a wider content of an individual's attitude towards a social state than that given until now. We quote: ≫In the present study the objects of choice are social states. The most precise definition of a social state would be a complete description of the amount of each type of commodity in the hands of each individual, the amount of labor to be applied by each individual, the amount of each productive resource invested in each type of productive activity, and the amounts of various types of collective activity such as municipal services, diplomacy and its continuation by other means, and the erection of statues to famous men. It is assumed that each individual in the community has a definite ordering of all conceivable social states in terms of their desirability to him. It is not assumed here that an individual's attitude towards different social states is determined exclusively by the commodity bundles which accrue to his lot under each. It is simply assumed that the individual may order all social states by whatever standards he deems relevant.≫[18)]

When emphasizing in Chapter II, Section 1 that the programming systems presupposed one economic will, one value scheme to prevail in the society studied, we built partly on Arrow's theorem showing the impossibility of aggregating one social ordering from many individual orderings.

[18)] Arrow [2], p. 117. The quotations are reproduced by permission of Cowles Commission for Research In Economics.

b) KOOPMANS EFFICIENCY POINTS

Instead of introducing a ≫social welfare function≫, welfare analysis sometimes works with the following condition: ≫In a synthesis of welfare economics we can derive first a set of production conditions which require for their validity the weakest of all ethical assumptions: *simply that more of any one output, other commodities or services being constant, is desirable; similarly less input for the same outputs is desirable.*≫[19] At an Efficiency Point as defined by Koopmans in *A Mathematical Model of Production* (1949)[20] — comprising more activities than commodities —, it is impossible to increase the output of one commodity without decreasing the output of other commodities. Thus it is possible to rule out combinations that do not correspond to Efficiency Points using only the ≫ethical assumption≫ in the last quotation from Samuelson, and all interests could agree to adopt such a kind of assumption. Unfortunately, this ≫ethical assumption≫ is not sufficient for a program of social action to be formed; it precludes certain alternatives which ought to be desired by none but in general a variety of possibilities are left between which the assumption permits no choice, and there the problem of aggregation of value schemes arises again. The principle of Efficiency Points, in one sense, loosens the connection between the production aspect and the absolute unity of interests by stressing cases on which all decision units could agree notwithstanding divergent interests, but in doing this it weakens the production theory to a very considerable degree and leaves in general the decisive decisions undetermined.

≫The present model is open: it is designed to study the technical possibilities of production, while leaving unspecified which particular bill of goods actually meets the demand and how that demand is formed≫[21] As we mentioned in connection with the programming analyzes,[22] the introduction of Efficiency Points amounts to teleological assumptions going further than the full-utilization-of-resources assumption but reaching only part of the way to the assumption of best-utilization-of-resources relative to the set of objectives of a society.

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[19] Samuelson [6], p. 230. Reproduced by permission of Harvard University Press.

[20] Koopmans [24].

[21] Koopmans [24], p. 74.

[22] p. 32 above.

c) THE GENERAL GAMES

We have discussed two attempts to bridge over the gulf between the absolute unity of interests of the pure production aspect and the absolute diversity of interests of the pure distribution aspect, starting from the production side. We now turn to an attempt to bridge over the gulf, starting from the distribution side.

We begin by the following quotation from the ≫Theory of Games≫: ≫In games — as usually understood — the total proceeds are always zero; i.e. one participant can gain only what the others lose. Thus there is a pure problem of distribution — i.e. imputation — and absolutely none of increasing the total utility, the 'social product.' In all economic questions the latter problem arises as well but the question of imputation remains. Subsequently, we shall broaden the concept of a game by dropping the requirement of the total proceeds being zero [or a constant].≫[23] What is indicated here is clearly the desirability of proceeding from a pure distribution point of view towards some considerations of productivity, of utility and of ≫social product≫. The step is taken in the chapter on ≫General Non-zero-sum Games≫; one of the authors comments on the result: ≫Another point of peculiar interest to economists — thus far apparently having escaped attention — is the statement of the theory that when complete information prevails the social maximum benefit is always reached. This observation belongs to the wide and important field of welfare economics, i.e., to the attempts to determine whether something like a social maximum can be defined, whether it exists, and by what price system and income distribution it would be assured. In other words, it is the problem of best taxation, of economic justice, of socialism versus free enterprise, etc. It is also the problem of the measurement and the interpersonal comparison and transfer of utility, both matters that are treated to some extent in the theory of games. The literature on these topics is immense and the interest in welfare economics has never abated. Therefore, the result that according to the present state of the theory of games the social welfare is — under the stated condition — always at a maximum may sound as strange as the assertion that this is the best of all possible worlds though it may be better founded.≫[24] The result is supplemented by the following reservation in the ≫Theory of Games≫: ≫This result is not as sweeping as it may seem,

[23] von Neumann and Morgenstern [1], p. 34, footnote 2. Reproduced by permission of Princeton University Press.

[24] Morgenstern [28], p. 306. Reproduced by permission of A. Francke A.G.

since we are assuming a numerical and unrestrictedly transferable utility, as well as complete information.≫[25)]

The outlines of the construction are as follows. Above we have only spoken of Zero-sum Games with $\sum_{i=1}^{n} a_i = v(I) = 0$, and Constant-sum Games with $\sum_{i=1}^{n} a_i = v(I) = s = a$ a constant, that is games where only the determination of the individual's shares out of a given total amount is a problem and nothing can be gained through agreement and collaboration between *all* participants. There is no difference in any essential respect between the Zero-sum Games and the Constant-sum Games, because every element of the latter is Strategically Equivalent to an element of the former. This is easily concluded from our exposition of the concept of Strategical Equivalence in the previous chapter, because in the case of Constant-sum Games we may let the n parameters a_i^0, which we have at our disposal for the reduction procedure, obey, in addition to the $(n-1)$ conditions (29:D) as before, also the condition $\sum_{i=1}^{n} a_i^0 + s = 0$ instead of the condition $\sum_{i=1}^{n} a_i^0 = 0$. In the General Non-zero-sum Games $\sum_{i=1}^{n} a_i = v(I)$ is a variable quantity; what matters is no longer only the determination of the relative income shares of the decision units but also the fixation of the total level, of the total amount to be distributed. How is then this variable quantity determined? The new concept introduced to achieve a fixation of the variable total amount is that of the Fictitious Player. A General Game between participants is considered as a Zero-sum Game between $(n+1)$ participants, the $(n+1)$:th member being ≫Nature≫ or the Fictitious Player, which loses, the amount that the totality of real players gain. This auxiliary Zero sum $(n+1)$-person Game has of course

$$\sum_{i=1}^{n+1} a_i = v(I) = v((1, 2, \ldots, n, n+1)) = 0.$$

What distinguishes it from a zero-sum $(n+1)$-person game with solely non-fictitious players is the fact that all possible Solutions of such a game are not admitted, but only a particular discriminatory Solution,[26)] in which ≫Nature≫ is by necessity assigned the amount $v((n+1))$, that is the amount which the $(n+1)$:th participant can secure for himself acting in isolation from the other participants — although the rules in general give him better objective possibilities than this. In other words a_{n+1} is put equal to $v((n+1))$; the Fictitious Player is excluded from taking part in agreements with the other units and his ≫income share≫ is always pushed to its minimum, $a_{n+1} = \min a_{n+1} = v((n+1))$ — recall the definition of an Imputation

[25)] von Neumann and Morgenstern [1], p. 513, footnote 3.

[26)] Cf. Chapter V, Section 4 b: γ.

given above on p. 52. This means from another point of view that the sum of the shares of the other n (real) players is always pushed to its maximum: $\sum_{i=1}^{n} a_i = -a_{n+1}$, because of the Zero-sum character of the auxiliary $(n+1)$-person game, and hence

$$\min a_{n+1} = \min\left(-\sum_{i=1}^{n} a_i\right) = \max \sum_{i=1}^{n} a_i \quad [= v((1, 2, \ldots, n))].$$

Thus the maximum of what the first n players can achieve is in fact also always realized. This is the production problem; it being settled, the distribution problem, that is the determination of the $a_1, a_2, \ldots, a_n$ themselves, is solved by the ordinary tools of the theory, somewhat modified to suit the new situation.

The restriction imposed on the Fictitious Player amounts to a considerable simplification of the General n-person Games, that is the Zero-sum $(n+1)$-person Games with a Fictitious Player, in comparison with the unrestricted Zero-sum $(n+1)$-person Games; at the same time the General n-person Games are understandably more complicated than the Zero-sum n-person Games. In the following three lemmas we are going to mention certain difficulties which seem to be connected with the General Non-zero-sum Games and hence with the synthesis between the production aspect and the distribution aspect at which the construction aims.

In the first lemma we must say a few words about the special position of the Zero-sum two-person Games within the theory of games. In these games the three basic concepts discussed in the previous chapter are of no essential importance; the relevant concepts are instead those of strategies, mixed strategies, best strategies, pay-off-matrices, values of the game, etc., everything grouped around the minimax concept. We are not going to dwell on these constructions here but wish to underline the important distinction between the *independent rôle* and *the auxiliary rôle* of the Zero-sum two-person Games. For the first-mentioned the existence of individual utilities is sufficient. For the last-mentioned the existence of a common interindividual unit of utility seems necessary, not because of traits inherent in the Zero-sum two-person Games themselves but because of characteristics tied to the elements of theory which are the non-auxiliary ones. We have adhered to the opinion that the search for such a unit looks futile for the time being and we have interpreted the relevant constructions in terms of money, a procedure which is in accordance with some of the statements on this topic in the ≫Theory of Games≪. We first turn to the auxiliary rôle of the Zero-sum two person Games.

The construction for which the Zero-sum two-person Games play an auxiliary rôle is the Characteristic Function — the existence of its constituent parts, the values of the various ≫coalitions≫, that is the $v((i))$ and the $v(S)$ is proved by the help of the concepts of these games. We have pointed out that the formation not of the $v((i))$ but of the $v(S)$ seems to presuppose that everything is counted in Utopian comparable utility or in everyday money, and thus, as always, the generality of the whole construction cannot exceed that of the evidently weak link — in this respect the elements from the Zero-sum two-person Games are ≫strong≫ as they presuppose no interindividual counting unit, and the necessity of taking into account ≫coalitions≫ of individuals is the weak link. This necessity arises, of course, for all Essential Games but also for the Inessential ones — it must be demonstrated that nothing can be gained for anybody through agreements, and hence all $v(S)$ must be formable, although the result is that only the $v((i))$ are of importance. Thus when it is said that the Zero-sum two-person Games form a subset of the Inessential Games, the potentialities of the first-mentioned games are not exhausted by this statement. For the three-and-more-person Inessential Games the money-assumption seems necessary. If this restriction be imposed on a game of two participants we certainly get a two-person Inessential Game.[27)] The money assumption is, however, not implied by the two-person setups, which are more general.

The Zero-sum two-person Games in their independent rôle, or as may also be said the *minimax-problems,* have been elaborated in various directions. We wish here to mention the very interesting equivalence of a linear-programming problem, consisting of a maximization of an Objective Function under the technical restrictions, to a minimax problem of a Zero-sum two-person Game.[28)] In this application there is a struggle between ≫Man and Nature≫. We note the partial similarity in problem statement with the General Games, partly waged also between ≫Man and Nature≫. In the application there is no constant-sum assumption but the problem is to increase as far as possible entities in the same category as ≫total utility≫, ≫social product≫, etc., but only one economic unit is involved in the analysis.

In our second lemma we therefore ask what becomes of the General n-person Game, if $n = 1$.[29)] Then obviously a pure problem of production, of

[27)] In the light of this remark footnote 2 on p. 44 has to be correspondingly modified — the part of the theory of games referred to also includes the Zero-sum two-person Games, viewed as a subset of the Inessential Games.

[28)] A. W. Tucker, "Linear Programming and the Theory of Games" (abstract), *Econometrica* 1950, pp. 189–190.

[29)] i.e., a game between one real player and one Fictitious Player.

increasing total utility, as far as possible remains, the distribution problem being completely absent, as only one real player is left. Do we even in this case get a minimax problem such as in the elaborated form of the pure production problem, which is labeled linear programming analysis? The answer is that the General one-person Game is an ordinary maximum problem. This circumstance indicates that the construction of the Non-zero-sum General Games with ≫Nature≫ represented by the Fictitious Player, is not general enough to comprise the straightforward cases of the pure production aspect represented by the linear programming analyzes. Certainly a General *two*-person Game may firstly be reduced to a Zero-sum Game and secondly specialized to a minimax-problem of the kind desired, but then the rationale of the General Games, which is to combine production considerations with distribution considerations, would be abandoned; no way would be left open to synthesize this minimax problem with distribution considerations and the whole general-game-construction would be superfluous. At least this contrasting of the minimax problems of production connected with the programming systems and the rudimentary maximum problems of production, implied by the General Games, directs attention to a point which ought to be further clarified.

In our third lemma we point out that all difficulties in connection with the special features of the General Games would be absent if a Utopian transferable utility existed. Total utility would then first be maximized; after this maximization had taken place the resulting optimal total amount would be distributed. The situation would be exactly the same if only one product — ≫corn≫, ≫real national income≫ — were produced in the society, i.e., were rendered by the services of ≫Nature≫: the total amount of this commodity would first be maximized and then the optimal total amount would be distributed. Under both these assumptions neither Arrow's theorem, nor the minimax character of the pure production problems could cause any trouble, as a special one-commodity case would present itself, escaping the particular difficulties tied to the two constructions. A community with only one commodity is, to be sure, extremely difficult to visualize. No doubt an imaginary society of this kind would show differences in relation to our world of such a qualitative and deep-going character that inference from it to reality would be highly doubtful.[30)]

The possibilities of transferable utility and the similar situation, with only one product being disregarded, the General Games meet serious difficulties. If several products exist in the society, the question arises about the

[30)] Cf. Arrow [2], pp. 69–70. It should be remarked, however, that Arrow's general attitude towards the theory of games as expressed in Arrow [2], particularly on p. 19, footnote 16, is different from ours.

valuation on the part of the decision units in the society of these diversified services of ≫Nature≫. In this case Arrow's theorem in general precludes the building up of a social valuation from many individual value schemes; the existence of only one value scheme of society becomes necessary, and thus absolute unity of interests prevails in the community, making all theories of distribution between various group interests superfluous. In addition, the difficulty stressed in the previous lemma reappears.

Finally, how does the situation develop, if an interpretation in terms of money is strictly adhered to? It is stated in the *Theory of Games and Economic Behavior* that all conceptual difficulties may be avoided through such an interpretation. There are no difficulties in visualizing a sum of money being divided between the individuals of a society, especially if it be added that the thing which matters is not the absolute amount to be distributed but the relative shares of the participants. But how is the condition $\sum_{i=1}^{n} a_i = \max$ to be judged, implying that the members of a (closed) society through combined effort always obtain the highest money amount admitted by the objective possibilities to divide between themselves? What meaning can be attached to the maximization of the total sum of money in a (closed) society? It seems to us that the significance of such a procedure must be highly doubtful.

Our lemmas, especially the last one, but also the preceding two, serve to emphasize our opinion that the problems of ≫productivity≫ and of ≫utility≫ are so intimately connected that difficulties in interpretation appear for a construction, which attempts to solve the production problem without attacking at the same time the utility problem, that is, above all works with the assumption of transferable utility. Bearing particularly in mind Arrow's theorem and the well-known difficulties connected with the establishment of a common interindividual unit of utility we may go even further and ask: does it make sense to speak of maximizing social product, real national income, etc. if no reference be made to a common set if objectives, a common value scheme for the society? If such a common preference scale exists there can be essentially no diversity of interests at all, and the distribution process can constitute no problem. Certainly there exist empirical considerations of this kind which need a theoretical expression. But diversity of interests is without hesitation empirically present too; cases where the formation of the distribution process is the only serious problem are perhaps more frequent than the other extreme, and these facts also need some theoretical expression.

d) THE BASIC DUALISM

As a result of our investigation of attempts to bridge over the gulf between the pure production aspect and the pure distribution aspect — the first-mentioned being concerned with the maximization of social product relative to a common value scheme of society, the second-mentioned due to the lack of a common value scheme of society, being forced to disregard any net-gain of the conscious cooperation of all citizens[31] — we must conclude that for the time being the formal possibilities of a synthesis do not appear bright. There seems to be a very deep-going dualism between the two aspects, and apparently only a revolution in utility theory could succeed in the synthesizing task. On the one hand interesting hints are given towards such a revolution in the last chapter of the ≫Theory of Games≪, on the other, Arrow's theorem is a serious obstacle to surmount. Thus both fields, the production field and the distribution field, seem to give a very large scope for interior developments but the way of synthesis seems to be strewn with conceptual difficulties.

The attitudes towards such a dualism — its existence assumed to be agreed upon — may vary. One attitude is to remain waiting for a unified theory, and in the meantime adhere to the sort of synthesis given by the Walrasian systems, acknowledging their most sincere simplifications regarding the quasi-isolation of the decision units, etc., and trying to improve the systems by special assumptions differing from case to case. Another attitude is to await a unified theory, and in the meantime provisionally accept the dualism.

It may, however, be that the dualism is not only of a formal character, arising out of the ≫weed and tangle of our economic thinking≪: it may be that the difficulties of synthesis correspond to difficulties inherent in the empirical reality, i.e. in the economic and social action of man. In fact, some reasons can be given for such a standpoint. The greatest socio-economic problem of our time is the choice or perhaps the optimal mixture between a so-called free enterprise economy, where the economic questions are settled through agreements, cooperation and conflicts between many independent economic subjects and social groups, and a so-called planned economy, centrally controlled by the government. The economic development during the period of modern industrialism is no doubt, at a first approximation, the result of technological innovations acting through the medium of profit-maximization; it has not mainly occurred as a result of the fulfillment of a plan of society with certain stated objectives. This development has caused a huge increase of the indices of total production and standard of living, an increase intermittently interrupted by severe crises and depressions and

[31] Cf. p. 93, footnote 32.

influenced by great wars; but this does not imply that the development in any deeper meaning corresponds to the needs and wishes of Mankind; in fact, the opinions on this point are widely divergent.[32)]

A strong belief in the progressive capacity of the free technico-economical process is still deeply rooted; this line of thought may in its pure form be characterized by the motto ≫every value scheme of society is an illusion≫. During the last three decades the centrally planned economy has been sponsored as an alternative to the free group-economy, both in discussions and, more or less tentatively, in reality. The basic rationale of this attitude is a strong disbelief in the meaning of free technico-economical progression; this line of thought may in its pure form be characterized by the motto ≫one common value scheme of society is the only thing needed≫. The success in establishing a formal synthesis between the production aspect and the distribution aspect should certainly have deep-going consequences for the practical solution of this major contemporary economic problem of a free group-economy *vs.* a centrally planned economy — this is, e.g. hinted at in the statement by Morgenstern quoted earlier in this section.[33)] On the other hand, the success in establishing a practical solution of the problem should indicate that a formal synthesis could be obtained. What are the prospects for such a practical solution? It is very significant in this context that some social scientists tell us about a cyclic movement in the history of Mankind, the oscillations taking place between the centrally planned society and the free group society. If such a movement exists, it supports the standpoint that the dualism between the production aspect and the distribution aspect is inherent in the empirical reality. That particularly traditional econometrics often overlooks the basic difference between a centrally planned economy and a free group economy, has already been touched upon in Chapter I and will be further elaborated in the following chapter.

[32)] The problem of ≫economic progress≫ in a capitalist society, that is in a society which according to our opinion is essentially covered by the pure distribution aspect, will be discussed in Chapter VII, Section 4 : b: γ; cf. also p. 103 below.

[33)] p. 86 above.

2) Some earlier observations related to the dualism between the production aspect and the distribution aspect

a) VEBLEN'S DISTINCTION BETWEEN ECONOMIC AND PECUNIARY VALUES

The idea of a deep-going dualism between two spheres of economic theory may at first seen to be completely alien to economic thinking. In this section we shall, however, continue our investigation of the dualism between the production aspect and the distribution aspect by inquiring briefly int some apparently related ideas held by three authors, namely Veblen, Myrdal and Johan Åkerman.

The problem in Veblen's writings to which we wish to call attention may be labeled ≫the theory of economic and pecuniary values≫, a fundamental point for his constructions.[1)]

Two types of values are according to Veblen created by the economic activities in a society, economic values and pecuniary values. This very idea of two types of economic values is fairly common in economics — we may only think of the familiar distinctions between real values and monetary values and similar setups. How then are the two types of values characterized by Veblen?

The *economic values* are created by industry, and are real physical products and commodities, which possess ≫usefulness≫. No doubt Veblen meets serious difficulties in defining this last-mentioned concept. It is not the question of being useful only to an individual, because an individual may satisfy his needs at the expense of his fellow-citizens: the usefulness must be of a social character. ≫A socially useful product must meet the 'test of impersonal usefulness — usefulness as seen from the point of view of the generically human.' The test is whether the product 'serves directly to enhance human life on the whole — whether it furthers the life process taken impersonally.'≫[2)] The individuals in a society best fitted to judge questions of social usefulness are the engineers, the technical experts, who are decisively governed by ≫the instinct of workmanship≫. In line with this technocratic attitude Veblen states that social usefulness has a ≫substantial foundation in material circumstances reducible to objective terms of mechanical, chemical and physiological effect≫.[3)] On this bias given to the conc-

[1)] We refer particularly to the exposition given in Gruchy [17], pp. 105–115.

[2)] Gruchy [17], p. 106. This quotation and the following ones in the subsection are reproduced by permission of Prentice-Hall, Inc. (Copyright 1947).

[3)] Gruchy [17], p. 108.

ept of social usefulness Gruchy comments : ≫[Veblen] is fully aware that there are other kinds of human needs beyond the material needs and that therefore there are other kinds of serviceability beside economic or material serviceability which are of interest to the community. But Veblen has no special interest in the satisfaction of the non-material needs of mankind. He appears to adopt the position that economic serviceability is basic to all other kinds of serviceability, and that once the community was well supplied with economic values or useful material products the satisfaction of the non-economic needs of humanity would then be a relatively simple task.≪[3)] In our somewhat divergent opinion the importance of Veblen's concept of economic values possessing social usefulness lies in the fact that he does not take refuge in referring to quasi-isolated individuals tastes which are always satisfied as far as possible through the working of some harmony mechanism, but instead he clearly sees that the participants in a society are so closely dependent on each other that ≫usefulness≪ has a distinct social aspect; the weak thing with this concept of Veblen is the fact that he tends to overlook the need of a value scheme even for establishing social usefulness, or more adequately expressed that he tends to overlook the possibility of quite other social value schemes than the rather ≫materialistic≪ one proposed by him. The technocratic movement, strongly inspired by Veblen, and to some degree even the Keynesian philosophy which is probably influenced by technocraticism, seem to share both the merits and the demerits of Veblen's concept of economic values.

The *pecuniary values* are created by business. They are the exchange values, the prices tied to physical goods and services, which we observe in the markets. They may, but need not necessarily, be directly determined from some considerations of social usefulness; instead they may reflect the attempts of the various decision units to secure highest possible monetary gains to themselves, and be influenced by numerous factors like power, monopoly, bargaining ability, mass psychology, everything directed towards the concept of ≫vendibility≪. Certainly this type of values is easier to define than the preceding one.

After having indicated what characterizes the two types of values we now come to the really interesting phase of Veblen's distinction. We quote Gruchy: ≫At certain times in the evolution of the economic system the streams of economic and pecuniary values have run close together. There has been such a high degree of coincidence between the two types of values that it could be said that they were harmoniously related. In such a situation pecuniary values reflect, or are rough measures of, economic values. At other times in the development of the economic system the streams of these two types of values have moved away from each other so that discrepancies between the two types of values have made their appearance.

In this circumstance pecuniary values are no longer adequate measures of economic values. Out of this lack of coincidence between pecuniary and economic values have come the basic economic issues that now confront society.≫[4)]

When are the economic and the pecuniary values ≫constitutionally in touch with each other≫? Veblen refers to the pre-industrial-revolution society of handicraft industry when ≫remuneration was a function of productivity; and productivity could be measured in terms of either economic or pecuniary values≫,[5)] but also to the future vision of a technocratic society with a clearly communistic imprint. It signifies that the existing, observable market values are determined chiefly so as to conform to a value scheme of society. At this juncture we recollect what was said above in our section on the programming analyzes about their still only incompletely explored one-to-one correspondence between real quantities and monetary prices. We particularly stressed that the prices did not play an independent rôle, that they were of an auxiliary or a ≫shadow≫ character: as soon as the set of quantities was determined, a corresponding set of prices was also implicitly given; the correspondence worked, however, even the other way, and as a means of reaching a given objective involving a set of real quantities the corresponding set of prices could first be established — this is said with the reservations stated loc. cit.[6)] Certainly in the society imagined by the programming analyzes the quantities and the prices, or let us say, the real values emanating from the Objective Function and the monetary values embodied in the prices, are ≫constitutionally in touch with each other≫, both types of values being based on the set of objectives of society. We repeat that the basic condition for such a society to exist seems to be the unity of interests, the similarity of value schemes, the presence of a common goal of society. Probably the days of the handicraft-society and the days of a Utopian communistic society referred to by Veblen fulfill this condition to a significant degree.

When are the economic and the pecuniary values of commodities, of labor, of capital ≫constitutionally out of touch with each other≫? Veblen considers this state of things to be characteristic for the whole period of modern industrial revolutions. It means that the existing, observable market values are not determined in order to conform to the value scheme of society visualized by Veblen or — as we think it permissible to add — any consistent value scheme of society whatsoever, because even the approximation of such a social value scheme does not in general exist during the

[4)] Gruchy [17], p. 106.
[5)] Gruchy [17], p. 109.
[6)] pp. 31 above.

period, but they are essentially determined by the numerous factors already referred to under the headlines of power, monopoly, bargaining ability, etc. As we see it the words ≫the lack of coincidence between pecuniary and economic values≫ ought to be interpreted as alluding to the lack of a common socio-economical goal, and ≫the basic economic issues that now confront society≫ which have arisen out of this lack ought to be put into connection with the working of the pecuniary values in their ≫independent≫ rôle, i.e., out of contact with a social value scheme. The society imagined by Veblen where the more ephemeral economic and the observable pecuniary values reflect each other we thus consider to lie remarkably close to a society where according to our terminology the production aspect is of main importance; a society where the economic and the pecuniary values diverge from each other we consider to resemble a society where the distribution aspect is dominating and the considerations of the production aspect play a minor rôle. We finally add our opinion that the observations made by Veblen and expressed in his theory of economic and pecuniary values must be completely uncongenial to an economist who tries to see reality through the Walrasian spectacles.

b) MYRDAL'S DISTINCTION BETWEEN POLITICAL ECONOMY AND ECONOMIC TECHNOLOGY

In his book on *Science and Politics in Economics* (1930)[7], Myrdal criticizes sharply and at length the *political economy* centered around the following three principal constructions: the ≫value ideas≫ of classical or neoclassical origin, the ≫free-trade idea≫, and the ≫social economy idea≫, that is the line of thought which, assuming the existence of many decision units in a society, still implicitly studies this as a big enterprise, as a unified government economy; the author considers this last-mentioned idea to be the most basic one, within the confines of which the two preceding ideas are developed; a research worker accepting the idea is said to submit himself to the ≫communistic fiction≫.

As a substitute for the political economy, Myrdal mentions some desiderata for an *economic technology,* abandoning the three principal constructions and taking into account the diversity of interests of various groups in a society, the changing institutional basis of price formation, etc. We only note the point here, and stress our opinion that the desiderata expressed for an economic technology are remarkably well met by the game, distribution theory — the economic technology sponsored by Myrdal ought clearly to be put under the headline of the distribution aspect.

[7] Myrdal [29] .

What about the other alternative, the political economy submitted to a ≫communistic fiction≫ and criticized by Myrdal? There is one significant difference between most of the works belonging to this category in Myrdal's book and the works which we explicitly refer to the pure production aspect: both treat society as a unified organization, but the latter openly contain an Objective Function or a similar entity which embodies value judgments, while the former try by every means to disguise all traits of a value judgment, hiding away the fact that the construction presupposes absolute unity of interests in the society imagined. Thus the latter may be as communistic as the former, but they are not so in a fictitious way. In comparing Veblen's distinction between economic and pecuniary values we find that the works criticized by Myrdal no doubt belong to a category of theories which show no discrepancy between a quantity side and a price side; the economic technology outlined by Myrdal precisely takes into consideration the forces mentioned by Veblen as governing the pecuniary values in a society where these are out of contact with any economic values.

Thus we emphasize in Myrdal's very important book the presence of a production aspect and a distribution aspect as clearly distinct entities, forming a contrast. He comes, however, unlike Veblen, close to denying the significance of the first aspect altogether.

c) JOHAN ÅKERMAN'S DISTINCTION BETWEEN THE CENTRALIZED ECONOMY AND THE FREE GROUP ECONOMY

In order to reach the problem group in Johan Åkerman's writings, which we consider to be related to the dualism between the production aspect and the distribution aspect, we begin by asking with the author: is it possible to sum individual micro-plans to a macro-plan for society?[8], or put in a somewhat more special form: ≫is it possible to sum individual anticipations to anticipations of society?≫[9] Johan Åkerman draws the conclusion that it is impossible to sum ≫micro-plans≫ to a ≫macro-plan≫[10]. As a first consequence of this methodological standpoint may be considered his statement that the course of economic events in some countries after 1920–1930 must essentially be viewed as the fulfillment of one ≫macro-plan≫, one government plan for society; this way of grasping the course of events is, however, basically different from the way to be used for the free enterprise economy of, we might say, the preceding one hundred years. Thus far the analysis is in principle in good accordance with a Veblenian harmony between economic and pecuniary values, with Myrdal's opinion

[8] Johan Åkerman [14], I, pp. 248–249.
[9] Johan Åkerman [14], II, p. 598.
[10] Cf. Arrow's theorem.

about the theory of a communistic state and with the pure production aspect. When it comes to the study of a society where free enterprise is important, more particularly to a study of business cycles, constructions are applied which definitely do not consider the course of events only as the fulfillment of one macro-plan; nor do they, partly as a second consequence of the basic methodological standpoint, view the socio-economic phenomena as resultants from some aggregation of microplans. The constructions are summed up in the two complementary, antinomistic explanations of the business cycle, called the α-thesis and the β-thesis. It is not our intention here to go into these explanations. We only mention that the α-thesis deals with the concepts of structure and group, which we have already touched upon in our previous chapter, and which clearly express ideas concerning diversity of interests and relative income-shares. Also the β-thesis, we would add, consists of distribution-considerations, regarding as decisive for the analysis partly the data giving the profitability of the individual enterprises, partly the data giving the opinions of the banks on the state of profitability in large sections of the economy.

In this section we wish to point out certain ideas related to the dualism between the production aspect and the distribution aspect. We have for this purpose mentioned Veblen's idea of a society where the observable ≫pecuniary values≫ reflect more basic ≫economic values≫ *vs.* a society where the ≫pecuniary values≫ mainly play an independent rôle; Myrdal's idea of the normative ≫political economy≫ constructions which he wants almost completely to cancel in favor of an ≫economic technology≫; and Johan Åkerman's idea of the basically different theory constructions needed for an economy where the government is dominating and an economy where free enterprise is of basic importance.

3) Economics, politics and rationality

The economic researches centered around the Marshallian and the Walrasian systems are usually said to be purely economic as distinguished from certain sociological studies which deal with society from more volatile political aspects. The opinion that the theoretical systems referred to represent ≫pure economics≫ seems to be highly justified. The basic economic activities depicted by the systems and believed to imply ≫rational economic behavior≫ are ≫choice activities≫, and the final consumers choices of commodities and assets influence the productive agencies through an automatic and harmonious distribution or pricing process. The complete absence of political factors from the constructions is most clearly seen from the rudimentary character of their distribution aspect, which has its origin in the quasi-isolation of the various decision units assumed. Thus

the concept of rationality in traditional economics is closely tied to the notion of ordering,[1] each consumer being able to order various bundles of commodities and striving to choose the most desirable alternative which the restrictions permit. Among the restrictions, however, are found the prices, and nothing is said about any rational attempt of the units to influence prices or the scheme of distribution, e.g. with more or less ≫political≫ means.

Now such a separation of ≫the economic sphere≫ from ≫the political sphere≫ is probably a most artificial trick, far from the reality to be grasped by the theories. As soon as the assumption of quasi-isolation of the various decision units in a society is removed, grave difficulties appear in defining ≫rational economic behavior≫. If we adhere solely to the production aspect the expression has a good sense, as these systems set up certain objectives to be fulfilled in the optimal way, the procedure involving a maximum or rather a minimax problem. An Objective Function to be maximized under certain technical restrictions and belonging to the same theoretical category as a social welfare function, seems to embody the traditional notion of rationality as an ordering of the various alternatives, the most desired alternatives permitted by the restrictions being chosen. The crux exists, however, that these constructions presuppose absolute unity of interests in the society imagined, and thus we can say that in this case the possibility of defining rational economic behavior rests upon a basic assumption — that about the absolute unity of interests — which comes close to what is usually considered a political element.

If more decision units with divergent interests be introduced *and* if the difficulties with transferable utility and other things, mentioned in our section on attempts towards a synthesis between the production aspect and the distribution aspect, be disregarded — reducing society to comprise only one type of — commodity the General Games give a unified definition of ≫rational behavior≫, but no longer solely of ≫rational economic behavior≫. The General Games have also a distribution side and comprise the concepts of Solution and Imputation, involving social activities — ≫negotiation activities≫ — which everybody would hesitate to label purely economical. It may further be remarked that the ≫rational behavior≫ defined by the General Games is radically different from the ≫rational economic behavior≫ of Marshal and Walras also in the respect that the result of the ≫rational behavior≫ of one decision unit is in general very much dependent on what the other units do. This is of course also valid for the special case in which a General Game reduces to a Constant-sum Game, that is to the pure distribution aspect.

[1] In the technical sense of the word introduced on p. 45 above.

If the difficulties of synthesis be recognized and consequently the distribution aspect be interpreted solely in terms of money, corresponding difficulties occur to a unified definition of rationality. Within the pure distribution aspect as expounded in the Zero or Constant-sum Games, rational behavior of one unit consists in getting as big as possible a relative share of the total monetary income of society; it is connected with elements of habit (Solutions, i.e. accepted standards of behavior)[2] and elements of power (Imputations). Thus rationality has one meaning within the pure production aspect, being tied to the notion of ordering of alternatives involving many different commodities, etc., and quite another meaning within the pure distribution aspect, being tied to the notions of Solutions and Imputations.

This dilemma seems to have its counterpart in the following not uncommon factual situation. A certain economico-political measure is discussed; to fix our ideas, let us say, an interest-rate question. The economist, quite possibly in an implicit way, reasons as if a complete unity of interests prevailed in the society and a common set of objectives for society existed, and gives his advice from these considerations. Let us suppose that his reasoning from the assumptions is correct. It may now happen that his recommendations are called ≫unrealistic≫ and politically not feasible, and that some interest-groups in the society with enough combined strength settle the matter in quite another way to that proposed by the economist. Are we here facing an example of how political considerations cause a loss to society through their impact on economic decisions, and can the political leaders be said to be wrong? This need not be the case at all. The political leaders may, possibly in a more or less implicit way, reason as if no common value scheme of society existed, and instead see the question from the distribution side and — rightly — conclude that the advice given by the economist would, if followed, decrease the relative income shares of their politically dominating groups. An issue of this kind is not always settled by the argument: the economist looks at the common good, the political leaders favor group-interests; for it is difficult to see how the common good — this worn phrase — could be defined if a set of objectives of society do not exist, implying absolute unity of interests, and this may not be and certainly is often not the case. A good illustration of the dilemma referred to would be a study of the development of social accounting and budgeting in a number of countries after the Second World War. The relation of these

[2] In this context it may be of interest to compare the criticism delivered against the institutionalists for their stressing the importance of ≫customary behavior≫ as opposed to ≫ rational behavior≫ of traditionalism. Cf. further Chapter V, Section 4: b: α.

pieces of research to the Leontiefian system and the programming analyzes is quite evident and recognized, but they are no doubt less explicit in various respects. They treat society, however, as a big enterprise and hence disregard the distribution aspect. In some countries the social budgeting analyzes may never have been of crucial importance (USA?): in other countries the belief placed in them after the war has diminished subsequently and (distribution-)problems have appeared, which fall outside their scope (Sweden, compare, e.g. the quotation from Lundberg below); in still other countries they seem to be of lasting importance (Norway?). Our opinion on the dualism would be stressed, if the result of the study turned out to be that the importance of the social budgeting analyzes were a function of the political structure of the countries, where in this particular case we mean by political structure the degree in which the government dominates the country, that is the degree in which one common value scheme and its resultants — absolute unity of interests and one common consistent plan of society — are enforced on or accepted by the community; and of course we imply such a ≫form≫ of the function that the more the unity of interests prevails the more the social budgeting signifies.

A very good exemplification of the dilemma between what we call the two kinds of rationality is given by Erik Lundberg: ≫[In a free democracy] the concentration of interest naturally tends towards the questions of inco_me distribution on the cost of positive planning. I like to draw a parallel with the development of planning in Sweden during the postwar years.

One of the main problems with regard to the future development of the Swedish economy refers to the stagnation in population growth. There will hardly be any increase for twenty years in our labor force. That means great difficulty for a capitalist system to work. One of the ways to solve that problem is to get labor from agriculture where there are certain reserves. An agricultural planning commission has been working for four years with thirty members.

The economists in that commission made a dynamic plan for agriculture put into a framework of the general Swedish employment development. The solution implied a continuous rationalization of agriculture during a period of a generation without danger for our food situation. We could calculate the marginal cost of having more of agriculture than according to that plan, that is to say the real price for security in food supply. But when the politicians got hold of the plan their main interest was the equalization of income between agriculture and the other parts of the population. I will by this example only remind you of the risk that politicians in a free society will be much more interested in inco me distribution than in long-term planning because of the fact that they to a large extent represent different pressure groups. And this of course is very disturbing for long-term

planning. My conclusion it that in order not only to have good plans made up by experts in different fields and consistent planning for the economy as a whole but also will and power to make the plans work, we have either to have an enlightened dictatorship, if I may say so, or a people very united about the aims of economic policy.

If you have a free society, then you can have people united about the aims of economic policy on the condition that these aims seem predominating to all of us. That was the situation after the war in countries with large rebuilding and reconstruction problems such as Poland and the Netherlands, as Professor Tinbergen said. Then you can get a general understanding of and common interest in the plans. In Sweden the needs have not been urgent to the same extent and therefore we get too much of the income- (and power-)distribution problems about which people cannot easily agree, with much room for politicians to play in. However, the Swedish economic policy has not been able to save Sweden from the dollar crisis (partly just because of its emphasis on the income-distribution aspects). Although we may regret it the new claiming difficulties, breeding urgent, needs, will perhaps make better prospects for efficient planning in the future≫.[3)]

Before concluding our argument in this section two minor points may be observed. *First,* we emphasize that in speaking above of, a ≫society≫ or a ≫country≫ we have thought of a closed society, as is by necessity the world economy. If an international view be taken on the production aspect and the distribution aspect it seems clear that the lack of a common value scheme of the world society is still much more apparent than it may be in a national case, perhaps to such an extent that the treatment in an international context of the various countries as unified organizations would be justified, although in many cases it is, e.g. probable that the interest group constellations cut the national boundaries. Not treating the various countries as unified organizations and looking on the world economy from the distribution aspect it seems, however, possible that *all* economic subjects in a country may gain through agreement and cooperation with each other and this may give a meaning to the term ≫the common good≫ without implying the existence of a common value scheme. *Second,* it may be asked: how can a society progress from any point of view if the distribution aspect be dominating and no value schemes, in traditional opinion consumers' value schemes, guide the production? An answer seems to lie in

[3)] E. E. Lundberg, Discussion of Oscar Lange: "The Practice of Economic Planning and the Optimum Allocation of Resources," *Econometrica* 1949, Supplement, pp. 175–176. Reproduced by permission of the Econometric Society.

the rôle of technological innovations and the resulting new investments. We shall return to the topic in Chapter VII and we only stress here the opinion that a boom of new investments may rightly be inspired by monetary profitability considerations, particularly because it may basically change the objective possibilities which the various decision units are facing in the distribution process, favoring relatively some groups; or from another point of view: the non-participating of some group in an investment boom may certainly distinguish the objective possibilities of this group in the distribution struggle. Thus the increase in quantities and standard of living may be more a by-product than an ultimate aim.

In the present state of economic theory economists should advise politicians and business men less and study politicians and business men more — this is our main and concluding argument in this section on economics, politics and rationality. Probably the position of the economist for the time being is best in a centrally planned economy, depending on the fact that economic theory has paid the bulk of its attention to the production aspect — none the less the contemporary economic theory — although this is often made in a somewhat disguised way. Notwithstanding this fact it would probably be wrong to conclude that the theory of the centrally planned economy is so developed that it is of no use for an economist to study the practical attempts, that is the attempts made above all by non-economists, to manage such a society.

When it comes to a society where diversity of interests is prevailing and the distribution problems are of primary importance the economist knows much less about the economic mechanism, and his theory is decisively still of so weak a character that an intensified study of how business men and politicians experiment with the problems that occur, treat inflations, crises, depressions, etc. — for which phenomena the distribution aspect seems to be dominating, as we hope to be able to indicate in Chapters V–VII — would certainly further the theory and bring that point closer, which every good theory must pass: the point where the theoretician can show himself to be superior to the most skilled practicians. The fact that the problems just mentioned: inflations, crises, depressions, often, no doubt, have been settled by the economically and politically acting individuals — although this may have taken far too long a time, and have been made in a defective way — indicates, strongly, that there is in general a solution to such problems which can be theoretically rationalized and made easier to grasp, more rapid to apply and more efficient to carry out.

When it finally comes to the problem of choosing a centrally planned state or a free group economy or some mixture of the two extremes, involving the problem of a synthesis between the production aspect and the distribution aspect and the connected topics on the nature of utility, the

economist knows almost nothing. Some economists expect much from the future in this respect: ≫Once a fuller understanding of economic behavior has been achieved with the aid of a theory which makes use of [measurable utility], the life of the individual might be materially affected≫[4] Other economists may point to facts of the type already exemplified by the hypothetical cyclical movement in history between government-regulated societies and free group economies, indicating that the practical experiments of Mankind on this vital topic have not given an approximate solution; and thus they may judge the problems involved with greater pessimism.

4) Summary

a) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS THEMSELVES AND BETWEEN THESE THEOREMS AND OTHER ECONOMIC THEORIES

The Walrasian systems present a synthesis between a production aspect: many coexistent value schemes guide, without conflicts between each other, the production of commodities and services — and a distribution aspect: the equilibrium conditions and related constructions depict a pricing process which is automatic and harmonious. The weak point in the constructions is constituted by the equilibrium conditions. These form a sufficient condition for the many value schemes to exist without any contradictions occurring; the present chapter first explores the topic whether this condition be also necessary, i.e. whether many value schemes can coexist under other forms of the distribution process than the peculiar automatic and harmonious one.

The analyzes belonging to the pure production aspect give no evidence that this is possible. They comprise one value scheme and no traits of any distribution process between independent units, thus seemingly depicting an absolute unity of interests. They are logically developed by deleting the doubtful pricing process from a Walrasian system, substituting balance identities for the equilibrium conditions; the resulting construction is seen to require the establishment of one explicit value scheme, lest most rigorous linearity assumptions be made. If a certain diversity of interests could in general be introduced into this firmly underbuilt type of analysis a door would be open for the possible inclusion of distribution considerations into the production analyzes, not suffering from the defects and opacities of the Walrasian distribution processes but taking into account explicit agreements between the decision units of society. In this way a synthesis

[4] von Neumann and Morgenstern [1], p. 20.

between the pure production aspect and the pure distribution aspect could eventually be reached.

The second summation theorem, Arrow's theorem, goes very far, however towards preventing the inclusion of any diversity of interests into the pure production aspect. It proves the general impossibility of building up a consistent social complete ordering of various alternatives — involving commodities and services — from many consistent individual complete orderings of the alternatives; i.e. the impossibility in general of aggregating many individual value schemes to a value scheme of society, for instance, to an Objective Function of a programming analysis or to a social welfare function. Thus strong evidence is found that the peculiar pricing process of a Walrasian construction is a necessary condition for many value schemes to coexist without conflicts — in other words that this coexistence is possible under no more general distribution processes than the automatic and harmonious ones. It follows that no prospects tor a synthesis between the production aspect and the distribution aspect are seen from this angle — beyond the Walrasian synthesis.

The analyzes belonging to the pure distribution aspect comprise no value schemes, being concerned only with the relative income shares of the various units and thus seemingly depicting an absolute diversity of interests. They are logically developed through abandoning the additivity characteristic of the Walrasian pricing processes, similar to those of the Inessential Games. The present chapter explores in the second place the topic whether certain value judgments may be introduced into the pure distribution aspect. If also the total income shares could be made of some importance in this firmly underbuilt type of analysis, i.e. if considerations of productivity and utility could be introduced, a way would emerge for weakening the diversity of interests prevailing in the constructions, admitting a positive value to arise from collaboration between all units of society. In this way a synthesis between the pure production aspect and the pure distribution aspect could eventually be reached.

The General Games attempt to extend the analysis in this direction. If Utopian transferable utility existed, the extension would meet no difficulties: total utility would then first be maximized; after this maximization had taken place the resulting total amount would be distributed. An exactly similar procedure could be applied if only one commodity were produced, i.e. rendered by the services of ≫Nature≫. The production of this would first be maximized and then the resulting total amount would be distributed. If these possibilities be disregarded, however, the extension meets serious difficulties. If more than one commodity be produced, the question arises about the valuation on the part of the decision units in the society of these diversified services of ≫Nature≫. In this case Arrow's theorem in

general precludes a social valuation being built up from individual value schemes and necessitates the existence of only one value scheme of society, thereby introducing into the community absolute unity of interests and making every theory of distribution between independent units superfluous. If the whole setup be interpreted in terms of money, a meaning could hardly be given to anything but the relative income shares of the decision units, and not to the maximization of the total sum of money in a (closed) society. It follows that the prospects for a synthesis between the production aspect and the distribution aspect look poor from this angle.

Thus we have provisionally to face a basic dualism in economic theory between a production aspect, dealing with the production of goods and services guided by one common value scheme of society, and a distribution aspect, counting in money and concerned with the fixation of the relative income shares of the decision units in a society; the first type of analysis is exemplified by the programming systems, the second type by the Zero or Constant-sum Essential Games. Such a dualism may at first seem very alien to economic thinking; we mention, however, three distinctions somewhat reminiscent of the dualism, namely Veblen's distinction between economic and pecuniary values, Myrdal's distinction between political economy and economic technology, and Johan Åkerman's emphasis on the different types of analysis required for studying a centrally planned economy and for exploring a free group-economy.

b) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS AND EMPIRICAL RESEARCH

The first summation theorem requires that economic values be tied to sets of economic subjects; the second summation theorem in general precludes the aggregation of individual consistent value schemes to a consistent value scheme for a set of individuals. We have provisionally proposed to ≫solve≫ this contradiction or paradox by accepting the basic dualism between the production aspect and the distribution aspect. It is suggested that the theoretical difficulties of synthesis between the two aspects may have an empirical counterpart in the difficulties of choice between a centrally planned economy and a free group-economy, or in the difficulties of establishing a stable mixture between the two extremes, experienced in history. This is closely related to the problem of economic planning in a free democracy, lacking a common value scheme of society. In this context the interesting question arises as to how economic ≫progress≫ is achieved in a free group-economy, having no resort to a social plan, based on a social value scheme.

Chapter IV

The Fundamental Problem

1) The discrepancy in traditional econometrics

a) THE BALANCE IDENTITIES AND THE EQUILIBRIUM CONDITIONS

In this chapter we intend to take up the main threads of Chapter I, utilizing our analysis in Chapters II and III. *In one inclusive theory, universally valid up to changes in structural coefficients, econometrics wants to realize three objects:* an explanation of the course of events, an instrument of decision for the government and an account of the behavior of the various non-governmental decision units or social groups in the society studied, preferably of their ≫rational behavior≫. The emphasis of the investigations is usually put on the ≫behavior equations≫. In Chapter I we mentioned a few further types of equations and ended up by a consideration of the connecting links between the various other relations in a system of simultaneous economic equations, the balance identities and the equilibrium conditions. These two relation-types are sometimes mixed up with each other, sometimes freely substituted for each other without altering the interpretation of the theory-construction and almost always taken to be self-evident, raised above every discussion of their empirical significance; in some of the rare cases where such a discussion is tried a certain hesitation about their significance is observable. We expressed the opinion that a clarification of the significance of the balance identities and the equilibrium conditions for an economic theory-construction would contribute towards a better understanding of the implications of the econometric systems.

In various exact socio-economic thought-experiments of Chapters II and III both balance-identities and equilibrium conditions are encountered. As a result of our investigations in this respect we shall now argue that the econometric systems, usually considered to be of a unified character, embody two tendencies which are irreconcilable, at least on the level of analysis involved. The first tendency, tied to the balance identities, leads, if followed up, to planning models — instruments of decision for a government in a centrally planned state, and the same time descriptive of the course of events in such a region. The second tendency, tied to the equilibrium conditions, leads, if pursued some way, to a harmony construction à la

Walras, depicting the (utility maximizing) economic behavior of quasi-isolated decision units, a construction in which a place for a government is difficult to find and the significance of which for an explanation of the course of events in any region is highly doubtful, due most apparently to the rudimentary distribution theory involved. Often both these tendencies are found in the very same system. Obviously this coincidence cannot be labeled an attempt towards a synthesis between the pure production aspect and the pure distribution aspect; it is instead a mixture of the pure production aspect and of the particular Walrasian synthesis between a production aspect and a distribution aspect, rudimentary in various ways. It is probably clear from what has already been said that we consider the econometric systems embodying the first-mentioned tendency to be of utmost importance — the empirical applications of the Leontiefian systems are representatives of this category, as are in a less pronounced manner the social accounting systems —, that we look at the empirical significance of the econometric systems embodying the second tendency with doubts, and adopt this attitude to a still more pronounced degree towards mixtures of the two tendencies.

Our argument will be carried out in two steps. Firstly, we are going to summarize — from the point of view of the econometric systems — our findings concerning balance identities and equilibrium conditions in the two preceding chapters, secondly, we are going to call attention to the transformation of Keynes ≫from Quesnay to Walras≫, of which the Keynesian literature of the 1930s and the 1940s gives evidence — the justification of the second step is immediately found in the well recognized crucial importance of the Keynesian systems for econometrics.

It is possible to express the difference between the balance identities and the equilibrium conditions in a simple way. The former are valid by assumption — the equality of supply and demand is assumed by the theory; the latter are satisfied through the working of some mechanism which has to be described by the theory — the equalization of supply and demand has to be evidenced by the theory. During our investigations of the elaborate socio-economic thought-experiments we first met with *balance identities* in the relatively simple Leontiefian system. Each balance identity depicts the sum of the requirements of all activities for a single commodity and states the equality between this sum and the amount of the commodity produced; that is the rate at which commodity i flows from activity I is equal to the rate of input of i to all activities in the system. Thus the combined system of equations expresses a full-utilization-of-resources assumption, this assumption being identical to the assumption of equality between supply and demand or output and input. The assumption is further the only one that has a certain teleological flavor, the other assumptions

being of a completely ≫mechanical≫ character. The question then arises: who achieves this equality, this full utilization of resources? The system gave no answer to this question; especially it was found that no prices play a rôle in this respect. On the contrary the fixation of prices is based on the very assumption of equality, the equality not holding, the prices not being defined; further the prices are of no decisive importance but only of a fictitious or shadow character. We then discussed extensions of the Leontiefian system in various respects. We observed firstly that the balance identities are retained in these still more elaborate systems and that they keep their function as embodying an assumption of full utilization of resources; secondly that the teleological element of the systems is widened, until finally the programming systems are reached, which quite explicitly introduce one economic subject, not only striving at full utilization of resources but even at the best utilization of resources implied by its value scheme, which is formalized in the Objective Function. This construction once being pointed out, it seems to us that there ought to be no hesitation in acknowledging it as the true logical extension of the introductory, relatively simple Leontiefian balance-identity systems. We labeled it the pure production aspect and emphasized its obvious and, in a certain sense as it seems, exhaustive importance for the theory of a centrally planned economy. Of special significance is the fact that no ≫behavior equations≫ at all, expressing the ≫rational≫ or ≫actual≫ behavior of various decision units, are found in constructions of this type, but the economic behavior of one single economic subject is described by the whole edifice.

In order further to elucidate the connection between the Leontiefian system and the programming analyzes proper, we introduced the distinction between the real and the virtual parts of the latter. The whole construction comprises more activities than commodities, i.e. several alternative ways of producing each commodity. The task of the planning unit is to choose from among these activities precisely that set of activities which maximizes the objectives. The choice being performed, certain activities are retained at positive levels, forming the real part of the programming system; the remaining activities, which were not chosen due to the particular shape of the Objective Function, belong to the virtual part of the system, existing only in the calculations of the planning unit and being not materialized; it seems convenient also to include the Objective Function in the virtual part. The real part of the system is the plan actually carried out; thus the real part also depicts the actual course of events in a centrally planned economy, disregarding some types of disturbances which may occur. In our opinion a Leontiefian system should be interpreted as the real part of some programming system, showing explicitly only a full-utilization-of-resources

assumption, but the whole system being based on a best-utilization-of-resources assumption which can only be explicitly formulated taking into account the virtual part. In case this interpretation be not adhered to, a most serious problem presents itself concerning the significance of the linear relations, stating that a commodity can only be produced in one fixed way; this problem is in the center of most criticisms leveled at Leontief's system. Our way of interpreting the setup gives a justification for at least one important aspect of the linearity involved by regarding the system as a specific part — ≫the real part≫ — of a wider system, the wider system not being based on the extremely restrictive linearity or fixed-way-of-production assumption.

We first met with *equilibrium conditions* in Mosak's system. They express the equalization of supply and demand for each commodity through the working of the price mechanism. We referred to the explicitly dynamic statements of this theory of equalization of supply and demand, elaborated above all by Samuelson. Drawing particularly on these statements we tried to discuss the implications of the construction and found it to embody essentially a rudimentary theory of distribution, the process of distribution depicted in its simplest form showing a certain resemblance to the procedure of markets with an auctioneer and being of an automatic and ultra-harmonic character: the basic assumption — identical to the assumption of equalization between supply and demand through the working of the price mechanism — is the quasi-isolation of the various decision units in the society imagined. The equilibrium conditions are retained in the extension of Mosak's system dwelt upon, namely, Marschak's system. Even there they are tied to a distribution process, the process being of a slightly more general kind than the ordinary one. The construction is, however, not quite clear on this very point, and a reference in the context is made to the game distribution theory. In this theory, however, the equilibrium conditions are completely absent, possibly disregarding the special case of the Inessential Games, which show close similarities to the traditional distribution theories. An equilibrium concept — the Solution — is retained, but it is quite radically different from the traditional equilibrium conditions. Of special significance is the fact that ≫behavior equations≫ are only found in constructions of this type, embodying the quasi-isolation assumption and the rudimentary theories of distribution.

Thus the balance identities dealt with, implying equality between supply and demand, are tied to the pure production aspect, while the equilibrium conditions referred to, implying the equalization of supply and demand through the working of the price mechanism, are tied to the particular Walrasian synthesis between a production aspect and a distribution aspect, rudimentary in various respects. The same types of relations are found

in the econometric systems. These are in general much simpler than the programming analyzes and the Walrasian constructions, and therefore it would be highly astonishing if the relations of the link category that they comprise were of a more general character than the similar relations found in the more elaborate theory-constructions. It seems straightforward to conclude that an econometric system with balance identities may share some of the merits of the programming analyzes but must possess to a full extent the limitations of these as disregarding everything but the production aspect; and that an econometric system with equilibrium conditions possesses to a full extent the limitations of the Walrasian constructions with their quasi-isolation of the decision units of the society, ≫which is not the empirically given [problem]≪,[1)] except probably for certain limited fields of consumption and consumers' demand;[2)] *and* that an econometric system with both types of relations or with uncertainty concerning the nature of the link relations used is of a still more doubtful character.

b) THE TRANSFORMATION OF KEYNES ≫FROM QUESNAY TO WALRAS≪

Klein, the expert on Keynesian theories, makes the following statement about the equations of *A Treatise on Money* (1930): ≫The 'fundamental equations' were merely definitions which did not rest upon refutable hypotheses — hypotheses which could be verified or refuted by available data. They were on a level with other famous equations in economics, such as $MV = PT$, which do not tell us anything about fundamental economic behavior.≪[3)] Thus the thing deplored by Klein in connection with *A Treatise on Money* is the absence at large of behavior equations and the dominance of balance identities. In both these respects Klein later in his book, praises the merits of *The General Theory of Employment, Interest and Money* (1936). The opinions on the relative merits, however, of these two works differ. Among others some Swedish economists stress the greater importance of the 1930-work and its fundamental equations of balance-identity-type for the social accounting systems and hence the greater significance of this work (Lindahl, Lundberg). *A Treatise on Money* we put up as the ≫Quesnay-extreme≪ in Keynes' writings, Quesnay being with his ≫Tableau Économique≪ the first (well-known) compiler of social accounts. Also in *The General Theory* a relation is found which may be suspected to be a balance identity, and which we have already mentioned in Chapter

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[1)] von Neumann and Morgenstern [1], p. 15.

[2)] Cf. Oskar Morgenstern, "Demand Theory Reconsidered," *The Quarterly Journal of Economics* 1947–48, p. 194.

[3)] Klein [13], p. 17. Reproduced by permission of Macmillan Company.

I: $Y = I + C$ where Y = (real) income, I = (real) investment and C = (real) consumption. This relation was in fact often interpreted as a balance identity in the early discussion following *The General Theory.* Besides the relation $Y = C + I$, the most important equations occurring in the mathematical reformulation of Keynes are first and foremost the consumption function, $(1')$ $C = C(Y)$, or in the simplest linear form $(1'')$ $C = \alpha Y$ and further the investment function, often put $I = autonomous$, sometimes, however, put $(2')$ $I = I(Y)$ or $(2'')$ $I = I(Y, r)$ (r = rate of interest), or, in a frequently encountered very special form $(2''')$ $I_t = \beta[C_t - C_{t-1}]$. The structural coefficient α is usually called the marginal propensity to consume or the coefficient of the multiplier principle; the structural coefficient β is sometimes called the relation or the coefficient of the acceleration principle. The behavior equations (1)–(2) are said to describe the activities of the group of consumers and the group of investors, respectively, in the society studied, and the parameters in the equations are supposed to be derivable from the observation of actual behavior mainly through time-series-analysis; the values of these parameters may in principle vary from region to region and from time to time.

Let us first stick to this balance-identity-interpretation of $Y = C + I$ and try to judge its implication. Then the crucial relation holds by assumption: it says that the part of income which is not consumed is always invested, that is income is always fully utilized — the particular form in this case of the full-utilization-of-resources assumption tied to balance-identities and referred to above; further it has two degrees of freedom, being one relation in three variables. We now introduce the principal Keynesian equation $C = C(Y)$, or for instance $C = \alpha Y$. Having done this and reduced the number of degrees of freedom to one, we cannot postulate $I = I(Y)$ or $I = \gamma Y$ as an independent relation, because it follows from the two relations already put up: $I = Y - C(Y)$ holds by assumption; in the linear case it follows for instance that $I = (1 - \alpha)Y$. But still the level of Y is undetermined in our system; it being fixed, let us say Y put equal to Y_0, the last degree of freedom is removed.

What does this fixation mean? It means that in order to complete the model a certain objective of society must be introduced — it is difficult to see how the condition $Y = Y_0$ could be interpreted otherwise than symbolizing a decision by society to fix the level of income or the level of the production of the universal commodity, or as we may say in this context the total amount of exertion, of labor effort, to be exercised by society.

The construction may be supplemented, e.g. by a relation showing income, Y, as a function of ≫employment≫, N: (3) $Y = Y(N)$, and then the level to be fixed is explicitly that of employment, but the modification

adds nothing essential. This decision of society seems to be *the* economic decision implied by the very simple model — the so-called behavior relations $C = C(Y)$ and $I = I(Y)$ taken at their face value do not distinguish themselves from technical relations and the logic of the model certainly does not require that some conscious economic decision process be visualized behind them; on the contrary obvious difficulties present themselves if this be tried, because of the fact that the two ≫behavior relations≫ $C = C(Y)$ and $I = I(Y)$ cannot be treated symmetrically, one being fixed, the other following, and hence *at most* one of them could be tied to an economic decision process: the possibility that they symbolize decisions taken by two groups of decision units, consumers and investors, is completely excluded in this setup.

Thus even this extremely simple construction brings out our main points on balance-identity-systems: they are planning models which must he supplemented by some statement of the objectives of the economy and the significance within them of behavior equations interpreted literally is highly doubtful, as the whole construction expresses a decision by society made under certain restrictions, and this decision seems to be the essential economic process of the models. Opinions differ as to whether this be a true description of Keynesian philosophy. We do not enter into that debate, but mention in conclusion that the Keynesian setup discussed here is closely similar to the second simplest social accounting problem given by Frisch and others in *A system of Concepts Describing the Economic Circulation and Production Process* (1948).[4] This fact does not contradict our general opinion and indicates that we are still at the ≫Quesnay-extreme≫.

Barring the obvious difficulties of reasoning within a construction so oversimplified as the Keynesian one we remark that the distinction between the real and the virtual part of a planning system could be applied to the Keynesian model outlined above, as follows: before the value of Y is fixed we have a whole planning system; when a particular value of Y is adopted out of the many possible ones this value constitutes together with the ≫behavior relations≫ the real part of the system. In a similar way our distinction applies to more comprehensive Keynesian setups with a number of ≫exogenous≫ or ≫autonomous≫ variables, said to be controlled by the government.

There may be room left for diverging views on details; we maintain, however, our opinion that the following general line of logical development exists:

Keynesian and similar balance identity constructions	→	Leontiefian and similar systems	→	Programming systems proper

[4] Frisch a.o. [30], p. 5.

the first two stages corresponding essentially to the real part of the third stage, although traces of a virtual part may be found in the Keynesian systems and, by the way, also in the Leontiefian ones, although we have not stressed this point.

We now turn to the second interpretation of $Y = C + I$, regarding this relation as an equilibrium condition, that is as a (somewhat incomplete) description of an equilibrating process, whereby supply (Y) is equalized to demand ($C + I$), or savings ($Y - C$) to investment (I); thus the relation is not valid by assumption, but instead the theory has to show how the equalization is achieved. This interpretation is that of Marschak referred to in Chapter I, Section 3 and that of the bulk of the recent literature on Keynes. In this case both a consumption function $C = C(Y)$ and an investment function $I = I(Y)$ or $I = \textit{autonomous}$ may be introduced and the difficulties confronting an attempt to derive these relations as ≫behavior equations≪ in the preceding setups are no longer present, because of the fact that the relations can be treated symmetrically; thus no argument of the kind applied there throws doubt on the interpretation that $C = C(Y)$ represents the behavior of the group of consumers and $I = I(Y)$ the behavior of the group of investors in the society imagined.

Marschak puts forward the question in which we are interested: whose behavior does the equilibrium condition describe — or, as we may equivalently say — of what character is the equilibrating process indicated by the theory?

This question stands in the center of attention of the outstanding recent contributions to Keynesian literature by Klein[5] and Patinkin.[6] We touched upon Klein's opinion in Chapter I when discussing ≫structural changes≪. He uses the equilibrium condition $Y = C + I$ in the form $Y - C = I$ or $S = I$ where $S = Y - C$ is savings. He further introduces r, the rate of interest, as a variable in the behavior equations and his whole main argument is based on the proposition, that in contemporary societies the equilibrating process $S(Y, r) = I(Y, r)$ does not work in a satisfactory way. We may ask why he introduces r as a variable and does not consider it sufficient for the elucidation of the equilibrium condition to use only Y. The adherence to this last-mentioned procedure leads easily, according to Klein, into a system with ≫a basic contradiction and indeterminacy≪.[7] Suppose that income is related to employment by the ≫production function≪ (3) $Y = Y(N)$. Let the number of workers, $N = N_0$, willing to work at

[5] Klein [13].

[6] Patinkin [31].

[7] Klein, "Theories of Effective Demand and Employment," *The Journal of Political Economy* 1947, p. 110.

or above a certain fixed minimum wage rate be ≫institutionally≫ given, or supplement the production function, as does Klein, by a ≫demand relation≫ for labor: (4) $\frac{dY}{dN} = \omega$, where ω is ≫the real wage rate≫, and by a ≫supply relation≫ for labor (5) $N = N(\omega)$; in both cases the system (3) or (3)–(5), ≫the background relations≫, determines a level of income, $Y = Y_0$, let us say the full-employment level of income or ≫output≫. If now the equilibrium condition $S(Y) = I(Y)$ be added to the system, this condition determines by itself a level of output $Y = Y_1$. being one relation in one variable, and Klein emphasizes that there is no obvious mechanism to insure that the two levels of output involved would be the same, i.e. that nothing insures the validity of $Y_0 = Y_1$. This reasoning does not, however, throw any light upon the meaning of the basic Keynesian equilibrium condition, until Klein introduces the rate of interest as a variable and concludes; the equation $S(r, Y_0) = I(r, Y_0)$ may very probably have no solution with a positive rate of interest, i.e: $r > 0$, when Y_0 is the full-employment level of income or output, determined from the background relations, and this is due to the interest-inelasticity of savings and investment, empirically found in contemporary societies. If the S and I-schedules were sufficiently interest-elastic, such a solution in $r > 0$ and for $Y = Y_0$ would exist, and (one of) the basic innovation(s) of Keynesian analysis is according to Klein the emphasis put on the small or absent influence of the interest rate on savings and investment. Thus Klein reasons in terms of the interest rate when he explores the character of the equilibrating process indicated by the Keynesian equilibrium condition $S = I$, and he maintains that due to certain characteristics of the economic behavior of consumers and investors, due to certain values of structural coefficients, the equalization does not work nowadays — the field is open for political recommendations to cure the evil.

The equilibrium condition $Y - C(Y) + I(Y)$ or $S(Y) = I(Y)$ seems to stand in an uncertain relation to the equilibrium conditions of Walras, which we encountered above — no prices are involved. However, after the introduction of a price — the interest rate — in order to throw light on the significance of the condition, and after the discussion of the equilibrating process in terms of this price, we immediately experience a closer resemblance to the Walrasian constructions and to our argument, that the economic equilibrium conditions essentially depict a process of distribution between quasi-isolated decision units.

Patinkin takes us a step further in the Walrasian direction. Also he starts from the ordinary setup: the behavior relations $C = C(Y)$, $I = I(Y)$ and the equilibrium condition $C(Y) + I(Y) = Y$. This condition he writes $E = Y$, where $E = F(Y) = C(Y) + I(Y)$ denotes ≫aggregate demand≫. He remarks that some features of the equilibrating process indicated by

the Keynesian theory are not explicitly shown in the ordinary setup, and starts the task of introducing them more openly, particularly the task of taking into account ≫the supply side≫. We note the similarity in attitude with Klein: both research workers feel that the basic equilibrium condition must be supplemented by some more arguments in order to be completely understood. As the result of his investigations he ends up with the following system of equations, of course admittedly oversimplified and too aggregative:

$$E(Y, r, p) = Y(N^D)$$
$$N^D \frac{\omega}{p} = N^S(\omega, p)$$
$$M^D(Y, r, p) = M^S$$

where E = aggregate demand for finished goods, Y = income or output, r = rate of interest, p = absolute price level, ω = wage rate, N^D = demand for labor, N^S = supply of labor, M^D demand for cash balances, M^S = supply of cash balances = a constant. Thus three markets are considered, those of finished goods, labor and money, with the corresponding, mutually dependent equalization processes. These processes are discussed in terms of reactions or changes not only in the rate of interest, but also in ≫the absolute price level≫; further, the wage rate is introduced. The basic innovation of Keynesian analysis, as contrasted with the classical position, is conceived of as follows: ≫From the framework just described, we can appreciate the major significance of the classical position. It is, in brief, that the behavior equations are so sensitive to price and interest changes that the market will automatically and quickly establish a position in which, everyone's desires are satisfied. That is, the market will make the desires of people consistent. Correspondingly, we can appreciate the nature of the Keynesian argument: its denial of the efficacy, and even existence, of these delicate balancing operations; and its insistence that the end result of leaving the market to its automatic functioning must inevitably be the frustration of desires somewhere in the system.≫[8)]

The desires of people are defined to be ≫satisfied≫ and ≫consistent≫ when all the equilibrium conditions of the system hold. There is no difficulty in tracing the ideas behind such a definition to the Walrasian harmony construction: through the effective working of the automatic and harmonic distribution process the various individual's preferences guide the productive activities under the given budgetary and technological restrictions. Patinkin freely admits this Walrasian influence even to the point of making the following strong statement: ≫To start from fundamentals, it is clear that a complete explanation of the economic system can be presented only

[8)] Patinkin [31], p. 383.

through a Walrasian general equilibrium system≫.[9] According to Patinkin, Keynes' chief merit lies in pointing out the fact that the equilibrating distribution process may either not work at all depending on the values of certain structural coefficients — compare Klein — or it may take undue time until it becomes settled. Patinkin's contribution we put forward as the Walras-extreme of Keynesian literature yet published.

The balance-identity-interpretation of Keynesian analysis views the Keynesian system as being the nucleus of a program of action for the government, disregarding other decision units. This line of research has already been admirably developed in certain social accounting studies and in the programming analyzes; and vast possibilities are still unexplored. The equilibrium-condition-interpretations of Keynesian analysis view the Keynesian system as a description of the course of events in a multi-unit-of-decision society, and as a diagnosis of certain maladjustments in such a society. The exhaustive fulfillment of this rôle is probably much more difficult than the creation of a program of action for one dominating decision unit. The equilibrium-condition-interpretation seems, if elaborated, destined to lead into the blind alley of Walrasian constructions.

Having thus completed the second step in our attempt to elucidate the discrepancy in traditional econometrics between one-decision-unit planning systems and general equilibrium systems comprising many quasi-isolated decision units, we make a few concluding remarks about the triple character of econometric systems, mentioned in Chapter I. Disregarding the particular aspects and limitations of economic equation systems with balance identities and/or equilibrium conditions and of simultaneous systems of equations in general, it seems that the most effective bar preventing for the time being the presentation of a unified theory, giving both a description of the course of events, a program of action for society and an account of the rational behavior of the decision units in society, is to be found in those difficulties of utility analysis, which undoubtedly lie at the root of the dilemmas meeting attempts to synthesize the pure production aspect and the pure distribution aspect. If these difficulties cannot be solved, or until they are solved, other possibilities than a unified theory ought to be tried. As regards the other main point in Chapter I: the permanence of economic laws, we have in our considerations up to this point tried to make plausible that the idea of a theory universally valid up to changes in structural coefficients only seems to fit within the field of programming analyzes and within the Walrasian constructions. When the problem of distribution is discussed, differences appear between various social states

[9] Patinkin [31], p. 365.

which seem to require much more elaborated elements of change of social theory than structural coefficients.

2) The constructive hypothesis

In our considerations we have taken a somewhat critical attitude towards traditional econometrics despite the fact that we have underlined the great econometric conquests in the study of the pure production aspect, not least through investigations of production functions, and the study of consumers' demand, a limited field where probably the Walrasian or rather the Marshallian assumptions are more significant than in other contexts. Our criticism is directed towards a particular trait of contemporary econometrics which will now be explicitly discussed.

There are various definitions of ≫econometrics≫. Two very good and essentially overlapping definitions are the following ones: ≫The Task of econometrics is ... to utilize statistical methods and data and exact problem statements in order to reach better knowledge of the changing reality.≫[1]; ≫Econometrics is a special type of economic analysis in which the general theoretical approach — often formulated in explicitly mathematical terms — is combined — frequently through medium of intricate statistical procedures — with empirical measurements of economic phenomena.≫[2] Thus econometrics directly attacks the facts of reality, particularly the measurable facts, and it does so, not blindly, but inspired and guided by a general theoretical framework. Such, a procedure promises to open up the fertile circular process of science,

$$\text{induction} \quad \overleftarrow{\longrightarrow} \quad \text{deduction}$$

where reoccurrently a theory, based on certain facts, is corroborated by other facts and helps, to discover new facts, which form the basis of a better theory. The trait of contemporary econometrics towards which we have directed our criticism is not mentioned in the definitions: the trait that econometrics is almost only inspired by the thoughts embodied in the Walrasian constructions — or in the Marshallian ones, as was the base of early econometrics; moreover, the universal validity of this particular source of inspiration is frequently not even questioned.

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[1] Johan Åkerman, "Det ekonomiska framåtskridandets problem," *Studier i Ekonomomi och Historia,* Uppsala 1944, p. 324, footnote.

[2] Wassily Leontief, *Econometrics, A Survey of Contemporary Economics,* Philadelphia and Toronto 1948, p. 388, footnote. Reproduced by permission of The Blakiston Company.

One fundamental problem — we label it ≫the fundamental problem≫ in this thesis — for contemporary economics is to let empirical research be inspired by constructive frameworks of interpretation other than the Walrasian systems. This opinion was expressed by Johan Åkerman in the article on "The Setting of the Central Problem" (1936),[3] where he calls for an epistemologically better oriented analysis of economic observations than the traditional one; we quote the concluding lines of his article: The ≫development of economic science does not consist in the transference of scientific attention from one set of concrete phenomena to another set of concrete phenomena but rather in the adoption of a different *attitude* towards the asking of a different set of questions about the same problem.≫[4] Here again Walras, writing at the eve of this century, anticipated the actual need when he spoke of the necessity of a symbiosis between philosophy, history, economics and mathematics.[5]

A review of recent contributions to economic theory tends to show — if attention is concentrated on the setting of the central problem, that in economics the time has not come to speak of many things, the time has come to speak of fundamental things.[6]

A similar opinion is held by Rutledge Vining in his recent discussion with Koopmans on "Methodological Issues in Quantitative Economics" (1949):[7] ≫... from [Koopmans] discussion it appears not unfair to regard the formal economic theory underlying his approach as being in the main available from works not later than those of Walras≫.[8] ≫Is the Walrasian conception not in fact a pretty skinny fellow of untested capacity upon which to load the burden of a general theory, accounting for the events in space and time which take place within the spatial boundary of an economic system?≫[9]

On this topic von Neumann and Morgenstern speak of ≫statistical economic research which holds the real promise of progress in the proper

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[3] Johan Åkerman [3].

[4] R. W. Souter, *Prolegomena to Relativity Economics* New York 1933, p. 163.

[5] L. Walras, *Éléments d'Économie Politique Pure,* Paris 1926, preface to 4th edition, p. XX.

[6] Johan Åkerman (28), p. 122. Reproduced by permission of the Econometric Society.

[7] Vining [32].

[8] Vining [32], p. 80.

[9] Vining (29), p. 82. Reproduced by permission of Harvard University Press.

direction≫,[10] and Morgenstern continues the line of thought: ≫This leads to a final brief remark on the relation of the theory of games to the empirical-descriptive part of economics. It is clear that decisive progress in economics is not possible unless the empirical-descriptive side is developed on a scale at present hardly imagined. The theory of games will become more useful to economists if this work is accomplished. While the theory of games also depends on the description it can — as any good theory — make some contribution to it even in its present state. It has provided us with certain precise concepts, such as imputation, solution, exploitation, discrimination, standard of behavior; etc., and they should guide new descriptive work. For example; it would be possible to look into the history of the formation of cartels, particularly into the preliminary negotiation of quotas there, into wage disputes, into the bluffing that goes on at financial markets, into the working of incomplete monopolies and the treatment of outsiders, with all these and still other concepts in mind. One would not only obtain desirable corroboration for the theory but also be led to the discovery of new facts of economic life that, requiring explanation, might help to give new directions to theoretical work which would otherwise be missed.≪[11]

In Chapter III, Section 3 we said a few words about social theories with the emphasis put on their rôle as instruments of decision in various contexts. In our discussion a distinction can be found between what may be hoped from future social theory and what can be accomplished with the now available material. Examining here social theories with the emphasis put on the rôle of interpreting and explaining reality, the same distinction is relevant. It may be debated how far future social theory can reach in this respect, if it can arrive at a level comparable to that of the most developed natural sciences — provided a meaning can be given to this assertion —, but it seems to us fairly obvious that for the time being a very modest position must be taken in economics and its neighboring fields, as there the circular process of science just referred to is still in its beginning. It is in conformity with this attitude that we use as synonyms for existing social theories, expressions like ≫socio-economic thought experiment≪, ≫general theoretical approach≪, ≫framework of interpretation≪ and even ≫epistemological orientation≪, indicating our belief that the time for a detailed and rigorous application has not yet come.

As a result of our analysis of the two summation theorems and of their relations to each other and to Walrasian and kindred constructions

[10] von Neumann and Morgenstern [1], p. 4.

[11] Morgenstern [28], p. 308. Reproduced by permission of A. Francke A.G.

we sponsor two general types of economic theories: theories belonging to the pure production aspect and theories belonging to the pure distribution aspect; we are, however, not able to point out a synthesis between the two extremes. These theories are satisfactory in the sense that they are completely derived from consistent assumptions on human economic action — the first type from the assumption that society in the form of a unified organization tries to make the best possible use of its resources of productive factors, the second type from the assumption that every decision unit in a society tries to make its relative money income as high as possible — they do not comprise any elements stated ad hoc like the Walrasian equilibrium conditions, the relation of which to an underlying principle of human economic action is highly uncertain. We now state our constructive hypothesis, foreshadowed at various places above: Theories belonging to the pure production aspect and to the pure distribution aspect should be substituted for the Walrasian constructions in the task of explaining economic observations, and of inspiring and guiding economic research; the course of economic and social events cannot be grasped by the constructions of the pure production aspect, nor by the constructions of the pure distribution aspect; both aspects are required for an explanation of reality, but, the relative importance of the two aspects varies from time to time and from region to region. Particularly we shall maintain the hypothesis in the form that the pure production aspect essentially covers a centrally planned society, while the pure distribution aspect covers the main features of a free-group economy. This methodological standpoint ought not to cause any particular conceptual difficulties as long as it is considered only provisorial, workable until a synthesis is found between the two dual aspects; and no more pronounced attitude is needed.

Accepting this constructive hypothesis we should firstly try to connect certain observed phenomena in planned economies with the relevant features of the pure production aspect: the distinction between the real and the virtual parts of a planning system and the elements of change consisting of the structural coefficients; secondly, we should try to connect certain observed phenomena in free group-economies with the relevant features of the pure distribution aspect: the various elements of change labeled Characteristic Functions, Imputations and Solutions, and subsets of these elements, i.e. concepts which are implications of the first summation theorem. From the point of principle the procedure is exactly similar to that applied in empirical research relying essentially on a Walrasian theoretical foundation: there it is attempted to establish a contact between certain observed phenomena — be they from planned economies or from free group-economies — and the elements of change consisting of structural coefficients.

Obviously we must in fulfilling our constructive hypothesis concentrate on one or two problems, and our main task will be the discussion of certain observations pertinent to interest rates, inflation and business cycles in multi-unit-of-decision economies. In examining these from the pure distribution aspect our approach will in many respects be fairly different from traditional analysis; it can at most be a modest beginning, utilizing only basic facts openly visible. We do not hope to solve problems but we hope to be able to point out neglected and essential traits of the phenomena studied. As regards centrally planned societies we shall not establish any connection between observations from such economies and the pure production aspect, but we shall discuss phenomena of interest rates and cycles within the aspect, thus preparing the ground for future empirical contact. In carrying out the last-mentioned task and comparing with the studies of related phenomena within the pure distribution aspect we shall shed further light in some special contexts on the distinction between the production aspect and the distribution aspect. For the sake of comparison a few words will also be said about the relations between the empirical phenomena studied and the Walrasian constructions.

Chapter V

The Rate of Interest

1) The Time-series

Tinbergen presents a program for econometric analysis of the rate of interest in the article "Some Problems in the Explanation of Interest Rates" (1947) :[1] ≫The final purpose of [econometric] analysis may be formulated as the explanation of the differences between: a) interest rates in different countries at one time; b) various types of interest rates in the same country and at the same time; c) the development in time of these interest rates≫[2] He further outlines the relevant facts to be explained: ≫What are the most conspicuous features of the level and the course of interest rates in the principal countries demanding explanation?≫[3]

We focus our attention on some of these conspicuous features of the level and the course of interest rates in different countries, listed by Tinbergen. *First,* we observe as a general background the great diversity of the interest rate phenomenon: China and India show pronounced dissimilarities, although ≫economically comparable≫; the First World War shows a marked rise in interest rates but not so the Second World War; etc. *Second,* we pause to regard the two characteristics of movements: the ≫falling trend≫ throughout the nineteenth and twentieth centuries and ≫the long cycles superimposed on this trend≫ which are positively correlated with the general price movement.[4] The falling trend Tinbergen very tentatively places in connection with ≫the marginal productivity of capital≫ and,

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[1] Tinbergen [33].

[2] Tinbergen [33], p. 397.

[3] Tinbergen [33], p. 431.

[4] Cf. John Maynard Keynes, *A Treatise on Money,* London 1930, Macmillan, II, p. 198: ≫the extraordinarily close correlation over a period of more than a hundred years between the rate of interest, as measured by the yield of Consols, and the level of prices, as measured by the Wholesale Index Number ... is one of the most completely established empirical facts within the whole field of quantitative economics≫.

more precisely, with a supposed fall in this entity.[5] As regards the other characteristic, Tinbergen holds that ≫It is doubtful whether the long-term correlation between the price level and the interest rate can be easily understood≫, because ≫An important feature of purely statical theory is that the *rate of interest is only dependent on 'real' or physical, not on nominal, factors.* For example, a doubling of the money in circulation will, according to this type of theory, only lead to a doubling of nominal prices and money items, but not to a change in any 'real' variables and hence not in the rate of interest.≫[6]

Let us now look directly at the time-series underlying Tinbergen's statement about the two characteristics. The following four charts give a comparison between the movements of the rate of interest (· · ·) and the price level (—) in USA 1890–1950 (I, II), in England 1785–1950 (III, IV), and in Sweden 1870–1950 (V). We do not think that the ≫price level≫ is a very useful concept but take it here only as a valid representative of movements in prices, other than interest rates. The charts at first sight seem to confirm Tinbergen's observations about the two characteristics, the falling trend and the positive correlation with general price movements, but a closer scrutiny suggests that a somewhat different presentation should be given to the characteristics. Possibly the most conspicuous trait in the charts is the sharp kink occurring around 1932. If the whole time-period 1870–1950, or even 1785–1950 is considered as homogeneous, the data show the falling trend, above all due to the pressing down of the interest level after ≫the year of change≫ 1932, and they let the positive correlation stand out too, but only so before the year of change; after these the correlation is zero or negative. To every unbiased observer it must seem very appropriate in this case to divide the time-span into at least two subperiods: the period before the year of change, and the period after the year of change; the first period being mainly characterized by the positive correlation between interest rates, and other prices, the second period being above all distinguished by low, steady interest rates despite rising prices. These two, widely different relations seem to be so clearly expressed by the data that no observation errors of any possible magnitude could reverse them.

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[5] Tinbergen [33], p. 436 and "Het verloop op langen termijn van de rentestanden," *Statistische en Econometrische Onderzoekingen,* ed. by Centraal Bureau v.d. Statistiek, The Hague 1943, nr. 2/3.

[6] Tinbergen [33], p. 434. The quotations are reproduced by permission of Harvard University Press.

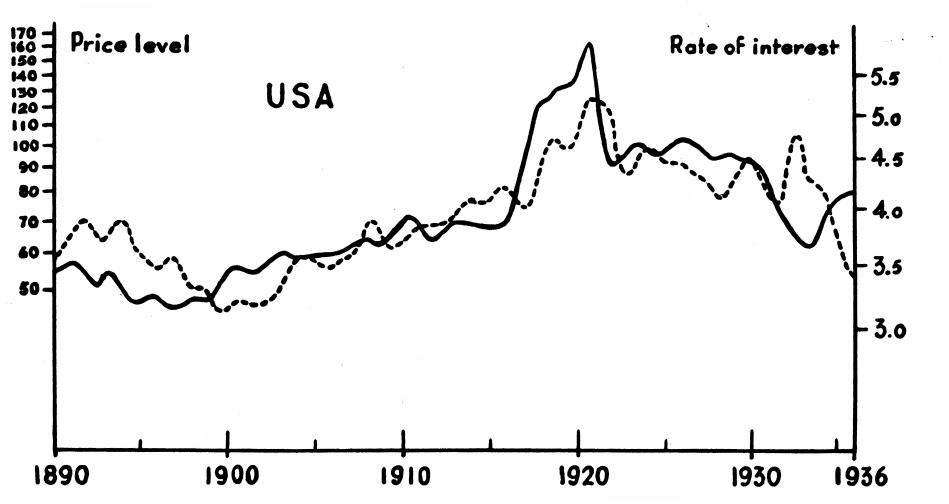

Chart I. American Railroad Bond Yields (· · ·) and U. S. Bureau of Labor Statistics Index Number of Wholesale Commodity Prices (—) 1890–1936. Reproduced by permission from *The Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856,* by Frederick R. Macaulay, National Bureau of Economic Research, New York 1938.

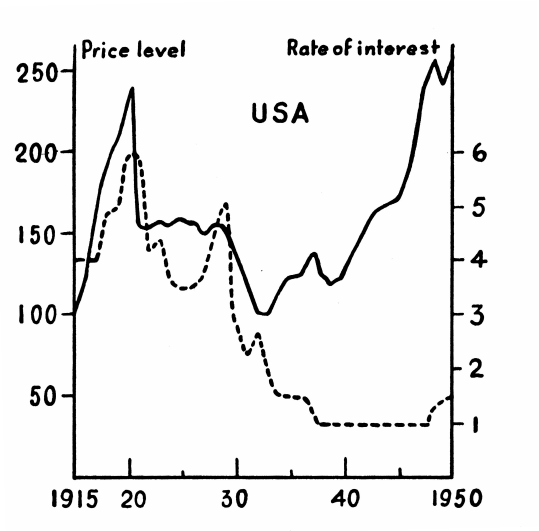

Chart II. Federal Reserve Rate of Discount (· · ·) and American Wholesale Prices (—) 1915–1950.

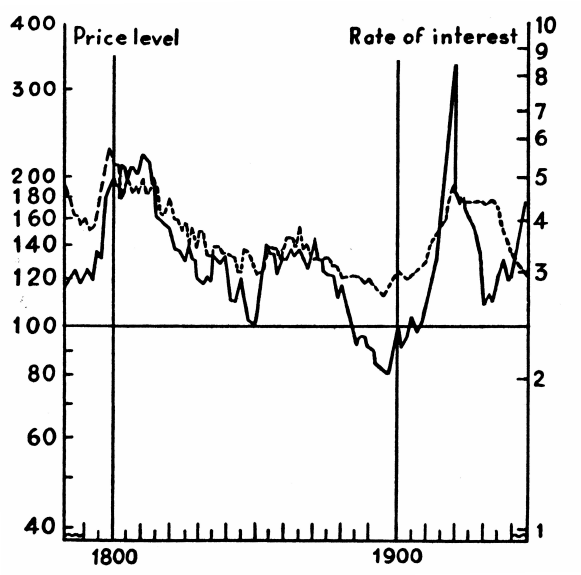

Chart III. Yield of British Consols (· · ·) and British Commodity Prices (—) 1785–1941. Reproduced by permission from *The ABC of Inflation* by E. W. Kemmerer, copyrighted in 1942 by McGraw-Hill Book Co.

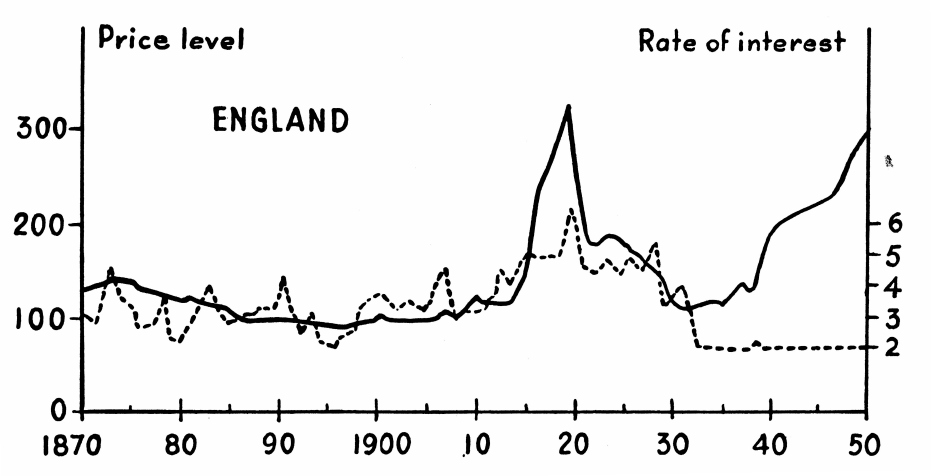

Chart IV. Bank of England Rate of Discount (· · ·) and British Wholesale prices (—) 1870–1950.

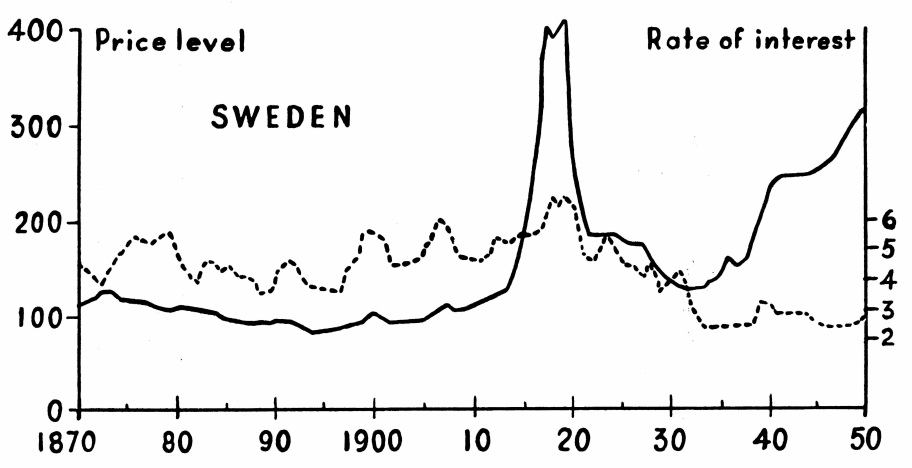

Chart V. Bank of Sweden Rate of Discount (· · ·) and Swedish Wholesale Prices (—) 1870–1950.

These are the two time-series facts which we wish to discuss: the positive correlation between interest rates and the general price level in USA, England and Sweden before 1932; and the sharp change to low, steady interest rates despite rising prices in these countries after 1932. Thus the time-series have suggested to us certain problems. We now turn to the theories of the rate of interest, based on other types of observations than time-series, that is broad observations of human motivations, of human activities in pursuit of economic and social goals. Our final purpose is to find a contact between the time-series observations and certain theories, this contact being supplemented by observations of a more qualitative character. The general procedure and the purpose are the same as those of Tinbergen. He discusses, in the article referred to, various interest theories of the simultaneous-equation types. In our ensuing discussion of interest theories we shall utilize our findings in Chapters I–IV about theories of these and other kinds.

2) The rate of interest and the rate of growth

a) THE INTEREST THEORIES OF JEVONS AND WICKSELL

Let us begin by quoting a famous passage in W. Stanley Jevons' *Theory of Political Economy* (1871),[1)] a pioneering work of interest theory: ≫We may obtain a general expression for the rate of interest yielded by capital in

[1)] Jevons [34].

any employment provided that we may suppose the produce for the same amount of labour to vary as some continuous function of the time elapsing between the expenditure of the labour and the employment of the result. Let the time in question be t, and the produce for the same amount of labour the function of t denoted by Ft, which may be supposed always to increase with t. If we now extend the time to $t + \Delta t$, the produce will be $F(t + \Delta t)$, and the increment of produce $F(t + \Delta t) - Ft$. The ratio which this increment bears to the increment of investment of capital will determine the rate of interest. Now, at the end of the time t, we might receive the product Ft, and this is the amount of capital which remains invested when we extend the time by t. Hence the amount of increased investment of capital is $\Delta t Ft$; and dividing the increment of produce by this last expression, we have

$$\frac{F(t + \Delta t) - Ft}{\Delta t} \times \frac{1}{Ft}$$

When we reduce the magnitude of Δt infinitely, the limit of the first factor of the above expression is the differential coefficient of Ft, so that we find the rate of interest to be represented by

$$\frac{dFt}{dt} \times \frac{1}{Ft} \qquad \text{or} \qquad \frac{F't}{Ft}$$

The interest of capital is, in other words, *the rate of increase of the produce divided by the whole produce;* but this is a quantity which must rapidly approach to zero, unless means can be found of continually maintaining the rate of increase. Unless a body moves with a rapidly increasing speed, the space it moves over in any unit of time must ultimately become inconsiderable compared with, the whole space passed over from the commencement. There is no reason to suppose that industry, generally speaking, is capable of returning any such vastly increasing produce from the greater application of capital. Every new machine or other great invention will usually require a fixation of capital for a certain average time, and may be capable of paying interest upon it; but when this average time is reached, it fails to afford a return to more prolonged investments.≫[2)]

Jevons' interest rate formula may be derived from the following considerations. Put $x = Ft$ and write

$$x = Ce^{at} \qquad C = \text{const.} \tag{1}$$

[2)] Jevons [34], pp. 245–247.

x, the quantity or value of the product involved, then depends on the variables a and t. It is almost trivial that the maximum growth of x, let us say over a definite time-span $0 \to T$, is achieved when

$$a = \frac{1}{x} \times \frac{dx}{dt} \tag{2}$$

where a is the rate of growth, identical to Jevons' rate of interest. It also follows that x is proportional to its time derivative,

$$x = b\dot{x} \tag{3}$$

and thus Jevons gave an early expression for the principle of acceleration in economic literature, if by principle of acceleration we denote the case when a variable is proportional to a first derivative or a first difference.

Formulas (1) and (2) immediately remind us of another crucial point in the development of the theory of the rate of interest — Wicksell's famous wine example in *Lectures on Political Economy* (1901)[3]:

$$W = f(t) = V_0 e^{\varrho t}, \tag{4}$$

where V_0 is the price of grape juice ÷ (wages + rent); W the value of wine which increases with time; ϱ the (real) rate of interest; and $f(t)$ a given function of time. There are, however, some more elements present in Wicksell's exposition than in that of Jevons.

Often Wicksell's example is presented thus. (4) describes the situation of an entrepreneur, to whom the market rate of interest, i, is given and whose objective is to maximize the net discounted money profit from the wine activity, that is $\frac{f(t)}{e^{it}}$. The only variable at his disposal in this maximization process is t, the length of juice storage before the wine is sold. We get:

$$i = \text{ exogenous}, \tag{5}$$

$$\max_t \frac{f(t)}{e^{it}}, \tag{6}$$

that is two relations in two variables. It is easily concluded that the maximization implies

$$\frac{df(t)}{dt f(t)} = i, \tag{7}$$

[3] Wicksell [35].

which means that at the optimum time of sale, $t = t_0$, the rate of proportional increase in the value of the good over time is equal to the market rate of interest. (Allen, Stackelberg, etc)

At this point our discussion of the marginal productivity theory of distribution in Chapter II, Section 1, may be recollected. The setup (5)–(7) amounts to a marginal productivity theory of a single firm but is no theory of distribution, as it does not even determine any rate of interest, which is supposed to be given. Certainly it was Wicksell's intention to elaborate such a last-mentioned theory. He mentions the interpretation (3)–(7): if $\varrho(\equiv i)$ had been considered as given, the formulas he presents ≫would have given the value of t which maximizes V_0, i.e. the storage period which the cultivators themselves would adopt, if they could borrow money at the rate of interest ϱ for their current expenses.≪[4] He wants, however, to achieve more than this. His variable quantities are three: V_0, ϱ, and t (the storage time); his equations are two, our relation (4) and the relation

$$\varrho = \frac{W'}{W}, \tag{8}$$

formal correspondencies to which we have already met in Jevons — (2) — and in the firm-problem — (7). A perusal of the relevant section in Wicksell's *Lectures* will give the obvious result that he does not succeed in letting the three unknowns be variable at the same time — except through introduction of another relation, containing an exogenous variable, the amount of ≫capital of the community≪ — which is an indeterminacy typical of the marginal productivity theory of distribution. This circumstance leaves Wicksell's theory of the rate of interest in a somewhat unsatisfactory state. It is, however, possible to develop his considerations in such a direction that a purely real rate of interest is determined.

b) THE INTEREST THEORIES OF HAWKINS AND VON NEUMANN

A purely real rate of interest was encountered already in our discussion of Jevons, relations (1)–(3). Because of the extreme simplicity of this setup its implications are difficult to judge, and we undertake immediately a straightforward generalization of it. Instead of (3) we write

$$X_i = \sum_{j=1}^{n} (a_{ij} X_j + b_{ij} \dot{X}_j) \qquad (i = 1, 2, \ldots, n) \tag{9}$$

[4] Wicksell [35], p. 179.

Here we have replaced the simple proposition that the output of one particular good is always proportional to the rate of increase of this good — equation (3) — by the more complicated statement that in a system of n productive activities the output of each particular good — X_i — is always equal to the sum of the inputs of this good, used in the various activities, both as factors to be immediately consumed in the processes — the terms $a_{ij}X_j$ —, and as factors to be stored in the processes — the terms $b_{ij}\dot{X}_j$ — all this under the assumptions that the amount required of a ≫consumed factor≫ in a certain activity is proportional to the output of this activity, and that the accumulated amount needed of a ≫stored factor≫ in a certain activity is also proportional to the output of this activity — which implies that the amount required of a stored factor is proportional to the rate of increase of the output. The solution of the system of linear differential equations (9) is

$$X_i = \sum_{j=1}^{n} C_j D_{ij} e^{\alpha_j t} \qquad (i = 1, 2, \ldots, n) \tag{10}$$

which is a substitution for (1). We now refer to Chapter II, Section 1: (9)–(10) are Hawkins system,[5] discussed there. This system is based on a certain weak teleological assumption, the full-utilization-of-resources assumption, which is formally expressed through the fact that the equality between the two members of equations (9) is postulated to hold; it is not to be achieved through the working of a price mechanism. Granted the full-utilization-of-resources assumption, consistency conditions for solutions of the system (9) can be established — corresponding to condition (2) — which determine the α's of the explicit form (10). It may further be shown as mentioned above that there is only at most one $\alpha = \alpha_1$ representing a steady-state solution. The other α's represent transient states, which are superimposed on the steady state and which vanish as time goes on, if the system be stable. In the steady state the relations between the quantities of the various products remain constant and α_1 is the maximal rate of growth of the whole economy, being implied by the full-utilization-of-resources assumption (and the obvious condition that all variables be positive).

This α_1 is a straightforward generalization of Jevons' rate of interest, and it stands at the extension of one main tendency in Wicksell's theory of the real rate of interest. Wicksell's wine example may lead to a production theory of one single profit-maximizing firm in an atomistic credit-market [(5)–(7)]. In that case it is no theory of the determination of the rate of interest. It may be utilized for argumentation along the lines of a marginal

[5] Hawkins [21].

productivity theory of distribution (Wicksell , op. cit.), but the setup remains undetermined until an assumption is introduced, conveying a certain kind of objectives of society. At this point the theory starts being a theory of the determination of the rate of interest [(9)–(10) and (1)–(3)]. The kinds of objectives of society introduced are the full-utilization-of-resources assumption, implying the maximal growth of the system, say, during a definite time-span $0 \to T$. The teleological element is only weakly marked. Before discussing its possible extension, let us turn from the quantity side to the price side.

It was observed in Chapter II, Section I, that Hawkins' system admits the definition of prices, which should not necessarily be introduced if the quantities be already established, standing in a one-to-one correspondence to quantities and based on the teleological assumption. The rate of growth α_1 was derived solely from consideration of real quantities. Does it have a monetary counterpart? The prices in the steady state are expressed thus

$$P_j = \sum_{j=1}^{n} P_i(\alpha_{ij} + b_{ij}\alpha_1) \quad (j = 1, 2, \ldots, n) \tag{11}$$

It follows from (11) that for a particular activity I

$$\alpha_1 = \frac{P_i - \sum_{j=1}^{n} P_j \alpha_{ji}}{\sum_{j=1}^{n} P_j b_{ji}} \tag{12}$$

Everything counted per unit of output, the numerator of the right-hand term in (12) is income less costs for the factors to be consumed, the denominator is value of the capital employed, and the ratio may be called profit rate. Thus the monetary profit rate is the same in all activities and equal to the real rate of growth. We could say that (12) expresses the equality in the steady state between the real rate of interest and the monetary rate of interest, but it must be emphasized that we have only attempted to trace the *real* rate of interest, α_1, back to the equally named entity in Wicksell's writings.

Also von Neumann's ≫model of general economic equilibrium≫[6] has a real ≫coefficient of expansion≫, giving the maximum technically possible rate of growth of the whole economy and this coefficient of expansion is equal to a monetary ≫interest factor≫. In this respect von Neumann's system gives the same interesting result as does Hawkins' steady state. In some other respects there are differences: von Neumann's system is discrete, not continuous, has only ≫stocks≫ and no ≫flows≫, and does

[6] von Neumann [22].

not build on the complete full-utilization-of-resources assumption, which presupposes that supply is equal to demand, but assumes that there are sufficient quantities of the various commodities to carry out the productive activities, which presupposes only that supply is equal to *or* greater than demand.[7] It seems, however, that the only essential difference, from the interpretation point of view, between the two systems consists in Hawkins having as many commodities as activities but von Neumann introducing more activities than commodities, thereby opening up the possibility of producing a product in more than one way and in fact abandoning the restrictive linearity assumptions of Hawkins (and Leontief). The consequences of this major innovation make themselves clearly felt.

In Hawkins' steady-state system the full-utilization-of-resources assumption implies the maximum-possible-growth characteristic. Von Neumann starts from the weaker assumption that sufficient quantities of the commodities are present, but he arrives at the same maximum-possible-growth consequence; moreover, he shows that for a given set of technological coefficients the real rate of growth and its monetary equivalent are uniquely determined, as they are in Hawkins' system. But the (relative) quantities [and the corresponding (relative) prices] are not uniquely determined in von Neumann's system contrary to what is the case in Hawkins' system, granted certain obvious conditions. The indeterminacy of the Neumannian construction is fundamental. Von Neumann comments (where x = quantities, y = prices, P = technical processes or activities, G = commodities, α = coefficient of expansion, β = interest factor): *Solutions of [the system] always exist,* although there may be several solutions with different $x_1, \ldots, x_m$ or with different $y_1, \ldots, y_m$. The first is possible since we have not even excluded the case where several P_i describe the same process or where several P_i combine to form another. The second is possible since some goods G_j may enter into each process P_i only in a fixed ratio with some others. But even apart from these trivial possibilities there may exist — for less obvious reasons — several solutions $x_1, \ldots, x_m, y_1, \ldots, y_m$. Against this it is of importance that α, β should have the same value for all solutions; i.e. α, β *are uniquely determined.*[8]

Later researches strongly suggest how this basic indeterminacy should be interpreted. When the extremely restrictive linearity postulate, implying that a product can only be produced in one given, fixed way, is abandoned, the teleological assumption amounting to the achievement of the maximal rate of growth of the whole economy is in general not sufficient. A device such as Dantzig's Objective Function must be introduced, establishing a

[7] Cf. Dantzig [23], pp. 206–207 and 202.

[8] von Neumann [22], p. 3.

value scheme of society, on the basis of which a choice between the various alternative activities available can be made and the setup be thus rendered determinate, barring indeterminacies of such trivial characters as those mentioned in the beginning of the above quotation from von Neumann. After the logical completion of the system by an Objective Function, there is not much sense left in interpretations which regard the system as describing ≫a competitive, capitalist economy with zero profits in all enterprises≫.

It may, however, be argued that even if a definite set of relative quantities can only be determined on the basis of a value scheme of society, the rate of interest is uniquely determined by the technical coefficients and the original very vague teleological assumption and hence not dependent on the setting up of a particular Objective Function. In our terminology: all feasible real parts of the system, i.e. all actual plans that may be fulfilled with a given setup of technical processes, show the same rate of interest = rate of growth. Is not then the interest theory in the model of a wider generality than the fixation of the other variables in the system? We give the answer that the interest rate of the construction is certainly invariant under changes of the value scheme of society, but it is part of a model which seems to require the establishment of a set of objectives of society in order to become determinate, and hence the interest theory given presupposes the existence of *some* value scheme of society, no matter which of the possible alternatives it be. Arrow's theorem shows that even the mere existence of *some* preference scale of society is a strong postulate. A more definite answer would be the evidence that in still more general models — where all sectors do not grow proportionally — the rates of interest are not invariant under changes of the objectives.

When we take the step from von Neumann's system to Dantzig's programming analyzes the Objective Function is explicitly added and the full-utilization-of-resources assumption is restored in its strong form: no further essential changes need occur. But Dantzig's setup can, as we mentioned in Chapter II, Section 1, allow a more general type of dynamics — it need not signify that the relations between the various quantities are constant, the whole economy growing at the same pace; it can signify that the relative amounts of the products vary over time, that some sectors of the economy expand more rapidly than other ones in the fulfillment within a definite planning period of the stated objectives. It seems that not until this point be reached does an interest rate theory of the category discussed in this section become really interesting. It is hard to see why interest-rate (and price) concepts could not be also developed in connection with the explicit analyzes, but as far as we know no effort has been exercised yet in this direction and possibly it has to wait for some more basic problems to be settled first. Probably the development of such concepts under contact

with the interest-profit(-price) experiences that are available from centrally planned economies would be a rewarding line of research.[9)]

We conclude that von Neumann and Hawkins have given highly interesting contributions to the theory of the rate of interest, logically extending the line originated by Jevons and Wicksell and possibly Wicksell's Austrian predecessors, and to a large extent overlooked by other economists. At first sight these interest theories seem to settle two much-disputed problems in economics. They show that under stationary conditions — when the quantities of the various products of the economy remain constant over time — the rate of interest is zero. They further give an interest rate determined by purely technical data, technical coefficients. In our view, however, these findings ought to be, at second sight, modified in the following respect. The very definition of ≫stationary conditions≫ or ≫statics≫ used here is tied to constructions which require, in order to be logically complete, the existence of a value scheme of society; this is only weakly marked in the rudimentary variants but quite explicitly underlined in the more general ones. Hence the conclusion about the zero interest rate applies only straightforwardly to economies which approximately possess one prevailing set of objectives. Further: in Hawkins' system the rate of growth and the profit rate of the steady state are determined not solely by the technical coefficients but in conjunction with the weak teleological assumption about full utilization of resources, implying the maximal growth of the system; in von Neumann's system the coefficient of expansion and the interest factor depend, not only on the technical coefficients but also on the efficiency condition about the maximal growth of the system, and, what is more important, are part of system which seems in general to require an Objective Function in order to be determinate and complete, even if the interest rate elements be invariant under changes of the Objective Function. Hence the conclusion about the technically determined interest rate seems only to apply straightforwardly to economies which approximately possess one prevailing set of objectives, and it may even be guessed that in more generally dynamic constructions the interest rate will turn out not to be invariant under changes of value schemes.

Can the interest theories discussed here give some clues towards the interpretation of the time-series facts listed in the previous section? As far as these theories have been developed they let the rate of interest change with technical conditions, with the technical coefficients. A proportional fall or decline in all prices leaves the rate of interest entirely unaffected. Granted

[9)] The arguments in this section partly coincide with certain arguments in Chapter II, Section 1, but they are adapted to the special case of interest rates.

that ≫the general price movement≫ is in essence such a common fall or decline, the first observation on the positive correlation between interest rates and the general price movement before ≈ 1930 is in disagreement with the theories at their present stage. The fairly constant interest rates and the rising prices after ≈ 1930 seem to fit better into the picture, but we do not think that anything would be gained by a further elaboration of this coincidence in one case; in Section 4 of this chapter we shall attempt an explanation which gives clues towards the interpretation of the positive correlation before ≈ 1930 as well as of the change in the relation and the ensuing fairly constant interest rates under rising prices.

Thus we have tried to indicate that interest rate phenomena appear in constructions belonging to the pure production aspect, although this side of the constructions is still only very incompletely elaborated; we do not think, however, that a connection could be established between these theories and the relevant facts.

3) The market rate of interest

Marshall gave a famous interest theory,[1)] letting the rate of interest be determined as any other price by the market equalization of supply and demand, in this case supply of capital (= credit) and demand for capital (= credit); in other words, savings and investment respectively:

$$S = S(i) = \sum_{j=1}^{n} S_j(i) \tag{13}$$

$$I = I(i) = \sum_{j=1}^{n} I_j(i) \tag{14}$$

$$S = I \text{ or } S - I \to 0 \quad \text{as } t \to \infty \tag{15}$$

where S = total savings, I = total investment, S_j = savings of saver no. j; I_j investment of investor no. j. The behavior equations S_j and I_j are summed over all units to form the totals; the market distribution process described by (15) is automatic and ultra-harmonic — compare Chapter II, Section 2.

[1)] Alfred Marshall, *Principles of Economics,* London 1890.

Keynes' equally famous interest theory[2)] may be presented in a form exactly similar to the three-equation setup (13)–(15), two behavior equations and one equilibrium condition, but the interpretation of his behavior equations is often believed to be different from that of Marshall's behavior equations, the Keynesian ones dealing with supply and demand of money or cash as functions of the rate of interest — this much may be said without intruding too deeply into a highly controversial field.

If presented solely in the form of such a three-equation setup, of such a Marshallian supply-and-demand-cross, the theories are very difficult to judge as to their significance; moreover, the presentation does obvious injustice to the verbal richness of the interest-discussions of the two authors. In our opinion the theories of the type (13)–(15) can be elaborated in two directions, the first coinciding with some important verbal arguments attached to the theories, the second implying the incorporation of the theories into a Walrasian construction. The first line will be discussed in our next section; the second, pertinent to much of the recent interest-discussion, will be very briefly considered in this section.

Attempts to include the Marshallian and Keynesian interest theories in a Walrasian system presuppose two things: *first,* and perhaps most obvious, the three-equation setup should be extended to comprise more markets than just-one, i.e. more prices than one rate of interest should be introduced and all behavior equations be written as functions of the whole set of prices; *second,* and most important, the behavior equations should be derived from considerations of utility or profit maximization on the part of the various participants composing the society imagined — a methodological standpoint particularly stressed by Marschak and being in agreement with our opinion that economic theory should start from certain broad observations on human economic and social activity in pursuit of some goal. Certainly, however, we would add that the economic activities pictured by a Walrasian construction are rudimentary in the respect that they let the distribution process take an automatic and a harmonic character. On looking at some of the very few attempts of inclusion which obey both our points the most conspicuous thing to be found is probably the uncertainty still prevailing about the rôle of the rate of interest in a Walrasian construction. Mosak's system, the most direct extension of a Marshallian one-market-setup, comprises no interest rate in its ≫static-equilibrium≫ or ≫uniperiod≫ version discussed above. In the less elaborated ≫intertemporal-equilibrium≫ version which tries to view the individual economic agents as planning for a sequence

[2)] John Maynard Keynes, *The General Theory of Employment, Interest and Money,* London 1936.

of time-intervals, extending up to the planning horizon, certain interest-phenomena arise, as they do at a similar stage of Marschak's early theory of assets. Unfortunately, however, the later, more firmly underbuilt elaboration of Marschak's system has not as yet been extended to comprise lending and borrowing — a step which is promised; the interesting point in such an elaborated extension would be the rôle of the ≫revenue functions≫ or ≫selling schedules≫, i.e. the imperfections of the various markets, for the interest-rate phenomenon. As will be subsequently pointed out, Marschak proves that positive stocks of money are essentially only compatible with a system including market imperfections, and absolutely incompatible with a uni-period perfect competition model; the interesting question, still unanswered as far as we know, would be if the same dictum may be made concerning the interest-rate phenomenon.

4) The rate of interest as a political price

a) TWO NORMATIVE INTEREST THEORIES

The interest rate constructions discussed, or mentioned up to this point could adequately be labeled ≫analytic theories≫. They start from certain basic assumptions about the economic activities in the society imagined and try to derive from these assumptions conclusions about the rate-of-interest-phenomenon — we particularly stress the postulate on the single value scheme of society which seems to be inherent in the ≫rate-of-growth≫ interest theories and the extreme compartmentalization of society which the Walrasian interest theories must imply.

We would further emphasize the somewhat preliminary stage at which the two types of interest rate theories still stand, the price phenomena in the firmly underbuilt programming analyzes having not been systematically explored and the interest rate phenomena in those Walrasian systems where they appear at all, being destined to run into the general difficulties of the systems. Granted that no insoluble difficulties of derivation appear, it is obvious that the theories involved are applicable to the degree that the basic assumptions coincide with reality.

The bulk of interest rate constructions, however, hardly belong to the analytic category. It was argued in the previous section that a distinction ought to be made between the Marshallian or Keynesian interest theories when more or less successfully incorporated into a Walrasian general equilibrium system and when discussed in looser, verbal versions. We are now going to follow the first line of development, mentioned in the previous section and coinciding with some important verbal arguments attached to the theories; in our opinion the line comes close to establishing a contact with

the econometric observations. The verbal arguments in question embody certain interesting and easily discernible normative elements; in fact, a large group of interest rate constructions could tentatively be called ≫normative theories≫ as contrasted with ≫analytic theories≫ — it is scarcely attempted to derive their propositions from some basic assumptions of human economic activity, the conclusions being stated almost completely ad hoc. Let us glance at these normative traits. We shall finally distinguish between two types of normative interest theories corresponding to the distinction between Marshall and Keynes, but for the sake of discussion we shall choose two still more extreme main representatives of the two camps.

As our time series only stretch back to the nineteenth century we are not going to dwell upon the fight against ≫Usury≫, but concentrate first on the nineteenth century interest rates. These are payment for ≫Abstinence≫: ≫Although Human Labour and the Agency of Nature independently of that of man, are the Primary Productive Powers, they require the concurrence of a Third Productive Principle to invest them with complete efficiency. The most laborious population, inhabiting the most fertile territory, if they devoted all their labour to the production of immediate results, and consumed its produce as it arose, would soon find their utmost exertions insufficient to produce even the mere necessities of existence. To the Third Principle or Instrument of Production, without which the two others are inefficient, we shall give the name of *Abstinence* ... To abstain from the enjoyment which is in our power, or to seek distant rather than immediate results, are among the most painful exertions of the human will ... of all the means by which man can be raised in the scale of being, abstinence, as it is perhaps the most effective, is the slowest in its increase, and the least generally diffused≫ (Senior).[1)] They also accrue to economic decision units for their ≫Sacrifice of Postponement≫ (Cairnes) or at least for their ≫Adjournment≫ (Bastiat) and ≫Waiting≫, that is ≫Postponement of Enjoyment≫ (Marshall). The interest rates fulfilling this task of payment are moreover ≫Normal≫ (Marshall, Wicksell) and sometimes even ≫Natural≫ (Wicksell), and ≫cannot be lowered by deliberate action of the community≫ (Cassel).[2)]

Disregarding insignificant differences of wording the rate of interest in all these cases is a price which the owners of money demand for putting their means at the disposal of others.

Let us now turn to the interest rates of the 1930's. In the then predominating political economy the rate of interest is a price which the owners of

[1)] N. W. Senior, *Outlines of the Science of Political Economy,* 5th Ed. pp. 58–60. (Quoted from Cassel [36].)

[2)] Cassel [36], p. 184.

money demand for *not* keeping their means in a liquid form. (Keynes 1936, Vogt). The existence of a positive rate of interest is due to such factors as ≫The Propensity to Hoard≫ and ≫The Speculation Motive≫ (Keynes), but the rate of interest may and should be fixed by deliberate action of the community, possibly lowered even to the zero point (Keynes), or lower (Vogt)[3] ≫The normative principles within modern economic theory have no place at all for rises of the interest rate, but only for a low interest rate, and hence only for downward changes of the interest rate too≫ (Vogt)[4] ≫The practical problem that appears on the basis of this new opinion about the significance of the rate of interest is the following one: how to proceed in order to establish and, keep established in the best possible way a low rate of interest≫ (Vogt)[5].

Of these two well-known contrasts, or ≫norms≫ as we shall tentatively call them, the nineteenth century construction is, above all, characterized by the fact that it places interest as a distributive share on a par with the other distributive shares like wages; the contemporary construction is characterized by the fact that it places interest as a distributive share in a special position, distinguished from other distributive shares like wages; and the special position implying that the rate of interest can be held at a constant, possibly very low, level which is not fixed by the same considerations as those determining the other distributive shares, not accruing to the savers.

As representatives of the two contrasts we choose Cassel's *The Nature and Necessity of Interest* (1903)[6] and Vogt's *Money Abundance and Interest Policy. The Modern vs. the Orthodox Viewpoints* (1947)[7] — in our opinion these works embody the norms in a somewhat more extreme and explicit form than do the writings of Marshall and Keynes, from which we started.

How are these constructions to be judged? — a question which we believe to be of great importance to economics. An economist skilled in the intricacies of the analytic theories treated in the previous sections may consider both constructions to be wrong, or at least to be prematurely made and weakly underbuilt heuristic guesses about relations, still almost unexplored by systematic scientific analysis. An economist supporting one of the constructions may consider the other wrong, and his own opinion as final and universally valid, let us say, at least for a non-communistic society.

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[3] Vogt [37], pp. 60–62.
[4] Vogt [37], p. 51.
[5] Vogt [37], p. 20.
[6] Cassel [36].
[7] Vogt [37]

There are no difficulties in finding evidence that such feelings are present in both camps. We confine ourselves to quoting Vogt: ≫According to this orthodox view the present policy of low interest rates is obviously a completely abnormal phenomenon.≫[8] — ≫There are now but few individuals wanting to identify their own opinions with those of the Austrian rentier-philosopher Eugen von Böhm-Bawerk, or the Bastiat of modern economics, Gustav Cassel.≫[9]

The opinion that both constructions are wrong or are prematurely made guesses, the opinion that the first-mentioned construction is right and the second-mentioned is wrong, the opinion that the second-mentioned is right and the first-mentioned is wrong, are all thrown into doubt by a look at our time-series. We know that the first ≫orthodox≫ construction was accepted by the majority of economists during a finite time-span, ending around 1930 — even if critics, like Marx and Lasalle, expressed practical conclusions putting interest payments in a special position and somewhat resembling those of the later epoch; the second ≫unorthodox≫ construction was accepted by most economists a few years after 1930, after the appearance of Keynes' *General Theory* — even if there are critics, who maintain the equal nature and necessity of interest and other distributive shares and approach the opinions of the earlier epoch. Thus the change in the theoretical constructions almost coincided with, or perhaps slightly lagged after, the change in the time-series relations. The opinion that both constructions are irrelevant may be questioned when taking into consideration this circumstance — is there a connection between the almost coincident changes in the two spheres: the sphere of economic theory and the sphere of economic activity, reflected by the time-series? The opinion that only the first construction is right seems difficult to reconcile with the course of time-series after 1930: at that time interest obviously holds a special position, the rate of interest being low and fairly constant despite rises in most distributive shares. The opinion that only the second construction is right seems difficult to get into agreement with the time-series facts before 1930: at that time there is no indication of a marked special position for interest, the rate of interest following fairly closely the fluctuations of the other prices.

We have already indicated the way we propose for escaping from this dilemma, namely, by labeling the interest theories involved ≫normative theories≫. We believe that neither construction is an analytic theory, capable of explaining the course of events, if its assumptions coincide with reality. Both constructions or ≫norms≫ are rationalizations of a ≫settled

[8] Vogt [37], p. 11.
[9] Vogt [37], pp. 9–10.

habit of thought common to the generality of men≫ during a definite time-period. *An analytic theory capable of explaining economic reality should take the existence of such norms into account;* it is necessary to step outside the constructions and fit them into a larger context in order to grasp their significance.

b) INTEREST-RATE NORMS AS ELEMENTS OF A WIDER THEORY

α) *The ≫norms of conduct≫ of institutionalism*

The econometric systems are claimed to be universally valid up to changes in structural coefficients; their elements of change are the structural coefficients and these elements of change we find also, of course, in the programming analyzes and in the Walrasian systems. The elements of change in the game distribution theory are the Characteristic Functions, the Solutions and the Imputations. In order to throw more light on our opinion regarding the relation between these analytic theories and the interest-rate normative theories we ask: Is it possible to express the change of norms within the analytic systems? Is it possible to sum up the change of norms in changes of structural coefficients or in changes of Characteristic Functions or Solutions or Imputations? Is it possible to couch in terms of one of these elements of change the variation from a norm placing interest on a par with the other distributive shares to a norm putting interest in a special position? If social norms of the kind indicated are important ingredients of economic activity, an analytic economic theory, the assumptions of which are realistic, should be able to comprise changes of social norms.

It is obvious that the structural coefficients of the programming analyzes and of the Walrasian systems are far from complying with these desiderata. The coefficients of the first type embody technological facts and describe the structure of the set of objectives of the one economic will in the society; the coefficients of the second type are only attached to individual decision units, and describe the structure of their preferences and of their technical equipment, except possibly for Marschak's revenue functions, ≫which do not form an independent set≫. In our opinion traditional analytic economics cannot comprise any social norms characterizing a certain society during a given period of time, but part of traditional economics consists of social norms or normative theories, ≫theories≫ which ought, among other things, to be the objects of an analytic theory explaining the course of events. Here we think particularly of the interest-rate norms indicated, but we believe the phenomenon to be quite general. It ought to be added that traditional economics itself seldom admits the normative character of certain of its constructions.

In order to find an explicit acknowledgment of the existence and importance of social norms we turn away from traditional economics, first, to institutionalist writings, developed in close contact with social reality but lacking a logical apparatus of any dimensions. Gruchy, in his abovementioned systematic discussion of institutionalist thought, speaks very much about the use of a social rather than an individualistic interpretation of human behavior and about the application of a new psychological theory to economics, which implies that against the background of the objective possibilities present, against the background of every ≫economic field≫, human beings set up certain collective standards of conduct: the economic field ≫conditions the behavior of the individual by setting up certain collective standards or norms of conduct. The individual is therefore treated as a social being whose behavior is largely collective and habitual≫.[10] Why is it so? Commons sheds some light on this question: ≫... modern man like his primordial ancestor is born into a world of 'discipline and obedience.' In this environment of socially sanctioned habits 'we continue as members of concerns already going, so that conformity to repeated and duplicated practices ... is the only way to obtain life, liberty and property with ease, safety, and consent'≫[11] ≫In all cultural areas 'custom and habitual assumptions are the underlying principle of all human relations. Each may be named a law, not in the sense of a law of nature, but in the sense of a law of human nature ... They are a law of human nature in that they go to a fundamental and ultimate principle without which man cannot live in society — the principle of Security and Expectations. It is not justice, nor even happiness, that is fundamental — it is security, even the security of injustice and poverty'.≫[12] Further: ≫The penalties for failure to learn how to adjust one's life to the demands of the working rules of the modern economy are so serious that most individuals offer little objection to the rule of custom. They are born into a world of customary transactions, and find little reason to resent the compulsions inherent in these customary modes of behavior≫[13] ≫Like Veblen, Commons was quick to catch the significance of customary behavior in economic life and to realize that, whenever economic activity is looked at from the realistic or cultural viewpoint, the fact of prime importance appears to be the tendency for economic affairs to fall into patterns of customary behavior.≫[14] ≫Patterns of customary behavior

[10] Gruchy [17], p. 19. This quotation and the following ones in the subsection are reproduced by permission of Prentice-Hall, Inc. (Copyright 1947).

[11] Gruchy [17], p. 161.

[12] Gruchy [17], p. 162

[13] Gruchy [17], p. 224.

[14] Gruchy [17], p. 225.

are the cement which effectively binds together individuals and groups, and provides coherence in society≫[15)]

The quotations indicate the rationale of ≫collective norms of conduct≫. Since an ≫economic field≫ has the basic characteristic that in general the economic actions of one participant or group of participants decisively influence the economic conditions of other participants and groups of participants, the adherence to certain collectively agreed upon rules of conduct is necessary for the society to be able to exist at all. In Gruchy's exposition a more or less clear perception of this discovery seems to be *the* uniting link between the various institutionalist writings. The institutionalists do not seem, however, to have inquired very deeply into what constitutes such a collective standard of behavior, if it can be chosen at random, or if it has to obey certain properties. They do emphasize the point on diversity and change: ≫unfortunately man is not able to enjoy his refuge in customary ways of behavior for any great length of time. There are forces at work which tend to disrupt his life, to alter his customary patterns of behavior, and to compel him to develop new ways of behaving ... In more recent centuries technological changes of many kinds have torn men away from the protection of pre-established modes of customary behavior. These disturbances of customary ways of living elicit independent thinking on the part of individuals. When men are sufficiently prodded by external events, they turn to intellectual inquiry for a solution of their distressing troubles. But once the necessary adjustments have been made and new social habits have been firmly established, the rôle of thinking again becomes minor in the life of the general population ... Economic change merely provides the setting for the development of new customs, any one of which may be selected by the human will to suit changing social conditions ... Thus, within certain limits prescribed by natural factors, there is room for the exercise of that human volition without which there could be no economic progress.≫[16)]

Thus the quotations stress the necessity of social norms or collective standards of conduct — each characterizing a certain society during a given period of time; they further stress the relativity of these standards — in particular, wide differences over time exist, the changes being caused by ≫external events≫; there is also some indication as to how a social norm, when first being established, is the result of penetrating intellectual inquiry, but later, when having prevailed for some time, becomes a matter of ≫habit≫.

[15)] Gruchy [17], p. 164.

[16)] Gruchy [17], p. 164.

β) Two attempts towards an ≫institutional≪ interest theory

The main points in the quotations are very applicable to our interest-rate problem. In our opinion, social norms are often expressed in economics, but seldom very consciously so. The interest-rate field presents certain examples of extreme cases of this rule — some recent writers come close to really recognizing a certain ≫political≪ character, particularly of the interest-rate theory and practice during the last two decades. This is partly the case with Vogt's publication, which has been already mentioned, but since the author so exclusively sponsors this theory and practice, he also comes close to denying the feasibility of all other theories and practices which are ≫wrong≪. Welinder in his *Outlines of an Institutionalist Interest Theory* (1942)[17] looks at the problem more as an outside observer; we quote: ≫Certainly all price formation is politically conditioned in the sense that politics determines the institutional framework within which the formation of prices takes place. But as long as this framework consists of legal statements or at least legal practices, which are commonly accepted as fairly self-evident, the circumstance is of no great practical importance. Conditions are changed when price formation is a direct function of economic policy. Then we have a political price, which cannot be analyzed solely with resort to such concepts as supply and demand but which is determined by the goals that the politically dominating parties have established, and by the means utilized for achieving these goals. The rate of interest has now become such a political price.≪[17a] Nyboe Andersen in the volume *The Rate of Interest* (1947)[18] expresses very much the same opinions; possibly he takes a further step in denouncing all statements which postulate ≫the existence of an 'unpolitical' rate of interest, to be explained by interest-theory, while expositions of interest-policy deal with the political manipulations of this rate of interest≪,[18] and in sponsoring theories which let the rate of interest directly stand out as a ≫political price≪.

These two elaborations of interest theory ought to be compared especially with the third-mentioned main point in our quotations from institutionalist writings, the point asserting that a social norm starts as a result of intellectual effort and grows into a habit. Both these authors look upon the interest rates of our time as political prices determined in the conscious fulfillment of some purpose, but at least Welinder openly hesitates in putting the same label on the interest rates of earlier epochs, which he apparently considers to be analyzable in terms of ≫supply and

[17] Welinder [38].
[17a] Welinder [38], p. 38.
[18] Nyboe Andersen [39].

demand≫ without the introduction of any political elements. The works suggest the hypothesis that the years immediately preceding the point of change around 1930 saw the old age of a social norm — with certain consequences among other things for the rate of interest, in which we are interested here —, which had been established through intellectual inquiry at an earlier stage and then had grown into a custom ≫accepted as fairly self-evident≫; and that the years following 1930 have seen a new social norm — with certain consequences for the rate of interest —, which is so young that the intellectual trouble in getting it to work is still felt as a quite conscious achievement.

γ) *The ≫accepted standards of behavior≫ or Solutions of the theory of games*

Exactly what constitutes a social norm, what properties has it to obey, and what characteristics must it possess? The general institutionalist statements referred to did not lead directly to this problem nor do the institutionalist writings seem to have treated it in a very precise way, nor even to have given many concrete examples of social norms or collective standards of behavior in the economic sphere. The interest rate constructions in which we are interested, e.g. as expressed by Cassel and Vogt, immediately display the following characteristics: the first-mentioned considers interest payments as a distributive share among other distributive shares or the rate of interest as a price among other prices; the second:mentioned gives to interest payments and to the rate of interest a special, excluded position among the distributive shares and the prices respectively. A glance at the time-series shows that the first-mentioned construction is not contradicted by the observations prior to the point of change around 1930, as the rate of interest follows the other prices in their general movements, and that the second-mentioned construction is not contradicted by the observations subsequent to the point of change, as the rate of interest is then fairly constant despite the rise of prices. It would be futile to investigate these empirical relations further without having explored deeper into the nature of social norms; we now turn to the theory of games for this purpose, more precisely to the concept of Solution.

In our second chapter we gave the definition of a Solution, translated in the *Theory of Games and Economic Behavior* by ≫accepted standard of behavior≫, and to a certain extent we discussed the concept.

It is usually very difficult to judge the implications of a concept on the basis of a mere definition and a general discussion; many detailed applications are also required. Here we are going first to look at the application of the Solution-concept to the Zero-sum three-person Game, three being

the highest number of participants for which every thing is so far settled in game theory. The phenomena that appear already in this comparatively simple setup must be expected to be of a very fundamental kind.

We recollect that the Reduced Essential Zero-sum three-person Game is unique, only one Reduced Characteristic Function being possible to form for $n = 3$, showing the objective possibilities of the decision units in the distribution process. Against this physical background, however, several standards of behavior may be established. The only essential line of distinction goes between ≫the objective Solution≫ and ≫the discriminatory Solutions≫. Let O be the sum to be divided between the three decision units and put $v((i))$ equal to -1 — in the reduced form the amount which can be achieved by one decision unit acting only in isolation is the same for all decision units. In the objective Solution two of the decision units get $1/2$ and one gets -1. All the decision units have the chance of getting $1/2$, but only the two decision units that really decide to collaborate and form an agreement get $1/2$ each. As three agreements may be formed — between decision units nos. 1 and 2, between nos. 1 and 3, and between nos. 2 and 3 — the objective Solution consists of three exactly delimited Imputations, every Imputation giving $1/2$ to each coalition partner and -1 to the remaining decision unit; and every attempt of one coalition partner to withhold more than $1/2$ leading to the breaking up of the coalition, as it then would become profitable for the second partner to join the third decision unit and with him establish a new ruling group. Thus the unique objective Solution of the Zero-sum three-person Game embodies the principle that every decision unit be given the same chance of entering into an agreement; every Imputation belonging to this Solution corresponds to an actually established power constellation in the society imagined, the shares being determined with regard to the maintenance of the constellation.

In a discriminatory Solution, one of the decision units, the Excluded Player, gets the amount c, c lying necessarily between these limits: $1/2 > c \geq -1$; the two other decision units together get $(O - c)$, but there is nothing to fix their relative shares of this amount. It follows that all decision units do not have the same chances within a discriminatory Solution. The Excluded Player is always assigned the amount c and nothing else. ≫The place in society of [the excluded player] is prescribed by the other players; he is excluded from all negotiations that may lead to coalitions. Such negotiations do go on, however, between the two other players: the distribution of their share, $-c$, depends entirely upon their bargaining abilities. The solution, i.e. the accepted standard of behavior, imposes absolutely no restrictions upon the way in which this share is divided between them ... This is not surprising. Since the excluded player

is absolutely 'tabu,' the threat of the partners desertion is removed from each participant of the coalition. There is no way of determining any definite division of the spoils≫[19] A discriminatory Solution does not consist of a finite number of exactly delimited Imputations. Every Imputation within such a Solution has the same c, but as only the total amount of what goes to the non-excluded players is determined — their relative shares of this amount being fixed by ephemeral factors like ≫the bargaining abilities≫ tentatively proposed in the *Theory of Games* — a discriminatory Solution does comprise infinitely many Imputations. Thus a discriminatory Solution of the Zero-sum three-person Game embodies the principle that one decision unit be given a special, excluded position; beyond this requirement, there is a fundamental indeterminacy as regards the scheme of distribution to be established. Moreover, there is no unique discriminatory Solution. Because c may be fixed everywhere in the interval $1/2 > c \geq -1$, first and essentially we get one discriminatory Solution for every possible value of c, that is a family of infinitely many, and from the point of principle very similar Solutions; secondly, we have the possibility of three such families of Solutions, since each of three participants could be excluded. The unique objective Solution and the three families of Discriminatory Solutions form all the Solutions of the Essential Zero or Constant-sum three-person Game.

We now contrast with each other the objective Solution and one family of discriminatory Solutions. When an Imputation — that is a scheme of distribution, a ≫state of society≫ — belonging to the first-mentioned Solution is actually established, this is likely to be a fairly stable one. For one of the two alternative Imputations to be put up, a basic change in the structure of the society, that is a new ruling group of decision units, a new power constellation, is required. An established Imputation belonging to the second-mentioned Solutions is apt to be of short existence, because it is changeable in two directions while maintaining the basic structure of the society, that is with the same participant excluded. Firstly, the share of the Excluded unit may be varied, let us say, lowered from the neighborhood of $1/2$ to the minimum point -1, the society thereby shifting from one Solution to very similar Solutions within the same family of Solutions, all members of the family of Solutions being based on the same qualitative principle of excluding one participant; secondly, the shares of the two other units may be varied even within the same Solution. Besides this ≫relative determinacy *vs.* deep-going indeterminacy≫ the most interesting fact arising out of a comparison between the objective Solution and one

[19] von Neumann and Morgenstern [1], p. 289. This quotation and the following ones in the subsection are reproduced by permission of Princeton University Press.

family of discriminatory Solutions is that ≫the discrimination need not be clearly disadvantageous to the player who is affected. It cannot be clearly advantageous, i.e. his fixed value c cannot be equal to or better than the best the others may expect. This would mean that $c \geq 1 - c$, i.e. $c \geq 1/2$ — which, is exactly what [the condition $-1 \leq c < 1/2$] forbids. But it would be clearly disadvantageous only for $c = -1$; and this is a possible value for c but not the only one. $c = -1$ means that the player is not only excluded, but also exploited to 100 per cent. The remaining c with $-1 < c < 1/2$ correspond to gradually less and less disadvantageous forms of segregation≪[20] In other words: the position of the Excluded participant in a family of discriminatory Solutions is better than or equal to the position of the defeated participant, that is the unit outside the ruling group, in the objective Solution — although the latter has the chance of coming into a position which is better than any possibility open to the Excluded unit.

This perhaps astonishing circumstance could tentatively be made somewhat more understandable if a sort of ≫dynamic≪ viewpoint be applied. Suppose that players nos. 1 and 2 form a ruling group within the objective Solution; external events occur to the society which make no. 1 ready to dissolve his collaboration with no. 2 and approach no. 3; it may then be that no. 2 is not immediately forced into a defeated position within the objective Solution, but that instead a discriminatory Solution is established, giving to no. 2 a share intermediary between his former place and the defeated position, the discriminatory Solution being followed by others out of the same family and the share of no. 2 gradually approaching the minimum point of c, i.e., a place equal to the defeated position within the objective Solution.

How is the distinction between an objective Solution and discriminatory Solutions preserved in Zero-sum (> 3)-person Games? The Zero-sum four-person Characteristic Function most similar to the unique Zero-sum three-person Characteristic Function is obtained by adding a Dummy to the three-person setup.[21] A Dummy is characterized by the fact that it can give no advantage to any agreement through joining it — it can contribute only with its minimum amount $v((i))$ to the sum of the shares of a coalition; i.e., the basic Characteristic Function formula becomes an equality:

$$v(S \cup T) = v(S) + v(T), \qquad \text{if } S \cap T = \Theta \tag{16}$$

when T is equal to the Dummy, let us say, participant no. 4. This Zero-sum four-person Game has the same Solutions as the Zero-sum three-person

[20] von Neumann and Morgenstern [1], pp. 289–290.

[21] von Neumann and Morgenstern [1], pp. 299 ff: Corner VIII.

Game, only with the condition added that every Imputation gives to the Dummy just its minimum amount $v((4))$, which is heuristically perfectly plausible and has also been formally proved. How is the Dummy related to the Excluded Player? ≫One might compare the dummy rôle of player 4 in this game with the exclusion a player undergoes in the discriminatory solutions [of the zero-sum three-person game]. There is, however, an important difference between these two phenomena. In our present setup, player 4 has really no contribution to make to any coalition at all; he stands apart by virtue of the characteristic function v(S). Our heuristic considerations indicate that he should be excluded from all coalitions in all acceptable solutions. We shall see [later] that the exact theory establishes precisely this. The excluded player of a discriminatory solution [of the zero-sum three-person game] is excluded only in the particular situation under consideration. As far as the characteristic function of that game is concerned, his rôle is the same as that of all the other players. In other words: the dummy in our present game is excluded by virtue of the objective facts of the situation (the characteristic function $v(S)$). The excluded player in a discriminatory solution is excluded solely by the arbitrary (though stable) prejudices that the particular standard of behavior (solution) expresses≪[22)]

We are not going to dwell any longer on this slightly, and very formally, changed three-person setup but will consider a modification of the discussed Zero-sum four-person Game with a Dummy, where the fourth player is of some importance but does not have the objective possibilities of the three other players.[23)] An agreement between two players from among 1, 2, 3 may even now be the most important strategic aim, but it is no longer the only aim: the formula (16) is not true, but instead

$$v(S \cup T) > v(S) + v(T), \qquad \text{if } S \cap T = \Theta \tag{17}$$

when $T = (4)$ and S being e.g. player 1 (or a coalition between 1 and 2). The left-hand member in this case always exceeds the right-hand member by $2(1 + x_1)$, x_1 being a parameter which takes the value -1 in the discussed Zero-sum four-person Game with Dummy, and moves to the value $+1$, every separate value belonging to one distinct Characteristic Function. At the point -1 the excess vanishes as it should, and in the neighborhood of -1, where our interest lies, it is slightly > 0. The corresponding excess of formula (17) with two players other than no. 4 is always $2(1 - x_1)$ which is 4 for $x_1 = -1$ and slightly < 4 in the neighborhood of -1.

[22)] von Neumann and Morgenstern [1], p. 301.

[23)] von Neumann and Morgenstern [1], pp. 305 ff.: The main diagonal from VIII to I up to the point $x_1 = -1/5$ and even further up to the center.

Thus the coalition between two players, other than player 4, is of a much stronger texture than any other where player 4 enters into the picture, — but the latter cannot nevertheless be disregarded. Since the first coalition is the stronger one, it may be suspected that it will form first and that once it is formed it will act as one player in its dealings with the two remaining players. Then some kind of a three-person game may be expected to take place for the final crystallization[24)]

The Characteristic Functions in the neighborhood of $x_1 = -1$ — more precisely in the interval

$$-1 \leq x_1 \leq -1/5$$

have not been completely explored, but it has been proved that they all have one Solution containing the Imputations

no. 1	no. 2	no. 3	no. 4
$\frac{1-x_1}{2}$	$\frac{1-x_1}{2}$	x_1	-1
$\frac{1-x_1}{2}$	$\frac{1-x_1}{2}$	-1	x_1
$-x_1$	$-x_1$	x_1	x_1

and the Imputations which originate from these by permuting the players 1, 2, 3 — in all 9 Imputations. It ought to be recalled that x_1 is negative, hence $-x_1$ is a gain and x_1 a loss, if the zero-sum condition be imposed.

If it be tentatively assumed that two players out of nos. 1, 2 and 3 first form an agreement of the stronger texture, decide to split even and then collaborate against the others in an unreduced three-person game, this Solution may easily be derived in a quite heuristic manner, employing the three-person discussion[25)] No other objective Solution of the games discussed has yet been found.

About possible discriminatory Solutions the authors of the *Theory of Games* state: ≫We do not question, of course, that other solutions of the 'discriminatory' type as repeatedly discussed before, always exist. But they are fundamentally different from the finite solutions which are now under consideration.≪[26)] Utilizing the same type of heuristic reasoning as applied in deriving the Objective Solution we *surmise* that the following set of Imputations is a discriminatory Solution of the games, although the proof may be of some complexity. The fourth and weakest player is assigned the amount c, which lies in this interval: $x_1 > c \geq -1$; i.e., between the highest amount no. 4 can get in the objective Solution, which it never quite

[24)] Cf. von Neumann and Morgenstern [1], p. 306.
[25)] von Neumann and Morgenstern [1], pp. 306–307.
[26)] von Neumann and Morgenstern [1], p. 307, footnote 1

reaches, and the minimum amount. Two of the remaining players, 1 and 2, or 1 and 3, or 2 and 3, form a first agreement, which according to the objective possibilities of the situation, i.e. the Characteristic Function, can secure them at least $-2x_1$ and at most $[0-c-(-1)]$, -1 being the minimum amount of the player out of 1, 2, 3 which does not succeed in joining the first agreement. There are equally strong reasons as in the objective case to believe that the two players in the first agreement must split even the total sum accruing to them — let us denote this by a —, but there does not seem to be anything to fix this amount between the limits already indicated: $1 - c \geq a \geq -2x_1$. Thus we get in our tentative discriminatory Solution the Imputations:

no. 1	no. 2	no. 3	no. 4
$\frac{a}{2}$	$\frac{a}{2}$	$-a-c$	c

c for a particular Solution fixed somewhere in the interval

$$x_1 > c \geq -1$$

a in a particular Solution varying in the interval

$$1 - c \geq a \geq -2x_1$$

and the Imputations which originate from these by permuting the players 1, 2, 3.

This tentative Solution consists of infinitely many Imputations and is above all characterized by the facts that the fourth and weakest player is excluded and that there is a fundamental indeterminacy as regards the division of the proceeds between the ≫first coalition≫ and the third non-excluded player, this income-relation depending, within limits, on such factors, as ≫bargaining ability≫; the tentative Solution, moreover, belongs to a family of Solutions, the members of which are distinguished from each other only through the value of c, the amount assigned to the Excluded participant. We find that the indeterminacy or variability extends in the same two directions as in a family of discriminatory Solutions of the Zero-sum three-person Game; and what was said there about changes of Imputations without changes of the basic group structure of society also applies here.

Can Characteristic Functions including only three or four participants be of any realistic importance? We have already discussed this subject in speaking about the hierarchy of distribution problems,[27] and mention

[27] Cf. pp. 60–63 above.

here that laborers, farmers and entrepreneurs, seem to be the *three* interest-groups of dominating importance in many modern societies supplemented by a *fourth* group which is of decidedly minor importance but cannot be altogether disregarded, namely the savers or the rentiers — in general the individuals who are dependent in some way on interest as a major source of income. It will be pointed out later that observations pertinent to modern economies show a remarkable similarity with the features of the tentative discriminatory Solution in the four-person game just discussed: the group of savers is in the position of the Excluded Player, the groups of laborers and farmers form ≫the first coalition≫ and the group of entrepreneurs is the remaining, non-excluded participant.

In the whole set of Zero-sum four-person Characteristic Functions, even outside the ≫main diagonal≫ with which we have dealt above, various objective finite Solutions have been established — they are there no longer necessarily unique. It has further been proved that every Zero-sum four-person Game has at least one discriminatory Solution — with one player excluded and in fact even pushed to his minimum in the cases explicitly considered — containing infinitely many Imputations and showing to various extents the characteristic indeterminacy of the division of the shares between the remaining three participants; i.e. an established Imputation may vary without causing any basic shifts in the power constellation of the society.[28)]

In the only Zero-sum n-person Characteristic Function thus far completely explored — the Simple Game $[1, \ldots, 1, n-2]$[29)] — there is a unique objective or finite Solution [Case (II)], giving to all players the chance of entering into agreements which is permitted by their objective possibilities and containing a finite number of strictly delimited Imputations, just as is the case in the three-person Characteristic Function; the remaining Solutions do not give ≫objective≫ chances to all decision units and they comprise infinitely many Imputations, the indeterminacy within a Solution extending from the Solutions where only the relations between one participant (the Chief Player) and a group of participants show a variability which may be called ≫some kind of indefinite bargaining≫ — the relative shares within the group being completely fixed and the participants outside the Chief Player and the group getting a fixed amount [Case II″ with 55.12.5 (a) or (b)] —, to the Solutions where the Chief Player is excluded, i.e. assigned a given amount and the relative shares of all the remaining participants are undetermined [Case (I)]; moreover, the most clearly discriminatory,

[28)] von Neumann and Morgenstern [1] : The section on General Games, pp. 550–555; see particularly a remark under 60. 4. 2.

[29)] von Neumann and Morgenstern [1], pp. 473–503.

infinite, Solutions belong to families of Solutions, the members of which are distinguished from each other only through the value of the amount assigned to one Excluded Player.

Thus this particular n-person Characteristic Function also shows the distinction between; on the one hand, an Imputation — within a finite Solution, giving to all participants their objective possibilities of entering into agreements — which is likely to be stable if established, as it corresponds to a certain power constellation in society, and its alternatives within the same Solution needing quite other groupings of the decision units in order to be adopted; and, on the other hand, an Imputation — within an infinite Solution, not giving to all participants their objective possibilities of entering into agreements but discriminating in various ways — which is apt to be of short existence, because it may vary without effecting any basic changes in the structure of society, depending as it does on ephemeral factors like ≫the bargaining ability≫ of some players or groups of players; to this indeterminacy must be added the variability arising out of shifts within a family of Solutions of the kind indicated.

c) THE OBJECTIVE VS. THE DISCRIMINATORY SOLUTIONS

No empirical identification of the Solution-concept has, so far as we know, been hitherto attempted in the literature. It is in the nature of things that an accepted standard of behavior is extremely difficult to observe at a given point of time; through the comparison of conditions at two points of time it may stand out quite clearly, however, that there exists such an entity which has changed from one time point to the other. We now venture the suggestion that the interest-rate norms of Cassel and Vogt express accepted standards of behavior, each prevailing at the point of time when the respective work was written; in other words, we believe that the concept of Solution can take into account empirical phenomena like the two alternative interest-rate norms, more so than the vague indications by institutionalist writers on ≫norms of conduct≫; and by the above-mentioned ≫institutional≫ theories of the rate of interest — which, however, move in the same direction. The first-mentioned social norm is ≫objective≫ with regard to the economic interest-group depending on interest as a main source of income, i.e. the savers: it gives to this group its objective possibilities in the process of distribution, i.e. treats it on a par with the other big social groups; the second-mentioned social norm, is ≫discriminatory≫ towards the savers: it excludes them from all distribution-negotiations, assigning to them a special position, within limits determined by the other big economic interest-groups of society. To explain why a certain social norm prevails during a definite time-period and why this norm changes in a certain manner at a definite time-point would possibly be a topic of

social psychology; the qualitative observations of the history-of-doctrines type indicate that a decisive change of social norms has taken place around 1930 and the theory of games gives a theoretical structure capable of comprising such sharp changes, which is remarkably different from the potentialities of traditional economics. We have labeled the time-point or rather time-interval when the change occurred a ≫point of change≫ or a ≫year of change≫. Synonymously with these expressions the term ≫structural limit≫ — borrowed from the writings of Johan Åkerman[30] — might adequately be utilized.

Thus far we have only dealt with the Solution element of change, more precisely with a distinction between an objective Solution and discriminatory Solutions. Our suggestions must be checked by introducing the Imputation element of change and comparing the observations before and after the point of change with the features of Imputations in an objective Solution and in a discriminatory Solution respectively.

During the reign of the first norm a particular institution took care of the savers' interests in many societies: an independent central bank, with instructions to preserve the value of (the savers') money. In order to attain this goal the central bank utilized certain codified monetary techniques. During the reign of the second norm the central banks have lost their independence and become subordinated in a first step to the government, in a second step to the economic groups dominating the political arena, the laborers, the farmers and the entrepreneurs. In order to fulfill their new instructions, e.g. to keep a constant, low, interest rate, the central banks have adopted new monetary techniques, which, when first applied, seemed extraordinary and unnatural in comparison with the old techniques, but which are perhaps apt to become as self-evidently accepted as these. What are the distribution consequences for the savers of these varying functions of the central bank under the two social standards? We now connect our analysis directly with the quantitative time-series observations. A Solution, i.e. ≫an accepted standard of behavior≫, or as we have said in some places ≫a social norm≫, comprises many alternative Imputations, i.e. schemes of distribution. A particular Imputation shows fixed relative income shares of the various decision units out of the total amount. An Imputation prevailing over time could perfectly well show variations of the nominal income shares of the units, but the relation between any two shares should tend to remain constant. Comparing time-series representing the shares, this constancy would obviously imply a close correlation between the series, extending to proportionality between their variations and equality between

[30] Cf. Johan Åkerman [14], e.g. II, pp. 568–569; [15], e.g. p. 516; [16], e.g. p. 797.

their percentage increases and decreases. An Imputation belonging to an objective Solution — at least as far as this concept has been used above — is rather delimited from its alternatives, as these correspond to quite other power constellations within the society; build on other ≫political premises≪; thus it is likely to show a certain persistence over time. In our opinion the positive correlation, or rather proportionality between the general price level and the rate of interest — basically unexplained by traditional theories according to, e.g. Tinbergen[31)] —, can adequately be looked upon as expressing the maintenance during the period, if not of a particular Imputation within an objective Solution, yet of a limited category of Imputations within an objective Solution giving to the savers — whose income-share may be represented by the rate of interest — in the long run a fairly unchanged position in the distribution process relative to the positions of the other big interest-groups of society, e.g. the laborers, the farmers and the entrepreneurs — whose combined income share may be represented by the general price level; in other words and using modern phraseology: giving to the savers an index-regulated income-share. Probably a closer scrutiny of the period before the point of change would reveal subperiods, each with a somewhat different relation between the savers income-share and the remaining combined income-share.

The course of the time-series after the 1930 point of change indicates that the position of the savers during the previous period was no ≫defeated≪ one with the relative income-share of the savers pressed to its minimum. The process, originated around 1930, has striking similarities with the above-mentioned shifting within a family of discriminatory Solutions of the Excluded Players assigned amount from a position close to a non-minimal position in an objective Solution down towards the minimum amount. The fairly constant interest rate combined with the rising prices has meant a gradual deterioration of the position of the savers relative to the positions of the laborers, the farmers, and the entrepreneurs, decreasing the relative income-share of this group. The variation has taken place despite fairly stable political conditions within countries, thus illustrating one of the basic indeterminacies of a family of discriminatory Solutions.[32)]

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31) p. 126 above.

32) In the article in Swedish by Erik Dahmén, "Bedrägeriet ot spararna" ("The Treachery against the Savers"), Balans 1949, some observations are made very congenial to our exposition. The author surveys the development of the income-shares of the Swedish big social groups from 1938 to 1948, and finds an improved relative share of the farmers and of the laborers, a fairly maintained share of the manufacturers, and a strongly diminished share of the savers; he even tries to calculate ≫the loss of the savers≪

Let us summarize our arguments concerning the rate of interest as a political price. Before 1930 we observe the strong positive correlation between the general price level and the interest rate, implying a fairly constant relative income-share of the savers; we supplement this quantitative observation by mentioning the independent central banks acting in the time-span concerned and the social norm prevailing, putting interest explicitly on a par with wages and other distributive shares. We explain these observations by connecting them with one Imputation or a limited category of Imputations within an objective Solution,[33] established on the background of a Characteristic Function or a limited category of Characteristic Functions involving only a low number of active units, corresponding to the big social groups on the highest level of the distribution hierarchy. An objective Solution or ≫accepted standard of behavior≫ treats the units symmetrically in the sense that it gives to each in the distribution process its objective possibilities as embodied in the Characteristic Function; and an Imputation, a particular set of relative income-shares, within an objective Solution is likely to show a certain stability over time as it is clearly delimited from its alternatives, which require for their establishment in general quite a different power constellation in the society.

After 1930 we observe a fairly constant interest rate and a rising price level, implying a gradual deterioration of the relative income-share of the savers; we supplement this quantitative observation by mentioning the cessation of the independence of the central banks during the time-interval in question and the social norm prevailing, putting interest explicitly in a disfavored position among the distributive shares. We explain these observations by connecting them with a number of successive Imputations, varying within a family of discriminatory Solutions, established on the same kind of Characteristic Functions as before.[34] A discriminatory Solution or ≫accepted standard of behavior≫ treats the units unsymmetrically in the sense that it assigns to one unit its relative income-share, thus excluding this unit from taking an active part in the distribution-negotiations; moreover, a discriminatory Solution belongs to a family of qualitatively exactly similar Solutions, distinguished from each other solely by the value of the relative share assigned to the Excluded unit, and thus a discriminatory Solution shows a basic variability; and an Imputation, a particular set of relative income-shares within a discriminatory Solution, showing another

during the period, due to the fairly constant interest rate combined with rising prices.

[33] We think here mainly of the three- and four-person games referred to above; no general statement can be made.

[34] Cf., however, Chapter VII, Section 4: b: δ.

kind of basic variability as there is nothing but ephemeral factors like ≫bargaining abilities≪ to fix the relative position between (some of) the non-excluded units.

In this chapter we emphasize particularly the rôle of the first-mentioned variability — a variability between qualitatively exactly similar social norms — for the gradual deterioration of the savers position; in our next chapter on inflation we shall further scrutinize the second-mentioned variability and its connection with the first-mentioned one.

5) Summary

a) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS

First, a type of interest rates is discussed, which is found in systems established by Jevons, Wicksell, Hawkins and von Neumann. This rate of interest is equal to the maximal rate of growth of the economy studied, granted the assumption that all sectors of the economy grow at the same pace. If logically completed, these systems — as was shown in Chapter II — turn out to be planning models, requiring the establishment of one social value scheme. So far the rate of interest = the rate of growth has only been introduced into planning systems in such a way that its magnitude must be invariant under changes of the social value scheme: all conceivable real parts of the systems, i.e. all conceivable actual plans that may e carried out with a given setup of production processes, greater in number than the products to be produced, show the same rate of interest = rate of growth. This is certainly due to the existence of only one parameter of growth in the models, letting all sectors grow proportionally at the maximal rate. If several parameters of growth be introduced, admitting a non-proportional development of the sectors of the economy, this is likely to make the magnitude of the various rates of interest = rates of growth dependent on the particular social value scheme adopted. It is surmised that the development of interest theory in this direction might establish a contact with empirical features of centrally planned economies.

Second, the rate of interest is discussed in connection with the distribution process of a multi-unit-of-decision society. It is assumed that the variations of the relative money income of the social group ≫savers≪ or ≫rentiers≪ can be represented by the variations of the ratio between the rate of interest and the general price level. The variations of the savers relative money income is examined, utilizing concepts and theorems of the Zero or Constant-sum Essential Games.

Thus the general dualism between a production aspect and a distribution aspect — in our opinion implied by the apparent contradiction between the two summation theorems — is exemplified in a special context: the field of interest rate analysis. Two widely different rôles of the rate of interest are distinguished: the rôle of the rate of interest in a planned economy, stating the maximum possible proportional rate of growth of the economy and the rôle of the rate of interest in a free group-economy, representing in its relation to other prices the relative income share of the savers.

b) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS AND EMPIRICAL RESEARCH

Our examination of the two summation theorems in relation to each other leads to the acceptance of the dualism between the production aspect and the distribution aspect. The general dualism is exemplified in the special field of interest rate analysis. The next logical step should be the investigation of empirical interest rate phenomena utilizing the production aspect — essentially for studying features of a centrally planned economy; and the distribution aspect essentially for studying features of a free group-economy. Only the second task is attempted here.

Naturally such an investigation of empirical phenomena in contact with a theory is centered around various elements of change of the theory — certain ≫empty boxes≫ of the theory have to be filled with the relevant data, i.e. certain elements of change of the theory must be identified with time-series and other observations. Comparing the distribution theory of the Inessential Games and the Walrasian systems, implying additivity of monetary economic values — we recall that this expression is equivalent to the following one: the distribution of no part of the money national income has to be fixed through agreements —, and the distribution theory of the Essential Games, implying non-additivity of monetary economic values — we recall that this expression is equivalent to the following one: the distribution of part of the national income has to be fixed through agreements —, the basic difference is seen to consist in the character of the elements of change of the theories. The additive setups embody variations between sets of structural coefficients and between non-reduced Characteristic Functions; the non-additive setups rule out variations of this type as insignificant and add variations between reduced Characteristic Functions, between Solutions and between Imputations.

Starting from time-series of interest rates and related variables, an additive setup expressed in terms of a system of simultaneous equations should attempt to derive the values of certain structural coefficients from statistical manipulations with the observed variables, thereby identifying

certain elements of change with the observed phenomena. There is an inherent difficulty in this procedure. If a short period of time be considered, the structural coefficients to be estimated could plausibly be assumed to remain approximately constant within the period — but then the statistical estimates become uncertain, due to the low number of observations involved. If a long period of time be considered the statistical estimates become more certain, due to the greater number of observations involved — but then the structural coefficients to be estimated could plausibly be assumed to vary significantly within the period. In part to avoid this dilemma it has been proposed by, e.g. Johan Åkerman, that ≫structurally homogeneous≫ time periods be segregated by certain criteria, and structural coefficients be then determined from variables only relating to one period of this kind. The procedure would imply that the values of sets of structural coefficients were likely to change abruptly at certain time-points. Nothing in the additive setups confirms such a view.

Starting from time-series of interest rates and related variables, supplemented, by observations of other kinds, we tentatively try to identify certain types of elements of change of Essential Games with the observed phenomena. Due to the character of the elements of change the process of identification is quite dissimilar to the determination of structural coefficients. It should be particularly stressed that the elements of change involved can naturally experience abrupt alterations at certain time-points or ≫points of change≫. The basic alteration considered is, on the empirical plane, the substitution of the ≫unorthodox interest theory≫ for the ≫orthodox theory≫ around 1930; with this alteration we have identified the substitution, on the theoretical plane, of a discriminatory type of Solution for an objective type of Solution. The first type of element of change embodies the principle that all claims for income shares considered — we think of the claims of the social groups including the savers — should be treated in an equal manner; the second type of element of change embodies the principle that at least one of the claims for income shares considered we think of the claim of the savers — should be placed in a special, discriminated position.

On the empirical plane, the prevalence of the orthodox interest theory is correlated with the action of independent central banks for the preservation of the value of (the savers') money and with a fairly maintained position of the savers' income share relative to the income shares of the other social-groups. The existence of the unorthodox interest theory is correlated with the cessation of the action of independent central banks for the preservation of the value of (the savers') money and with a gradual deterioration of the savers' income share relative to the income shares of the other social groups. On the theoretical plane, an objective Solution is not likely to be easily changed if it embodies a distribution principle

qualitatively different from other feasible principles; in any case it implies that an established, constituent Imputation should be fairly stable, as this Imputation is clearly delimited from other Imputations within the set, being changeable only together with a basic alteration of the power constellation of the society. This calls for a relative stability over time in the relations between the various income shares during the reign of an objective type of Solution. A discriminatory Solution can easily be changed as it belongs to a family of qualitatively exactly similar Solutions, distinguished from each other only through the value of the amount assigned to the unit, excluded from active participation in the distribution process; moreover, it implies that an established, constituent Imputation should be fairly unstable, as this Imputation is only insignificantly different from the other Imputations within the set, being continuously changeable without any basic alteration of the power constellation of society. These features taken in conjunction call for an instability over time in the relations between the various income shares during the reign of a discriminatory type of Solution; the first-mentioned variability sets the direction of the changes as regards the Excluded unit: its income share may be gradually shifted through less and less favorable positions towards the minimum point. Thus the empirical phenomena correlated with the two normative interest theories, accepted during distinct time-periods, fit quite well into the theoretical consequences of the type of Solution which we have identified with the relevant interest rate norm.

c) CONCLUSIONS ON THE RELATION BETWEEN THE FIRST SUMMATION THEOREM AND OTHER ECONOMIC THEORIES

It is sometimes attempted to cramp the two normative interest theories into the mold of a Walrasian system, thereby depriving them of important, verbally expressed features. We have viewed these ≫theories≫ as condensed expressions for empirically prevailing social norms of conduct or accepted standards of behavior, and identified them with certain types of elements of change of game theory. We introduce the term ≫time-limited theories≫ to denote economic theories which are valid only as long as a particular element of change or category of element of change of game theory prevails.

Chapter VI

Inflation

1) The Time-series

Dealing with general price movements our analysis in the previous chapter partly intrudes into the field of inflation — which is not surprising as the connection between interest rate phenomena and inflationary phenomena is generally recognized. We shall now explore further into the related field of inflation. The extension is partly a necessity in order to fulfill the preceding analysis, but covers also completely new ground. We take our starting-point in two types of observations.

The first type coincides with the charts of the previous chapter,[1)] but here we place the emphasis on somewhat different traits of the time-series. We observe that upward movements of the price level, followed by downward movements, took place during the period of positive correlation between the price level and the interest rate, and particularly we stress the pronounced and persistent upward movement of the price level during the period of approximately constant interest rates. This study of the time-series leads to a question about the connection between inflation and interest rates, which was only implicitly treated in Chapter V.

The type of observations mentioned pictures an interior aspect of inflation: it deals with the movements of a national interest rate relative to a national price level. The second type of observations deals with an exterior aspect of inflation. Chart VI illustrates the well-known and very pronounced positive divergence between the internal value and the external value of the Mark in the early twenties, analyzed in Bresciani-Turroni, *The Economics of Inflation* (1931).[1a)] The chart is in principle constructed as follows. Relative to a certain base period the changes of the ratio between the price level of region A and the price level of region B are compared with the changes of the corresponding exchange rate, i.e. the value of the currency of B in terms of the currency of A. If the movements of the price-ratio tend to coincide with the movements of the exchange rate, then equality persists between the internal and the external values of each

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[1)] pp. 127–129 above.

[1a)] Bresciani-Turroni [40].

currency; if the exchange rate tends to surpass the price ratio, then the external value of the currency of the favored country B exceeds the internal value, and the external value of the currency of the disfavored country A falls short of the internal value — granted the fact that the base period chosen shows approximate equality between the two values. Hence in Chart VI region A corresponds to Germany and region B corresponds to the rest of the world — as a representative of this area we may choose USA: American (export) prices, in dollars or gold, tend to surpass German (export) prices, in dollars or gold.

It is seen how the positive divergence between the internal and the external values of the Mark rapidly rises to an extreme peak in the beginning of 1920, from which it rapidly falls, no doubt due to the impact of factors exogenous to the German inflation: the crisis of 1920 and the ensuing international deflation. In 1921 the positive divergence begins to fluctuate again around a rising trend, until in 1923 it fairly abruptly disappears.

There is a second way of expressing the facts of Chart VI: a time-lag may be said to have existed between the rises of German import prices and the rises of German (domestic and) export prices during the period of the German inflation, all prices measured in the same currency, e.g. in dollars.[2)] At the time-point where the positive divergence between the internal and the external values of the Mark disappears, the time-lag must be zero: the adjustment of export prices to import prices is instantaneous.

We express the facts of Chart VI in still another manner. The period of German inflation shows a very pronounced decline of the German ≫terms of trade≫, i.e. the ratio between the German export prices and the German import prices at the same moment of time. When the positive divergence between the internal and the external values of the Mark rapidly increases, the terms of trade rapidly deteriorate; when the divergence disappears, the terms of trade are restored to their pre-inflation value. It must be emphasized that this deterioration of the terms of trade can be no result of a long-run change of productivity or similar real entities; it is coupled with the short-run monetary changes of the inflationary period, and hence also with the internal political and economic disorder and the external weak position of Germany.

[2)] Cf. Jaques J. Polak, "European Exchange Depreciation in the Early Twenties," *Econometrica* 1943, p. 155, and equation (5), p. 154.

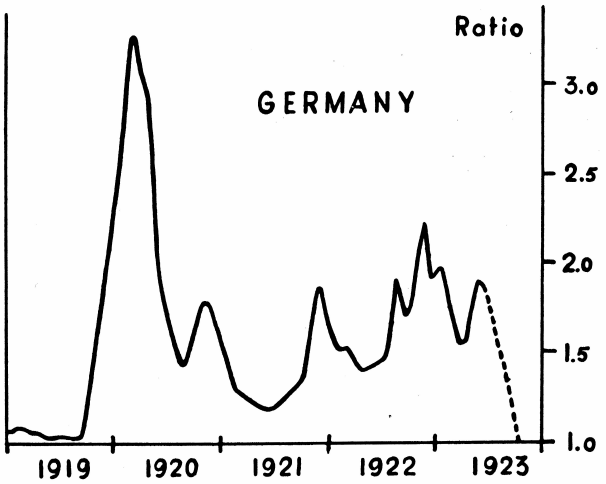

Chart VI. Ratio between Internal Value and External Value of the MarK 1919–1923 (as measured by the ratio of gold prices of German Import goods to gold prices of German domestic and ex port goods). Reproduced by permission from *The Economics of Inflation* by Costantino Bresciani-Turroni, George Allen & Unwin Ltd, London 1937.

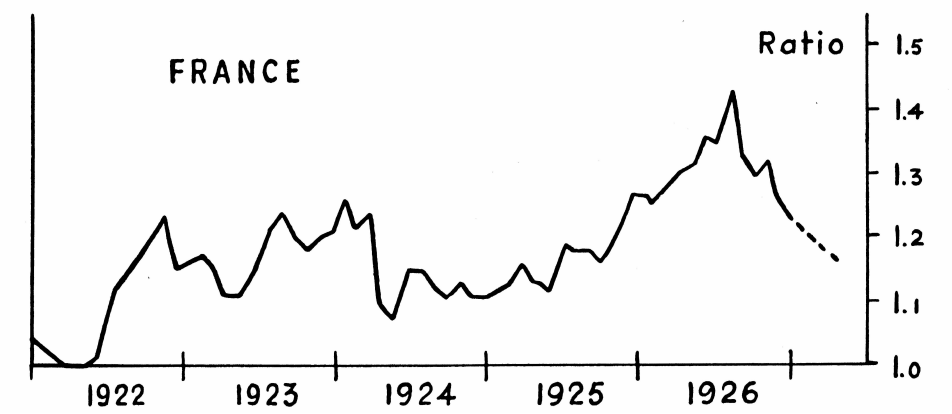

Chart VII. Ratio between Internal and External Value of the franc 1922–1926 as measured by the ratio of the value of the dollar in terms of the franc to the relation between French and American wholesale prices). The figures are taken from James W. Angell, *The Theory of International Prices,* Harvard University Press, Cambridge 1926, and Albert Aftalion, *Monnaie, prix et change,* Recueil Sirey, Paris 1927.

All the three different modes of expression lead up to a recognition of the most important international aspect of the German inflation: the change of the distribution of income between countries. The ratio between the German money income and, let us say, the American money income — both incomes measured in the same currency — is apt persistently to change in American favor as long as the positive divergence between the internal and the external values of the Mark increases, and the German terms of trade deteriorate; when the divergence disappears and the terms of trade are restored to their pre-inflation level, the relation between the two money incomes reaches its previous value. It should be stressed also in this context that the changes of the income relation take place during a process which nobody would hesitate in calling inflationary — there are certainly other types of changes of income-relations between countries.

A comparison between the observations relating to Germany 1919–1923 and observations from some neighboring regions and periods gives the following result. During the 1920's Austria and other Central European countries experienced an inflation similar to the German one; also, for instance, France showed inflationary periods with a marked positive divergence between the internal and the external values of the franc, implying a deterioration of the ratio between the French money income and the American money income, but the movements were not so violent as in the German case.[3)] Chart VII measures the ratio between the internal and the external values of the franc during one of the periods in the twenties which showed a positive divergence: 1922–1926. The general rule during the 1920's was, however, an approximate coincidence between the internal and the external currency values. During the period of the internationally accepted gold standard in the twentieth century before the First World War and in the last decades of the nineteenth century a pronounced and persistent divergence between the internal and the external values of a currency, tied to an inflationary process, can hardly be imagined. In fact, it seems that the gold standard was based on the very coincidence of the two values, a divergence leading to gold movements, causing the disappearance of the divergence. This general tendency is illustrated by the approximate equality between American and British prices in gold from 1880 to 1930.[4)]

[3)] Cf. Albert Aftalion, *Monnaie, prix et change,* Paris 1927, pp. 47, 49 and 271–289, and James W. Angell, *The Theory of International Prices,* Cambridge, Mass. 1926, p. 438.

[4)] See Fig. 4: "Index Numbers of Wholesale prices in Great Britain and United States in Terms of Gold, 1830–1930," p. 148, in Hugh B. Killough and Lucy W. Killough, *Economics of International Trade,* New York, etc. 1948.

There are, however, many examples from earlier periods of a divergence between the values in times of suspension of the gold standard.[5)]

In the 1930s the conditions for measuring divergences between internal and external values of currencies under processes of monetary change grew worse relative to the conditions of the 1920's. The first part of the decade was dominated by cyclical factors, which added to the picture of monetary change completely new traits; moreover, certain quotations of exchange rates were only nominal, and the selection of a base period for the calculation became more troublesome as the distance to the pre-war period of the gold standard lengthened. It is, however, commonly concluded that there existed towards the end of the period an approximate equality between the internal and the external values of most currencies.

After the Second World War, the statistical difficulties as to the nominal character of quotations of exchange rates and the selection of a base period are still more stressed. But certain observations display features strongly resembling earlier experiences of a divergence between the internal and the external values of currencies: more particularly, there seems to have existed in the years following the Second World War a marked pressure towards an increase of American dollar prices relative to European dollar prices, implying an excess of the dollar's external purchasing power over its internal purchasing power, a gradual deterioration of the European terms of trade relative to USA and a gradual change in American favor of the ratio between the money incomes of USA and of the European countries. The tendency is clearly indicated by the rise of American prices of export goods to Europe in relation to American prices of import goods from Europe — in 1948 and 1949 the ratio between these export and import price indexes has been calculated for England and found to average about 150 as compared with 100 in 1937 and 1938[6)]; this trend certainly led to the crisis of the pound and other European currencies with the ensuing devaluation in 1949. The economic impact of the Korean War seems partly to have reversed the direction of the movements, but certainly it is too early to judge whether we here face a definite reversion and a solution of the ≫dollar problem≫ or only a minor deviation from the dominating trend.

This study of various observations leads to a question about the connection between inflation and international economic relations.

[5)] Cf. Bresciani-Turroni [40], Chapter III.

[6)] United Nations Department of Economic affairs, *Economic Survey of Europe in 1949,* Geneva 1950, p. 151. It should be observed that the ratio does not measure the actual deterioration of the European terms of trade, due to the possibilities utilized of transactions in third markets.

These are the two categories of facts which we want especially to discuss: the pronounced upward movement of prices during the period of approximately constant interest rates after 1932; and the divergence between the external and the internal values of certain currencies observed during certain periods of the 1920's, accompanied by heavy upward movements of prices — a divergence to be viewed in connection with the pressure after the Second World War towards a faster increase of American than of European prices, all prices measured in the same currency, and contrasted with the general conditions prevailing especially during the period of the internationally accepted gold standard, showing an approximate coincidence between the external and the internal currency-values. Thus the time-series have suggested to us certain problems. We now turn to the theories of monetary phenomena, based on other types of observations than time-series, i.e. broad observations of human motivations, of human activities in pursuit of economic and social goals. Our final purpose is a contact between the time-series observations and certain theories, this contact being supplemented by observations of a more qualitative character. The general procedure and the purpose are the same as those in the previous chapter.

2) Money and the Walrasian constructions

a) THE QUANTITY THEORY OF MONEY

The disposition of this chapter is slightly different from that of Chapter V — no section is here found corresponding to Section 2 in Chapter V, which discusses the relevant problem from the pure distribution aspect. The reason for this difference is simply that we have not succeeded in finding any connection between the pure production aspect at the present stage of its development and inflationary phenomena, absolute prices, e.g. playing no rôle in the production analyzes and no income distribution problem appearing within the aspect. Hence we continue directly by considering the relation between the Walrasian constructions and monetary, particularly inflationary, phenomena.

The quantity theory of money may be presented in a fairly loose, heuristic form, stating, e.g. simply that ≫the price level tends to vary proportionally to the quantity of money in the community≫; and it may be somewhat further developed from this position, if formalized in the equation of exchange: $MV = PT$, where V, the velocity of circulation, and T, the volume of trade, are introduced besides M, the quantity of money and P, the price level. Such variants of the quantity theory of money are extremely mechanical and unsatisfactory in the sense that they do not refer to any human economic action of any kind; they only postulate a relation to hold without deriving it from some basic assumptions on human economic

behavior. This does not exclude that the quantity theory of money in its simplest form be in accord with observed facts during certain periods; it indicates, however, that the theory gives no sufficient and satisfactory explanation of these facts. Thus inflation — put identical to a rise in the general price level, P, defined in some way — is, according to the theory, caused, by an increase of the total quantity of money, M, barring certain variations of V and T.

So far as we know the idea to rationalize the quantity theory of money through combining it with an ordinary Walrasian system first explicitly occurred to Divisia[1)]; a Walrasian system proper determines the relative prices which equilibrate supply and demand of ordinary commodities; the equation of exchange could be superimposed on this system to determine the ≫multiplicative factor≫ of prices, i.e., the absolute price level. The basic rationale — in our opinion — of a Walrasian system must be its building on certain assumptions of human economic behavior, i.e., of utility (and profit) maximization, even though this maximization is carried out by quasi-isolated decision units. The question now appears about the compatibility of the superimposed monetary equation and the assumption of individual utility maximization. Not until recently was this problem scrutinized by Patinkin and Marschak, completing earlier works by Lange and Mosak. We shall now *first* examine Mosak's cautious combination of the quantity theory of money and a Walrasian construction in order to find a good formulation of the quantity theory (and subsequently of the purchasing power parity theory); *second,* we shall dwell briefly on the researches by Patinkin and Marschak.

Mosak[2)] states the well-known result that the n-commodity Walrasian system only determines $(n-1)$ relative prices, i.e., it determines all prices relative to the price of one ordinary commodity, *the numéraire*: it does not determine the value of the numéraire in terms of the eventual ≫token money≫, and hence it cannot determine the n absolute prices in terms of the token money. To determine the absolute prices Mosak introduces an equation of exchange in the following way. We denote by m the sum of money held by an individual, by K the proportion of m to his income r, and put $M = \sum m$, $R = \sum r$ — thus M is the fixed total quantity of money in the community, and R is the total value of all incomes; we further let q_i represent absolute prices, p_i relative prices, s_i quantities with

[1)] F. Divisia, *Économique Rationelle,* Paris 1928, pp. XXI–XXII, 402–415; cf. the works of Gustav Cassel, although the latter did not derive his Walrasian behavior equations from an assumption of human economic action, but stated them ad hoc.

[2)] Mosak [25], pp. 36–38.

a single individual and S_i quantities summed over all individuals. For one individual is valid:

$$r = \sum_{i=1}^{n} q_i s_i \qquad \text{and} \tag{1}$$

$$m = Kr = K \sum_{i=1}^{n} q_i s_i = q_n K \sum_{i=1}^{n} p_i s_i \qquad (p_n \equiv 1). \tag{2}$$

Summing over all individuals we get:

$$M = \overline{K} R = \overline{K} \sum_{i=1}^{n} q_i S_i = q_n \overline{K} \sum_{i=1}^{n} p_i S_i. \tag{3}$$

Given M and $\overline{K}$, q_n is uniquely determined:

$$q_n = \frac{M}{\overline{K} \sum_{i=1}^{n} p_i S_i}. \tag{4}$$

q_n is the multiplicative factor of prices or the general level of prices; thus the general level of prices is directly proportional to the quantity of money and inversely proportional to the fraction of income which is held in the form of money.

Mosak's particular introduction of a monetary equation — (4) — into a Walrasian system has the chief merit that it does not resort to any index-number-constructions. It lets the general price level depend on two constants, M and $\overline{K}$, and one variable sum, $\sum_{i=1}^{n} p_i S_i$, determined by the non-monetary part of the whole setup. If the constant M be increased, it follows that the general level of prices must increase proportionally but no further consequences necessarily ensue — no quantities, no relative prices, nor the ratios of money incomes are changed as a result of this increase. Thus in Mosak's extended system inflations — identical to upward movements of the general price level, i.e., of the multiplicative factor of prices — only cause all money incomes to increase proportionally. We underline this fact and express the opinion that the quantity theory of money is significantly characterized as a theory of inflation which lets the ratios of money incomes within a country remain fixed throughout the inflationary process. If $\frac{1}{q_n}$ be called ≫the value of money≫, an increase of M, the total quantity of money, would of course imply a decrease of the value of money, but this decrease would be a completely harmless one, because all money incomes increase automatically in the same proportion so as to ≫compensate≫ for the fall of the value of money. It may be surmised that

inflations would hardly have attracted any major attention if the core of the phenomenon be revealed by the insignificant variations of q_n in proportion to variations of M.

Researches by Patinkin[3] and Marschak[4] have shown that even Mosak's cautious superposition of a monetary equation on a Walrasian construction is untenable. This gives a situation somewhat reminiscent of the relevant problem in Chapter V, Section 3. There we spoke of the inclusion of Marshall's and Keynes' interest theories in a Walrasian system — we maintained that the possibility of an inclusion was highly doubtful, and attempted later on to develop the heuristic interest theories in quite another direction. Here we discuss the quantity theory of money and its inclusion in a Walrasian setup — we have just mentioned the existence of results which show the impossibility of including the theory in an ordinary Walrasian system and we shall subsequently utilize the theory in another context. Its rôle will, however, be much less significant than that which we attribute to the normative interest theories.

Patinkin sets out to study if people really under Walras-Mosakian conditions should keep money-sums m in proportion to their incomes. His conclusion, further substantiated by Marschak, states that the combination of an equation of exchange with such a Walrasian construction, or more widely the introduction of paper or other token money into such a system, is not only superfluous but also creates an inconsistency of the inclusive system.

The proof of this very interesting fact may be sketched thus, following Patinkin and Marschak. Suppose that we add a $(n+1)$:th commodity — paper money — to Mosak's system. For the sake of simplicity we consider the supplied or ≫initial≫ quantities s_i for all individuals of the various commodities as given. By their very nature all demanded quantities for all individuals are non-negative i.e., $d_i > O$. This restriction may be written in the following convenient form

$$d_i = r_i^2 \qquad (i = 1, 2, \ldots, n+1) \text{ for all individuals} \tag{5}$$

where r is a real number. The peculiar characteristic of the $(n+1)$:th commodity is its lack of direct utility to the consumer; hence, if we write an individual's utility function

$$u = u(d_1, d_2, \ldots, d_n, d_{n+1}), \tag{6}$$

[3] Patinkin [41].
[4] Marschak [27].

the partial derivative or ≫marginal utility≫ with respect to the $(n+1)$:th argument is always zero for all individuals:

$$u_{n+1} = 0. \tag{7}$$

Introducing the budgetary restriction:

$$\sum_{i=1}^{n+1} q_i d_i = \sum_{i=1}^{n+1} q_i s_i \tag{8}$$

and maximizing the utility functions with respect to the d_i under the restrictions (5) and (8), we obtain for each individual:

$$\left.\begin{aligned} u_i + \mu_i &= \lambda q_i \quad (9) \\ \mu_i r_i &= 0 \quad (10) \end{aligned}\right\} \qquad (i = 1, 2, \ldots, n+1)$$

where μ and λ are Lagrange multipliers. It follows that for any two goods — say l and s :

$$\frac{q_l}{q_s} = \frac{u_l + \mu_l}{u_s + \mu_s} \tag{11}$$

If s is put equal to $n+1$ — paper money — (11) reduces to

$$q_l = \frac{u_l + \mu_l}{\mu_{n+1}} \tag{12}$$

The right-hand member of (12) is finite provided $\mu_{n+1} \neq 0$; for this condition to be fulfilled it follows from (10) and (5) that the demanded amount of the commodity ≫paper money≫ must be zero for all individuals:

$$d_{n+1} = 0. \tag{13}$$

Hence m and M in Mosak's system ought to be put equal to zero. If this be not accomplished the prices of all goods $1, \ldots, n$ — as seen from (12) with $\mu_{n+l} = 0$ — are not determinate and finite. Here we find the inconsistency of the inclusive model which tries to synthesize the traditional theory of money and the simple Walrasian construction. The result outlined seems completely to remove every possibility of discussing inflationary phenomena in terms of a simple Walrasian construction[5] — and this is the fact in which

[5] Nota bene, if the behavior equations in this Walrasian construction be derived from assumptions on individual utility maximization — we do not discuss systems where the total supply and demand equations are stated ad hoc.

we are interested. It precludes the utilization in this connection of concepts like absolute prices, general price level, value of money, token money, and leaves only quantities and supposedly strictly determinate relative prices, changeable solely when the data are altered. No doubt the result is very serious to those looking upon the simple Walrasian construction as *the* economic theory.

After having contributed his important criticism Patinkin tried to present an extended Walrasian system, capable of comprising monetary phenomena. As it is of great relevance to one of our main themes we quote Marschak's criticism of this extension: ≫Patinkin followed the more superficial approach of Cassel whose aggregate demand and supply equations emerge from nowhere. Had these equations been derived from the maximization of each individual's utility subject to the limitation of his and the society's resources, these limitations would result in certain restrictions upon the [aggregative] system. Unless one derives these restrictions precisely, the determinacy of the system cannot be discussed.≪[6] These words ought to be combined with Marschak's dictum about ≫the classical theory of market equilibrium≪. ≫It fails to relate the desired quantity of paper money and the exchange ratios between paper money and various commodities (the 'absolute' prices of commodities) to assumptions of rational (utility-maximizing) behavior. As a makeshift, an 'equation of exchange' was used by later authors. It appealed to the alleged existence of an 'institutional' constant (velocity of circulation) superimposed upon the assumption of rational behavior of individuals. More recently, this makeshift device was generalized, and slightly rationalized, into Keynes' 'liquidity preference' equation. Like the rest of [aggregative] economics, this equation is still in need of being related to assumptions of rational behavior. Should the 'liquidity preference equation' and other Keynesian equations have purely empirical claims, these would be hard to establish: the observed time-series of the relevant variables (quantity of money, interest rate, price level, consumption, national income and, possibly, its distribution) are no doubt consistent with a large number of equation systems other than the Keynesian one.≪[7]

The quotations essentially express the idea of the two basic sources of economic knowledge — on one side, the observed time-series; on the other, the theoretical deductions, starting from certain assumptions of human economic behavior — and of the interaction between the two sources. Marschak, however, clearly limits the theoretical constructions considered to those expressed as equation systems, although he strongly

[6] Marschak [27], p. 72.
[7] Marschak [27], p. 71.

emphasizes that these constructions should be derived from assumptions of rational economic action. The Walras-Mosakian system is derived from such assumptions according to Marschak; but as Patinkin and he himself have shown, it does not comprise any features showing even a superficial similarity with observed inflationary phenomena.

Marschak's own extended system no doubt comprises certain features in rough correspondence with observed inflationary phenomena and it is derived from assumptions which are in one sense broader than those underlying the simple Walrasian construction. The thing added is — as we know from previous discussions — the ≫market functions≫, or ≫revenue functions≫, relating the prices of commodities to the amounts transacted in a way characteristic of each decision unit.

We have already discussed and characterized the general features of this extended system. It solves simultaneously the production problem and the distribution problem, as does the simple Walrasian construction, without touching the sincere dilemmas which are met in Arrow's theorem and in the transferable-utility discussion of the theory of games. Its solution of the distribution problem may be labeled automatic, because it does not suppose any agreements or negotiations between the decision units to be necessary in order to establish a certain scheme of distribution; and harmonic, because it does not comprise any real conflicts of interest, assuming the scheme of distribution to be uniquely determined by the data of the system, thus one data constellation allowing only one distribution of income; both basic traits recall the Inessential Games. Besides, the whole construction shows a certain formal difficulty, expressed by the fact that the revenue functions ≫do not form an independent set≫, the restrictions caused by this condition being claimed by Marschak to be discussed in the theory of games.

The special features of the Marschakian system thus far established and relating it to inflationary phenomena, may be summarized as follows. Let x_s^t denote an amount of commodity s transacted at instant t. The revenue functions of a particular decision unit at time-point zero

$$q_i^0 = f_i(x_i^0) \qquad (i = 1, 2, \ldots, n), \tag{14}$$

— where q as before denotes absolute prices — replace the set of prices q_i common to all individuals in the simpler system; if we allow for more than one marketing date expected prices $q_i^1, \ldots, q_i^T$, specific to each individual, are added — ≫a multi-period model≫; these are replaced by revenue functions

$$q_i^t = f_i^t(x_i^t) \qquad \begin{bmatrix} t = 1, 2, \ldots, T, \\ i = 1, 2, \ldots, n, \end{bmatrix} \tag{15}$$

assuming that imperfect future markets are expected. The revenue function of the n:th commodity, paper money, is for all individuals assumed to have the special form:

$$f_n^t(x_n^t) = 1; \tag{16}$$

consequently

$$\frac{df_n^t}{dx_n^t} = g_n^t = 0, \tag{17}$$

expressing in Marschak's terminology ≫perfect liquidity≫. In the simple Walrasian construction all commodities have ≫revenue functions≫ of the form: $p = f(x) =$ a constant, and are thus perfectly liquid. In Marschak's system ≫The degree of liquidity of any commodity $[i]$, with respect to a given individual, can be identified with the 'elasticity of demand to the individual,' $\eta = \frac{q_i}{x_i g_i}$ or the degree of 'illiquidity' can be defined identical with the so-called degree of 'market imperfection', $1 - \frac{1}{\eta}$. ≫[8)]

Introducing into the individual's utility function not only consumed quantities at instant $t = 0$ but also planned consumed quantities at instants $t = 1, 2, \ldots, T-1$ — consumed quantities obviously need not coincide with transacted quantities, as stocks may intervene — *and* stocks planned to be retained at $t = T$, the end of the horizon, Marschak maximizes this function with respect to the various variables involved and under the constraints imposed and arrives at certain interesting consequences.

First, in a Marschakian construction — with more than one marketing date and with some consumption goods imperfectly liquid — stocks of paper money can take values other than zero if all prices are not expected to remain constant.

Second, the prices which matter in the construction are the absolute prices and not the relative ones, that is the prices of the $(n-1)$ consumption goods in terms of money, the n:th commodity, and not the ratios between, let us say, the prices of the first $(n-2)$ consumption goods and the price of the $(n-1)$:th consumption good. Thus the supply and the demand of the various goods are dependent on the absolute prices, there is ≫money illusion≫ in the system — an increase of the absolute prices, i.e., the cheapening of paper money in terms of goods, induces the individuals to change their money holdings relative to their holdings of consumption goods. Thus Marschak has extended the simple Walrasian construction into

[8)] Jacob Marschak, Cowles Commission Discussion Paper: Economics 272, Section 4.

a ≫theory of assets≫, and shown that absolute prices, positive money stocks and ≫money illusion≫ are consistent with the model, possibly explaining phenomena like the flight into real values during times of inflation and apparently giving no simple connection between the movement of the total money stocks and a general movement of the absolute prices. It seems, however, to say very little or nothing about the conditions under which inflations arise, why at certain times there is no marked general trend in the movement of absolute prices and at other times there is a strong upward movement. Possibly such matters are supposed to be approached in a future dynamic version, treating, e.g. how expectations arise — expectations of constant future prices implying zero money stocks, expectations of changing future prices being capable of implying positive money stocks, etc.

In our opinion the really fundamental reason why the Walrasian constructions essentially fail — the Walras-Mosakian system completely so — to show any significant features, mirroring observed inflationary phenomena, must be found in their rudimentary treatment of the distribution aspect, which rudimentary treatment also makes it possible for them to gloss over the difficulties of combining the production aspect and the distribution aspect into one harmonic system.

b) THE PURCHASING POWER PARITY THEORY OF EXCHANGE RATES

The purchasing power parity theory of exchange rates is usually presented in an unsophisticated form, stating that the exchange rate between two national currencies tends to be equal to the ratio between the national price levels — which according to the quantity theory of money is directly proportional to the ratio between the corresponding quantities of money. Attempts have, however, also been made to superimpose the theory on a Walrasian construction, taking into consideration international trade. Naturally Mosak's volume[9)] should be instructive in this respect. The extension of a national Walrasian system to an international one — disregarding such special factors as transport costs, and unilateral payments — simply consists in letting the total supply and demand equations for each commodity be summed not only over all individuals $1, \ldots, n$, but over all individuals $1, \ldots, n$ in all countries $1, \ldots, v$, the equilibrium conditions then stating the equality of total demand and total supply from all countries. The various national levels of absolute prices are determined thus. We denote by e_{1j} the number of units of the token money of country (j) obtainable

[9)] Mosak [25], pp. 52–55.

for one unit of the token money of country (1). The following relation is then valid between the prices of (j) and those of (1) :

$$e_{1j}q_i^{(1)} = q_i^{(j)} \quad (j = 1, 2, \ldots, v; \quad e_{11} \equiv 1) \tag{18}$$

We further assume the quantity theory of money to be valid in each country:

$$M^{(j)} = q_n^{(j)}\overline{K}^{(j)} \sum_{i=1}^{n} p_i S_i^{(j)} \tag{19}$$

If each country determines its total quantity of money, then the national levels of prices are fixed by equations (19), while equations (18) give the values of the rates of exchange. If the factors determining the relative prices be given, equations (19) state that the ratio between two national price levels is directly proportional to the ratio between the corresponding quantities of money; equations (18) add that the exchange rate between two currencies is inversely proportional to this last-mentioned ratio.

Exactly as the price level in a national Walras-Mosakian system is without any real importance, so are both price levels and exchange rates insignificant in an international system of this type. If the quantity of money be increased in one country, the particular national absolute prices increase proportionally and the number of units of the token money of another country obtainable for one unit of the own currency diminishes. This change of exchange rates, implying a fall of the exterior value of the currency, is, however, without the slightest importance as all quantities and relative prices remain unchanged. An inflationary movement consisting of the increase of the total quantity of money in one country and the consequential rise of absolute prices and fall of exchange rates would be a completely harmless phenomenon, leaving intact the ratios of money incomes not only within but also *between* countries. We underline this fact and express the opinion that the purchasing power parity theory of exchange rates is significantly characterized as a theory of inflation which lets the ratios of money incomes between countries remain fixed throughout the inflationary process, all money incomes being measured in the currency of one country.

As we know, a system like that presented, combining the ordinary Walrasian equations with monetary relations, is not consistent. We think, however, that it is possible to utilize the purchasing power parity theory in another context and we shall return to the topic later. Referring to Patinkin's inconsistency result, Tinbergen concludes in his article on ≫Models

of International Trade≫ (1948)[10] that a Walrasian system with demand and supply functions containing only relative prices does not determine one single specific equilibrium rate of exchange: ≫any figure could be chosen, the choice is arbitrary≫. We prefer to say that the choice is absent as all monetary phenomena according to the theorem by Patinkin and Marschak are alien to a simple Walrasian construction; it can comprise no money quantities, no absolute price levels, no rates between absolute price levels.

Tinbergen presents, however, another system which is said to determine unique exchange rates. The supply and demand equations of this system show ≫money illusion≫; their variables are the absolute money prices and not the relative prices, just as is the case with Marschak's construction. The familiar exposition of the determination of exchange rates due to Joan Robinson and others follows the same Marschakian line — although no explicit motivation of the inclusion of absolute prices in the equations is formulated. Such ≫Marschakian≫ systems give no simple connection between the ratio of money quantities or price levels and the uniquely determined exchange rates.

3) A sociological theory of inflation

a) GENERAL VIEW-POINTS

The quantity theory of money and the purchasing power parity theory of exchange rates cannot consistently be superimposed on a Walrasian construction — a supporter of such constructions must reject them. An economist, aware of the severe limitations of the Walrasian constructions, is free to accept the two monetary theories despite their incompatibility with the former, if he can justify them on other grounds.

In the previous section we sponsored the following particular formulations of the two theories: the quantity theory pictured an inflationary — or deflationary — process, during the course of which all money incomes within a country changed proportionally; the purchasing power parity theory described an inflationary — or deflationary — process, during the course of which all money national incomes within the set of countries involved changed proportionally. The consequences of such processes are thus obviously fairly unimportant from all points of view. Utilizing these formulations we think that it is possible to give both to the quantity theory and to the purchasing power parity theory, a very modest validity as ≫time-limited theories≫ — we remember that this expression has appeared once before, namely in connection with the normative interest theories.

[10] Tinbergen [42].

Examining the relevant facts we find that the quantity theory does at least not apply to the period after 1932, characterized by a fairly constant interest rate-level and rising prices, because then the money income of the savers evidently does not change in proportion, e.g. to the income of the laborers. We further find that the purchasing power parity theory does at least not apply to the periods of divergency between the internal and the external currency-values in the 1940's and to the international situation after the Second World War, because then the money incomes of the United States and the European countries, measured in the same unit, do not tend to change proportionally. These periods, during which the two monetary theories thus stand in contradiction to the facts, are not the only ones commonly labeled inflationary within the time-span considered, but certainly nobody would hesitate to attribute to them inflationary characteristics, strongly pronounced. To explain the course of events in these periods we should have, instead of a theory letting inflation be a process of proportional income alterations, a theory expressed in terms of changes in the distribution of income, i.e. of changes in the relative income shares of the various decision units within a country and/or in the relative national income shares within a set of countries. Obviously a process of this kind has important consequences for the economic units involved.

The core of inflationary phenomena is in our view a distribution struggle between the big social groups within a country and/or between several countries, consisting of attempts by one or more units to increase their relative shares of the total income, i.e. to improve their relative income position, followed by reactions of other units, and with a result as to the distribution of income depending on the power of the various units; this idea will be elaborated below in two special contexts.[1)]

[1)] A similar general attitude towards the phenomenon of inflation is taken in the interesting article by Henri Aujac, "Une hypothèse de travail: L'inflation, conséquence monétaire du comportement des groupes sociaux," *Économie Appliquée* 1950, pp. 280–300. Aujac rejects an analysis in terms of inflationary gaps or of flows and stocks and sponsors an analysis in terms of the behavior of social groups. He considers inflation as ≫the monetary consequence of the behavior of the social groups≫. An inflationary movement begins by a certain group changing its ≫monetary constraints≫ (≫liaisons monétaires≫) — or as we think it permissible tentatively to interpret: its relative income position — utilizing some means: the government, e.g. new money issues, the entrepreneurs credits, coalitions, political pressure, the laborers strikes, political pressure. The action of the initiating group may be followed by reactions of other groups until finally an agreement is enforced or accepted. The form of the agreement is dependent on the

The opinion is alien to the quantity theory of money and the purchasing power parity theory of exchange rates, and to the Walrasian constructions, but related considerations are found throughout the literature. As an example of this we mention here a definition of inflation, famous in Scandinavia, and, put forward by Jörgen Pedersen in his treatise on ≫Monetary Theory and Monetary Policy≫ (1949)[2)]: inflation means ≫the financing of a rising level of wages≫, while deflation implies ≫the lowering or the attempt of lowering the level of wages≫. ≫We have here both the necessary characteristics: the monetary change and the time-process or disequilibrium state.

Such a financing of rising wages is as a rule identical to a general rise of prices and a lowering of the level of wages is identical to a decline of prices; but a rise of prices may also occur without a corresponding rise of wages, and a common decline of prices may happen without the wages to be lowered.

Conditions are similar in relation to exchange rates. A rising level of wages may be financed without varying the exchange rates ... Thus it is impossible to express inflation or deflation, as changes in the general price movement or in the exchange rates.≫[3–4)]

This definition of inflation is not centered around an impersonal concept like ≫total supply of money≫; it is tied to the action of a big interest group (labor) among other interest groups within a society; but evidently it pays attention only to a very limited aspect of income redistribution. In all probability the definition is based on definite observations from the last decades.

Two remarks conclude this introductory subsection. *First,* the following questions are often asked: ≫Is it possible to avoid inflation?≫, and ≫Is it desirable to avoid inflation?≫

It seems to us that an answer to the problem about the *possibility* of preventing inflation must of necessity be hedged with conditions, being connected with factors in the particular region and period considered, such as the strength of labor unions, and the capacity of the labor and entrepreneurial organizations, in agreement with each other, jointly to achieve higher wages (and prices). The problem about the *desirability* of

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power of the various groups and its aim is once more to make compatible the behavior of the various social groups. Cf. also Franklyn D. Holzman, "Income Determination in Open Inflation," *The Review of Economics and Statistics* 1950, pp. 150–158.

[2)] Pedersen [43].

[3–4)] Pedersen [43], p. 213.

preventing inflation must, as we see it, be answered with reference to the various social interest groups involved: from the point of view of labor it is desirable to get a higher money group-income relative to the other money group-incomes; from the point of view of the savers it may be most undesirable to experience higher relative wages; from the point of view of one country it may be most desirable to change in its favor the relative income position, etc. This should be supplemented by the reservation that pronounced inflationary processes may comprise such a heavy element of disequilibrium or instability as to become essentially undesirable from all points of view.

Second, in looking upon inflation as basically implying a change of the relative income shares of various economic units we do not think that all such changes should be called inflationary. The particular distinction applied by us will be made clear in the course of the following two subsections containing the two special applications of our ideas concerning the nature of inflation. Here we only state that the discussion in Chapter V was centered on a change in the distribution of income tied mainly to a change between two types of standards of behavior or Solutions; in Chapter VII we shall discuss changes in the distribution of income which we consider fundamentally caused by changes in the objective possibilities or Characteristic Functions. Finally, in this chapter on inflation, the emphasis is put on variations of Imputations, leaving Solutions and Characteristic Functions essentially unchanged, and especially on variations of Imputations within certain particular types of Solutions.

b) INFLATION AND THE RATE OF INTEREST

α) *The initiating inflationary change*

It was argued in Chapter V, Section 4 that the determination of the rate of interest in a multi-unit-of-decision society could only be understood from the point of view of the distribution aspect and that in particular the evidence suggested both by the varying ≫normative interest theories≫, by the time-series observations, and by some qualitative observations, brings out a change around 1930, signifying a decisively altered relative position in the social economics for the group of people depending upon the rate of interest as a main source of income. Prior to the point of change the ≫orthodox normative interest theory≫ emphasized that interest was to be considered quite on a par with wages and other distributive shares; the time-series observations, showing parallelism in the movements of interest rates and of other prices, indicated that the relative share of the savers had a tendency to be maintained, the qualitative observations on banking policy pointing in the same direction and underlining above all the rôle of an

independent central bank as an agency striving to preserve the value of the savers' money. After the point of change the ≫unorthodox normative interest theory≫ implies that interest is to be put in a special position as a distributive share, being determined quite apart from the field where laborers, farmers and entrepreneurs negotiate for their relative shares; the time-series observations, showing fairly constant interest rates despite rising prices of other kinds, indicate that the relative share of the savers continually diminishes, the qualitative observations on banking and other economic policy pointing in the same direction and showing particularly the position of the savers as a group, almost without influence in society after the cessation of the independence of the central bank, and not taking active part in any distribution agreements.

The observed features including, e.g. the sharp change, if taken in their totality, are very alien to traditional economic theory but show similarity with phenomena arising in game theory, more particularly with a change from an Imputation, or a limited category of Imputations, belonging to an objective Solution or accepted standard of behavior to Imputations belonging to discriminatory Solutions with the group savers in the excluded position. A standard of behavior of the first-mentioned kind appears in the games referred to in the previous chapter as a distinct entity, in a marked degree separated from other standards; a standard of behavior of the second-mentioned kind belongs to a family of standards, the members of which are distinguished from each other only in assigning different values to the share of the Excluded participant, permitting, for instance, a continuous decrease of this share through going from one member of the Solution family to others, all based on exactly the same main qualitative principle and separated solely with respect to the value of a quantitative parameter. As long as the discussion only deals with the determination of the interest share the chief attention should be paid to the adoption of the two types of standards of behavior, the objective standard and the family of discriminatory standards, this family of standards introducing into the picture a certain kind of variability between its members, the thing apt to be varied being the relative share of the Excluded participant and consequently also the combined relative share of the other participants. We also mentioned, however, in the previous chapter some consequences for the income distribution of the adoption of the different standards: the income distributions compatible with the objective Solution are finite in number, clearly delimited from each other and seem to be changeable only together with a basic alteration of the power constellation of society — compare above all the completely worked out case of three participants; the income distributions compatible with a discriminatory Solution are infinite in number, go over into each other continuously and change with

alterations of factors of an apparently ephemeral character, labeled the non-excluded participants ≫bargaining abilities≫ in the *Theory of Games* — the only thing fixed within a particular member of a family of discriminatory Solutions is the amount assigned to the Excluded participant, at least the relative shares of the remaining amount accruing to two groups of participants are freely variable. This last-mentioned fact introduces a second kind of variability into the picture, a variability of Imputations within a particular discriminatory Solution. In this chapter our attention should be chiefly concentrated on this indeterminacy of Imputations viewed, however, in conjunction with the first-mentioned variability within a family of discriminatory Solutions. Our main conclusions concerning interest rates and inflation will be based on contrasting with each other these two variabilities of discriminatory or indefinite Solutions, taken in conjunction, and the relative fixity of an Imputation within an objective or finite Solution, be this trait reinforced or not by the qualitative uniqueness of the objective Solution.

Let us survey a few facts after and before the point of change around 1930. We first consider what has been called ≫the crisis of the theory of wages≫[5], observed by many authors. The crisis consists in a struggle between two divergent types of wage theories. During the pre-1930 epoch a simple form of the marginal productivity theory of wages was devised and widely taught, namely the Cobb-Douglas function: $P = \alpha L^{\lambda} C^{\mu}$, where $P =$ quantity of product; $L =$ quantity of labor employed; and $C =$ quantity of capital used — see above all Douglas, *The Theory of Wages* (1934).[6] As is easily concluded this theory prescribes to labor a constant relative share of the national product($L\frac{\delta P}{\delta L} = \lambda P$); the same consequence is valid for ≫capital≫ ($C\frac{\delta P}{\delta C} = \mu P$). Thus labor income and capital income are proportional to total income: the shares are fixed, independently as well of the prices $\frac{\delta P}{\delta L}$ = unit wages and $\frac{\delta P}{\delta C}$ = unit profits as of the quantities L and C. No accumulation of capital could change the shares; neither could any trade union action do so. Referring to pre-1930 data Tinbergen comments on the theory: ≫Now it is a rather well-known statistical fact that the shares of labour and capital have been about constant, through a number of decades, both in the United Kingdom and the United States. This does

[5] François Perroux, "Note sur le dynamisme de la domination," *Économie Appliquée* 1950, p. 252; cf. also Paul H. Douglas, "Are there Laws of Production?," *The American Economic Review* 1948, p. 1–41.

[6] Douglas [44].

not hold good, however, for Swedish industry, for which also a long series of figures is available.≫[7)]

This deterministic and, from the point of view of labor, pessimistic theory was disputed by the Webbs at an early stage, maintaining that labor union action was capable of influencing the relative share of labor in the national income and sponsoring a ≫bargaining-theory≫ of wages. During the post-1930 epoch this opinion is certainly widely accepted and the earlier theory confined to the domain of text-books. This need not be of any importance.

What is more important, however, is the intensive creation during the later epoch of new institutions for the purpose of settling the relative position between wages and profits, like arbitration procedures and arbitration councils and price controls and even attempts of administratively fixing or ≫freezing≫ both wages and profits. This fact lends strong support to views holding that distribution disputes between the labor group or groups and the entrepreneurial group are of increasing importance both for the groups directly involved and for the whole national economy. Thus indeterminacy of the relative position between laborers and entrepreneurs or if we prefer the dependency of this relative position on bargaining abilities seems to be a most conspicuous and recognized feature of the period; there are no signs that very stable factors determine the relation between wages and profits. A characteristic trait of the period is seen in wage increases followed by price adjustments, compensating to an uncertain degree for the wage increases; obviously there may be cases of both undercompensation and overcompensation.

The farmers constitute a third powerful big social group which cannot possibly be disregarded. There does not seem to be marked signs of the same type of open distribution question between the farmers and the other groups as between the laborers and the entrepreneurs; no institutions of an extent comparable to the arbitration institutions exist for settling distribution disputes between the farmers and other groups, no attention comparable to that paid to wage — industrial price disputes is attached to farm price — wage disputes or farm price — industrial price disputes. The aim of the farm group after the considerable improvement of its relative position, achieved in political coalition with the labor group (Roosevelt 1933, Sweden 1936, etc.) seems largely directed towards ≫parity≫[8)], which may in essence mean

[7)] Jan Tinbergen, "Professor Douglas' Production Function," *Revue de l'Institut International de Statistique* 1942, pp. 39–40.

[8)] Cf. John D. Black and Charles A. Gibbons, "The War and American Agriculture," *The Review of Economic Statistics* 1944, pp. 1–55; cf. also the quotation from Erik Lundberg, pp. 102–103 above.

increase of the income share of the farmers proportionally to the increase of the laborers share; this parity aim appears to be fairly sanctioned by legal and administrative practices, lying outside the sphere of intensive struggle.

Having thus above all underlined the basic indeterminacy of the relation between wages and profits, which in our view is a most outstanding feature of contemporary societies, we now finally turn to a fourth social group — the savers or the rentiers. In Chapter V we emphasized the excluded position of this group in the post-1930 world: no agency of any importance was working to favor its interests; it did not take part in any distribution negotiations with the other big groups, but obtained its share as a residual, assigned to it by the other groups — all this in agreement with the normative interest-theory prevailing. Due to these and other circumstances we ventured the opinion that the basic observed interest rate phenomena of many countries during the last three decades fit well into discriminatory Solutions of a game between a very low number of participants — the big social groups of society. A discriminatory Solution in the simple cases so far explored has two implications, repeatedly mentioned above. One implication is the far-reaching indeterminacy or variability of Imputations within a discriminatory Solution. We have now indicated that a certain indeterminacy is also discernible in the empirical observations emanating from the post 1930-period, namely the indeterminacy of the relation between wages and profits, admitting a chain of settling, opening, resettling and reopening the wage-profit question.

The accepted standard of behavior at a certain point of time during the last decades has as inherent characteristic a basic variability of the relative income position between the laborers and the entrepreneurs, this relative position depending on ≫bargaining abilities≪ and being changeable without any basic modifications occurring to the grouping of the units in the society, i.e. to their participation in coalitions and power-blocks. The indeterminacy allows the question about fixing the relative position to be frequently reopened, sometimes perhaps as a direct consequence of some outside shock impinging on the national economy. We call this reopening the *initiating inflationary change*. The strict adherence to one particular discriminatory standard of behavior should require the combined relative income share of the non-excluded participants to remain constant; hence a change in the position between laborers and entrepreneurs should imply the increase of the relative share of the national income on the part of one group and a corresponding decrease on the part of the other group; in this way the relative share of the Excluded savers should remain unaltered, as should ≫the value of money≪. Probably changes of this kind have been experienced. Consideration should, however, also be paid to the other implication of a discriminatory Solution.

β) *The consequential inflationary change*

This second implication is the membership of every discriminatory Solution in a family of very similar Solutions, based on exactly the same qualitative principle of excluding the savers, but distinguished from each other through the value of a numerical parameter. A change from one member of the family to another involves a break of principles; this break, however, must be judged to be of an insignificant kind. Thus the special type of the accepted standard of behavior admits without the sacrifice of any main principles the following alternative to decreasing the relative share of the laborers or the entrepreneurs after an initiating change: the standard may be slightly altered to another member of the same qualitative type, ascribing to the savers a lower relative share, and hence permitting the imitating change to take place without the deterioration of the relative share in the total income of one of the first two groups — or at least making this deterioration smaller than would have been the case on complete adherence to the accepted standard of behavior. In other words a discriminatory Solution opens up the possibility of promoting the settling of a distribution dispute between two powerful groups through a slight break of principles, this break disfavoring the weak, Excluded group, implying a fall of the value of (the savers') money — but also introducing an element of instability in the whole economy. In agreement with these remarks the post-1930 period has witnessed the low-interest policy under rising prices, which has been termed ≫the expropriation of the rentiers≪, implying a marked gradual lowering of the relative share of the national income on the part of the savers or rentiers, a politically weak, almost unorganized interest group of society. We call this gradual lowering the *consequential inflationary change.* At this point the reader should recollect the tentative discriminatory Solutions of the four-person game discussed in Chapter V, pp. 153–154. One player is excluded — compare the savers —, two players form a ≫first coalition≪ which fixes the relation between their incomes — compare the laborer and the farmers — and negotiate with the remaining player — compare the entrepreneurs. Two basic variabilities occur: there are nothing but ephemeral factors to determine within certain limits the relation between the incomes of the ≫first coalition≪ and the remaining non-excluded player — compare the initiating inflationary changes — and the amount assigned to the Excluded Player may vary in a certain interval — compare the consequential inflationary changes.

The initiating change — admitted by the basic indeterminacy of Imputations within a discriminatory Solution —, and the consequential change — admitted by the variability within a family of discriminatory Solutions —, are inextricably interwoven features of the state of things during a regime of discriminatory Solutions. The phenomenon of inflation of the kind

considered we connect with both changes, i.e. with both types of variability or indeterminacy; this inflation thus consists in price rises, accompanied by decisive changes in the distribution of income between the big social groups. The process outlined could be called cumulative: as soon as a discriminatory Solution is adopted, a basic indeterminacy of certain income-relations arises; this indeterminacy constitutes the setting for a distribution struggle, which may be most easily solved through pushing the Excluded unit to a less favorable position; this change does not, however, diminish the indeterminacy.[9)]

The game theoretical variabilities are consequences of the adoption of a discriminatory type of accepted standard of behavior. If there be some truth in our manner of connecting the observations with this theory, the inflationary phenomena observed should be a consequence of the unorthodox interest-rate norms adopted. This statement should not be taken in so strong a form as to imply the inevitability of inflation — of the type considered — when discriminatory Solutions are accepted. We intend it to mean that inflation is extremely difficult to avoid under such conditions, due to the two implications of a discriminatory Solution, i.e. the two types of indeterminacy or variability. Inflation could be avoided if the position between wages and profits were rigidly fixed by administrative means — wage and price control, etc., but no stable political or ≫power≫ basis is to be found for such a rigorous fixation under the reign of a discriminatory type of Solution, owing to the insignificant differences of Imputations within a discriminatory Solution, these Imputations going continuously over into each other. Inflation could also possibly be avoided in the lack of controls of this type, if a completely strict adherence to one particular discriminatory Solution were carried out, enforcing the maintenance of the relative income share of the Excluded group and hence increasing the rate of interest when wages and prices go up — but no strong moral justification is to be found for such a complete adherence under the reign of a discriminatory type of Solution, owing to the insignificant differences within a family of discriminatory standards, based on exactly similar principles and going

9) In the articles by Aujac and Holzman, referred to in footnote 1), p. 181 models are presented showing approximately our two inflationary changes: the change of the relation between wages and profits and the lowering of the savers' relative income share; no doubt these numerical and arithmetical models are based on experiences from contemporary economies. Compare also in this context the article by Dahmén, mentioned in footnote 32), p. 158, which among other things tries to calculate the magnitude of the consequential change consisting in the deterioration of the savers' relative income share.

continuously over into each other. The last point is particularly stressed, if the strain of initiating changes on the norm of conduct be taken into account; a combination of rigorous fixation and strict adherence would be most likely to lead towards the goal of preventing inflation, but the fixation would be without a stable political basis and the adherence without a strong moral background.

We find some support of our position in the following dictum pronounced by the eminent monetary theorist D. H. Robertson, although there are differences of emphasis and expression and tendency: ≫Hence it has come about that in England since the war ended, in spite of the acknowledged existence of a state of inflationary pressure, rates of interest have been driven down to an even lower level than that maintained during the war... I suspect that the success with which the monetary authorities of the world brought down the rates of interest in the early 1930's in conformity with the real change which had come over the world economic situation has led to an exaggerated idea of the extent to which, if real economic forces are *against* them, they can fly with impunity in the face thereof. I have some fear that attempts to do so may pitch them in the long run into trying to impose a more detailed and permanent system of controls... ≪.[10)]

γ) The pre-1930 conditions

An objective standard of behavior in the games referred to does not comprise the same variability of Imputations (nor does it belong to a family of standards based on the same main qualitative principle). A change from one Imputation to another seems to have to be correlated with a decisive regrouping of the decision units into new coalitions and power blocks; thus a particular Imputation established depends on more stable factors than ≫bargaining abilities≪. A change from one objective Solution to another, if this be possible at all — the objective Solution may be unique — can at least imply the adoption of quite a new qualitative principle and thus be a very difficult step to perform. For an objective Solution has two implications, the first and most important calling for a certain stability of an established Imputation belonging to the Solution, the second making plausible a certain stability of the very norm adopted, both features as contrasted to the two basic variabilities of a discriminatory Solution.

In view of these considerations it would appear that the acceptance of an objective standard of behavior be the most radical means of avoiding inflation. Our next task is to discuss pre-1930 conditions, let us say

10) D. H. Robertson, *Money,* 4th edition, London and Cambridge 1948, Nisbet and Cambridge University Press, pp. 219–221.

conditions between 1875 and 1932 — in Chapter V we interpreted the relevant interest-rate facts to mean the existence of such a type of Solution during this period. We already touched upon this subject in the previous subsection when mentioning ≫the crisis of the theory of wages≪: the very heuristic (Cobb-) Douglas theory attributing to wages — and to profits — a constant relative share over time of the national income, and being in agreement with certain important pre-1930 facts according to Tinbergen. This evidence may suggest that the initiating type of inflationary change was absent or weak during the period. No doubt wage disputes were also frequent before the point of change, but the characteristic great importance attached to them under the later era seems to have been absent, and so were many of the institutions created to deal with wage-profit fixation and the continuous chain of opening, settling, reopening and resettling the relation between wages and profits. These signs of a certain stability (the term must not be taken in an absolute sense but only as relative to the very variable post-1930 conditions) do not contradict our hypothesis about the objective type of the accepted standard of behavior.

As regards the relation between the income share of the savers and of the other groups it may be instructive to refer to a certain famous economic theory, devised during the period and based on broad knowledge of the empirical conditions prevailing — we think especially of Wicksell's analysis, as first presented in his *Interest and Prices* (1898),[11] and later in his *Lectures.* In Chapter V, Section 1, a Wicksellian real rate of interest was discussed and placed in the same category as researches belonging to the production aspect. This real rate of interest is explicitly treated in Part I of Wicksell's *Lectures;* it stands in a somewhat uncertain relation to the real or natural rate of interest mentioned in *Interest and Prices* and in Part II of *Lectures.* The second-mentioned real rate of interest more nearly corresponds to what is usually called profits, i.e. it represents the income share of the entrepreneurial group and comes under the heading of the distribution aspect.[12] Thus from our point of view the two real rates are in essence quite different. Two more interest rates are mentioned by Wicksell in *Interest and Prices* and *Lectures II:* the bank or loan rate of interest and the normal rate of interest. The bank rate of interest presents no particular additional problems — it represents the income share of the rentiers or the savers. The normal rate of interest on the other hand has

[11] Wicksell [45].

[12] This interpretation corresponds to that employed in Johan Åkerman [15], I, and [16], p. 519, an interpretation which no doubt appears strange to a reader only thinking of the real rate of interest in Part I of Wicksell [35].

been very much discussed in the literature and various interpretations have been put forward.

According to Wicksell a bank rate of interest which is an immediate expression for the real rate of interest is a normal rate of interest. If the real rate of interest tends to outrun the bank rate of interest, i.e. if the income share of the entrepreneurs is favored relative to the income share of the savers, the entrepreneurs begin to increase their demand for labor and commodities; they become capable of paying higher wages and prices and the incomes of laborers and farmers rise; this increases further the prices of various kinds, and the Wicksellian upward cumulative process goes on. It is stopped through the action of the banks, raising the bank rate of interest and thereby restoring the relation between the income shares of the entrepreneurs and the savers. In a similar way a Wicksellian downward cumulative process starts, if the bank rate of interest tends to outrun the real rate of interest, i.e. if the income share of the savers is favored relative to the income share of the entrepreneurs; and it is stopped if the banks lower the bank rate of interest and thereby restore the relation between the two income shares. The Wicksellian upward cumulative process shows a kind of instability between profits and wages which resembles the variability tied to the initiating inflationary changes of the previous subsection. In the Wicksellian case, however, no lasting consequential inflationary changes occur, as the banks strive to adapt the bank rate of interest to the real rate of interest, although this process of adaptation may sometimes be carried out somewhat hesitatingly; when it has been completed, the equilibrium is restored and the entrepreneurs can no longer afford to grant any wage increases — the relation between wages and profits remains fixed until the next deviation appears between the real rate of interest and the bank rate of interest. The Wicksellian downward cumulative process shows the same kind of instability, but the direction of the potential consequential changes is reversed.

The central point in Wicksell's theory is the normal rate of interest — we have interpreted this much-debated normal rate as prevailing when a particular, fixed relation between the income shares of the entrepreneurs and the savers is maintained or restored through the action of the (central) banks. The approximate stability of this relation during the period is reflected in the time-series observations on the strong positive correlation between interest rates and price levels.

Thus we have discussed the Douglas theory of wages and the Wicksell theory of profits and interest, both being in agreement with important observations from the pre-1930 period, and both emphasizing the stability of the relative income shares of the social groups involved. A certain stability of the relative income shares of the various participants in a game is

implied by an objective Solution. If there be some truth in our combination of game theoretical concepts with the pre-1930 observations, the stability to be found should be implied by the orthodox interest rate norm prevailing during the period. Accepting this point of view the following comments to the theories of Douglas and Wicksell may be made. The particular relations, existing between the various income shares build on political premises, and they also presuppose a definite power constellation in the society, because the standard of behavior accepted is of a type which admits only a limited number of alternative income distributions, each corresponding to a certain grouping of the units in coalitions and power blocks. Moreover, but less important, the standard of behavior is likely to be strictly maintained as it is distinctly different from possible alternative standards, e.g. discriminatory ones. Thus changes of the wage-profit ratio are in general not likely to find a political basis and if they occur they are correlated with definite political reversions. If, however, the wage-profit question be opened in the absence of such political events and certain initiating inflationary changes ensue, the accepted standard should prompt the central banks to maintain the position of the savers by raising the bank rate of interest; thus precluding any consequential inflationary changes. The power element and the norm element are interwoven features of the situation, both calling for a certain constancy. The two economic theories are completely valid so long as one Imputation within the objective Solution prevails; they are somewhat modified, if a new Imputation within the Solution be established, as then the relations between the shares change decisively; they lose their validity, if a discriminatory Solution with its basic variabilities be adopted.[13)] These are the reasons why we label the theories in question ≫time-limited≫.

Inflation of the post-1930 type is unlikely to arise during the period treated because it is apt often to lack a political justification; moreover, if open distribution problems appear, a strong moral unwillingness to promote their settlement by modifying the accepted norm, due to the fact that a change of norm would imply a major alteration of qualitative principles, can be expected. We think, however, that the pre-1930 period has its own particular type of inflation; we shall return to this topic in our next subsection on time-limited theories.

Also as regards pre-1930 conditions we find some support of our position in a dictum by D. H. Robertson: ≫If we are to tell the blunt truth, was it not the menace of credit stringency which ultimately set a limit to

[13)] Compare Johan Åkerman's interpretation of Wicksell's theory, in [16], pp. 520–521, somewhat related to our interpretation.

the power of employers to concede increases of money wages and therefore to the determination of work-people to demand them?≫[14)]

Two further observations on the difference between objective and discriminatory Solutions conclude this subsection. *First,* in our discussion of the hierarchy of distribution problems we noted the increased strength of organizations after 1930 — the big social groups are, except for the savers, explicitly organized during the post-1930 period; during the pre-1930 period the presence of explicit organizations is not so strongly marked but despite this fact we considered it legitimate to speak of the groups as units, participating in the distribution process, in the same manner as we speak of the unorganized savers as such a unit. Our suggestion is now that the increased strength of explicit organizations is due to the discriminatory type of standard of behavior after 1930. The variability of such a standard is, as we know, likely to lead to a frequent resettling of the most important distribution relations; it thus calls for negotiations, repeated so often that it may pay to maintain a huge apparatus of organization. Only the savers need not be organized, as they are excluded from participating in the negotiations by the very type of norm accepted. This should be contrasted to the relative stability of an objective type of standard of behavior likely to require much less frequent discussions of basic distribution problems; forms other than strong organizations may therefore be sufficient to represent the different interests.

Second, the distinction between the two types of Solutions would gain in importance, if it be established that they imply different responses to changes of data. As pointed out in Chapter II, the games with more than four participants may show variations between significantly different Reduced Characteristic Functions, each Characteristic Function describing a particular ≫physical background≫, i.e. a particular set of objective possibilities of the various decision units in the distribution process. In Chapter VII we shall maintain that new investments are factors likely to imply alterations of the objective possibilities of the decision units, i.e. of Characteristic Functions; we are also suggesting that economic crises and depressions be processes of adjustment, caused by changes of Characteristic Functions. On the background of a Characteristic Function an Imputation is established, belonging to a particular Solution. Changes occurring to this Characteristic Function certainly have consequences for the established Imputation, which has to be more or less modified. It is now plausible, although this aspect has so far only been very incompletely

[14)] D. H. Robertson, *Money,* 4th edition, London and Cambridge 1948, Nisbet and Cambridge University Press, p. 222. Cf. D. H. Robertson, "Où en est le taux de l'intérêt?," *Économie Appliquée* 1949, pp. 204–205.

explored, that the prevalence of a discriminatory type of Solution with its continuity and indeterminacy makes the transition from one Characteristic Function to another with the ensuing adjustment of Imputations develop more smoothly than is possible under the reign of an objective type of Solution with its discontinuity and relative determinacy. The positive side of indeterminacy may be flexibility; the negative side of determinacy may be inflexibility. In this way it is possible that the strict adherence to an objective Solution has the tendency to necessitate adjustments so difficult as to provoke crises and depressions, while the adherence to a discriminatory type of Solution makes the adjustments easier. Thus the problem of labor unemployment, so important in the economic-political discussion, may be intimately related to the two types of accepted standards of behavior. We leave, however, the matter here with these brief remarks, which will be somewhat qualified in Chapter VII.

δ) Time-limited theories of interest and money

We have already on a few occasions above spoken about time-limited economic theories, a concept due particularly to Johan Åkerman.[15)] In our thesis it was first used in connection with the two interest norms, the orthodox, objective norm accepted before 1930, and the unorthodox, discriminatory norm accepted after 1930 — both often presented either as false or as universally valid theories, but in certain recent researches, mentioned on pp. 147–148, examined from a somewhat different point of view. In the previous subsection we further labeled as time-limited Douglas' theory of wages and Wicksell's theory of profits and interest — both usually presented either as false or as universally valid; compare, however, the opinion of Johan Åkerman on Wicksell's theory in the reference just given. In our opinion these two theories apply well to the relative fixity of income distribution existing during the reign of the orthodox interest norm. We also called the quantity theory of money time-limited. Our view is that this theory of inflation, letting the relative income shares remain intact during the inflationary process — it may also be utilized as a theory of deflation with the same implication — is valid for the period of the orthodox norm, with its strong factors working for a stable income distribution. If our interpretation be adopted, inflation (and deflation) were during this period — barring short-run adjustments and exceptionally violent cases — phenomena with fairly limited consequences. We accept in this context essentially the empirically strongly underbuilt explanations by, e.g. Cassel,

[15)] See, e.g. Johan Åkerman [16], pp. 521–522.

Warren and Pearson, and also Wicksell,[16] who attribute the long-run inflationary (and deflationary) movements of the period — identical to common variations of all prices — to movements of the quantity of money, more particularly the quantity of gold (in relation to the total, physical volume of production), but we consider the validity of the explanation and of the quantity theory in general not to extend further than 1930. During the period of the unorthodox, discriminatory interest norm long-run inflation takes another form, possessing widely differing consequences for the social groups; it is partly grasped by constructions like the bargaining theory of wages, Jörgen Pedersen's wage theory of inflation, and particularly the models attached to the articles mentioned in footnote 1, p. 181, showing both the initiating and the consequential inflationary changes. In our view all these theories are time-limited, confined to the post 1930 period, but they are partly presented as universally valid, although the last-mentioned articles apply a somewhat wider standpoint in their general discussion, putting inflation identical to changes in the distribution of Income between the big social groups.

In our analysis the time-limited economic theories have been fitted into the analytic theory of games. We have connected the interest norms with two types of Solutions, and the other time-limited theories with particular implications of these Solution-types for the Imputations included. Relating in this way certain traditional economic theories to elements of change of the theory of games, this theory could in one sense be called a ≫theory of theories≫. We are, however, completely aware of the fact that strictly speaking the word theory in an economic context should be confined to constructions like the theory of games, which start from explicit assumptions on human economic action; the time-limited theories discussed by us can hardly be said to do so, but are rather condensed expositions of empirical observations. Also in the section that follows we shall give examples of the phenomenon outlined here.

Thus comparing certain broad observations with deductions of the theory of games we have presented the first part of our sociological theory of inflation, attributing one type of inflationary phenomena to the excluded position of the rentiers in many modern societies. To avoid misscomprehension we stress emphatically that no value judgment, no political opinion of

[16] See, e.g. Gustav Cassel, "Om förändringar i den allmänna prisnivän," *Ekonomisk Tidskrift* 1904, pp. 312–331; Gustav Cassel, *The Theory of Social Economy,* London 1924, p. 447; George F. Warren and Frank A. Pearson, *Gold and Prices,* New York and London 1935, Chapter V, especially the graphs; Knut Wicksell , *Lectures on Political Economy II,* London 1935, p. 215 ff.

ours lies in the theory. We are not attacking nor defending the existing state of things. This may be inferred from the fact that the theory utilized by us predicts the consequences for the course of events, notwithstanding which interest group of society is placed in an excluded position. It would be of the greatest interest to our investigation if another period of inflation could be found during which a social group other than the savers was discriminated against. We think, however, that inflation of the type discussed is quite in accord with intelligent human action, if held within certain limits.

Our main interest, in this section as well as in all sections of the thesis, is the capacity of various economic theories to explain certain fairly simple facts from the socio-economic field. After having seen our attempts — to connect the orthodox normative interest theory with an Objective standard of behavior of the theory of games; to compare the tendency of interest rates to move proportionally to other prices with a maintained position of the rentiers within a fairly delimited type of Imputation; to connect the unorthodox normative interest theory with a discriminatory standard of behavior; to attribute the gradual decrease of the rentiers' relative share to the movement from one discriminatory Solution to others within the same family; to look at the frequently reopened wage negotiations in the light of the characteristic indeterminacy of Imputations belonging to a discriminatory Solution; and to find the essence of one type of inflation in the two last-mentioned circumstances — some readers may find our results to be an elaboration of the obvious. Why take the trouble to go into the theory of games when the outcome is merely a confirmation of well-known observations? Perhaps we may *first* find in all modesty some support for our position in the following words by von Neumann and Morgenstern: ≫We do not at all worry if the results of our study conform with views gained recently or held for a long time, for what is important is the gradual development of a theory, based on a careful analysis of the ordinary everyday interpretation of economic facts. This preliminary stage is naturally *heuristic,* i.e. the phase of transition from unmathematical plausibility considerations to the formal procedure of mathematics. The theory finally obtained must be mathematically rigorous and conceptually general. Its first applications are necessarily to elementary problems where the result has never been in doubt and no theory is actually required. At this early stage the application serves to corroborate the theory. The next stage develops when the theory is applied to somewhat more complicated situations in which it may already lead to a certain extent beyond the obvious and the familiar. Here theory and application corroborate each other mutually. Beyond this lies the field of real success: genuine prediction by theory≪.[17] *Second,* these words are further underlined by the extreme

[17] von Neumann and Morgenstern [1], pp. 7–8.

scarcity of attempts to connect some main concepts of the theory of games like Solutions, Imputations, and discriminatory Solutions with observed phenomena, i.e. to corroborate the theory with economic facts. *Third,* the doubtful capacity of Walrasian constructions to comprise the same facts may justify modest results; or if this wording be preferred: the dissimilarity of our discussion with explanations in terms of behavior equations may provoke interest.

c) INFLATION AND INTERNATIONAL ECONOMIC RELATIONS

α) *The game theory of Excess*

The previous discussion dealt only with closed economies, no attention being paid to the relations between national economies. Inflationary phenomena are, however, very commonly placed in connection with international features, e.g. changes in the rates of foreign exchanges. We have also mentioned one economic theory examining a certain type of inflation from an international point of view, namely the purchasing power parity theory of exchange rates. This theory pictures inflation as taking place without changing the relations between the national incomes in the set of countries studied — if two countries show parallel price movements the rate of exchange remains unaltered; if the price level of one country rises faster than the price level of another country, the value of the currency of the first country diminishes in terms of the currency of the second country so as to leave the relation between the two money national incomes intact, if these be measured in the same unit.

The purchasing power parity theory is often considered either to be false or to be universally valid — we shall suggest that it is time-limited. In order to accomplish this we must firstly try to fit it into a wider theoretical construction, and secondly, refer to the relevant econometric facts. The theoretical construction which we shall tentatively utilize is the theory of Excess in the *Theory of Games,* elaborated in the Chapter on "Composition and Decomposition of Games." The aim of this somewhat complicated chapter is to investigate which phenomena appear if two separate games[18)] are composed, that is viewed as one game — the constituent subgames are played by two distinct sets of players and completely separated from each other as far as the rules are concerned; or, we may also say, to investigate under what conditions and with what consequences it is possible to decompose one game into two constituent subgames, which are completely separated

[18)] We repeat that the word game is used synonymously with Characteristic Function.

from each other as far as the rules are concerned. The procedure of Composition and Decomposition may at first seem trivial and formal. There are, however, economic analogies which suggest themselves immediately: the subgames could tentatively be identified with two separate countries — the problem to be investigated is then what phenomena emerge, if the two countries are no longer considered as isolated from each other but viewed in relation to each other. This, may only be the preliminary step before introducing closer bonds between the economies, which require more than their being considered as two distinct but composed Characteristic Functions. It is plausible however, to think that the preliminary step has a great deal to say as an approximation to the more complete and difficult treatment that may finally ensue; especially it may be surmised that it can reveal the most elementary and basic additional features of an international economy in comparison with a strictly closed economy. Having discussed inflation from the point of view of the interaction between the big social groups of a national economy we are in this section precisely interested in the most obvious additional features which appear, if a national economy be examined in relation to other national economies.

The point is now that the procedure of Composition produces definitely non-trivial phenomena, due to the fact that the complete separation of the subgames so far as regards their Characteristic Functions need not imply the same for their Solutions. That is: ≫Although the two sets of players cannot influence each other directly, nevertheless when they are regarded as one set — one society —, there may be stable standards of behavior which establish correlations between them≪.[19] The deductions are centered around the concept of Excess.

Let us consider the game Γ composed by the constituent subgames Δ and H. We denote by J the set of all players in Δ and by K the set of all players in H. The basic fact of decomposability so far as the physical background — the Characteristic Function — is concerned, is expressed by the following condition

$$v_\Gamma(S \cup T) = v_\Delta(S) + v_H(T) \qquad \text{for } S \subseteq J,\ T \subseteq K \tag{20}$$

stating quantitatively the intuitive notion that Δ and H are composed to Γ without influencing each other; nothing can be gained through an agreement between a group of players S out of the participants in Δ and a group of players T out of the participants in H, because the combined amount

[19] von Neumann and Morgenstern [1], p. 341. This quotation and the following ones in the subsection are reproduced by permission of Princeton University Press.

which both groups can secure for themselves in cooperation arises additively from the two amounts which the groups can obtain acting separately. Thus interpreting the subgames as two separate countries and the players as the big social groups: there is no basis for any distribution agreement directly between a certain group in the first country and some group in the second country. Letting Γ be constant-sum, the following condition is valid

$$\sum_{i \in J} \alpha_i = v(I) = s \qquad \text{where } I = J \cup K \tag{21}$$

and we should expect the following relations also to hold

$$\sum_{i \in J} \alpha_i = v(J) = l \tag{22}$$

$$\sum_{i \in K} \alpha_i = v(K) = m = (s - l) \tag{23}$$

where s, l and m are constants. It is, however, a very important result of the Composition discussion in the theory of games that (21) does not in general imply the validity of (22) and (23), i.e., the constant-sum character of the composed game Γ does not in general imply that the total amount going to the participants in Δ — $\sum_{i \in J} \alpha_i$ — and the total amount accruing to the participants in H — $\sum_{i \in K} \alpha_i$ — are equal to given constants: $v(J) \equiv l$ and $v(K) \equiv (s - l)$. Instead these amounts may vary between certain limits: the case when

$$\sum_{i \in J} \alpha_i > v(J) \tag{24}$$

— and consequently

$$\sum_{i \in K} \alpha_i < v(K)$$

$$\text{as } \sum_{i \in J} \alpha_i + \sum_{i \in K} \alpha_i = v(J) + v(K) \quad —$$

being termed a positive Excess for the participants J in Δ and a negative Excess for the participants K in H;

the case when

$$\sum_{i \in J} \alpha_i < v(J) \tag{25}$$

— and consequently

$$\sum_{i \in K} \alpha_i > v(K)$$

being termed a negative Excess for the participants J in Δ and a positive Excess for the participants K in H. Introducing ε to denote the Excess we get

$$\varepsilon = \sum_{i \in J} \alpha_i - v(J) \text{ and } -\varepsilon = f = \sum_{i \in K} \alpha_i - v(K) \tag{26}$$

where ε may be positive, zero, or negative. The zero case obviously coincides with the old notion of a constant-sum game. The Imputations $\sum_{i \in J} \alpha_i$ and $\sum_{i \in K} \alpha_i$ of (26) are called Extended Imputations. We further denote the limits between which the Excesses must lie by $-|\Delta|_1, |\Delta|_2$ and $-|H|_1, |H|_2$ in such a way that:

$$-|\Delta|_1 \leq \varepsilon \leq |\Delta|_2$$

$$-|\Delta|_1 \leq f \leq |\Delta|_2$$

or as $f = -\varepsilon$

$$\left\{ \begin{matrix} -|\Delta|_1 \\ -|H|_2 \end{matrix} \right\} \leq \varepsilon \leq \left\{ \begin{matrix} |\Delta|_2 \\ |H|_1 \end{matrix} \right\} \tag{27}$$

Why is the extension of the theory to comprise Extended Imputations necessary? How are the limits of the Excess determined?

The first question is crucial to the whole discussion. If Extended Imputations did not appear, no non-trivial phenomena would arrive in the process of Composition because in that case the two subgames would be as separated with regard to their Imputations and Solutions as with regard to their Characteristic Functions.

The answer to the question in the theory of games is established through extensive and partly complicated reasoning letting heuristic plausibility arguments and mathematical procedures corroborate each other. We may indicate one line of reasoning. Having defined the concept of decomposability so far as the Characteristic Function is concerned — condition (20) — the next task is evidently to investigate the Solutions of a decomposable game, that is to discuss the concepts of Solution and Imputation in relation to the particular properties of the Characteristic Function of a decomposable game. We write Imputations for the subgames $\overrightarrow{\beta_J}$ and $\overrightarrow{\gamma_J}$ respectively, and Solutions = sets of Imputations $\mathbf{V}_J$ and $\mathbf{W}_K$ respectively. It is then natural to ask: May any two Imputations $\overrightarrow{\beta_J}$, $\overrightarrow{\gamma_K}$ for the

subgames be composed to an Imputation $\overrightarrow{\alpha_I}$ for the composed game; and conversely: may any Imputation for the composed game be decomposed into two Imputations $\overrightarrow{\beta_J}$, $\overrightarrow{\gamma_K}$ for the subgames? Similarly for Solutions: if $\mathbf{V}_J$, $\mathbf{W}_K$ are Solutions of Δ, H, is then their composition $\mathbf{V}_I$ a Solution of Γ; and conversely: if $\mathbf{V}_I$ is a Solution of Γ, can it then be decomposed into Solutions $\mathbf{V}_J$, $\mathbf{W}_K$ of Δ, H? (We call $\mathbf{V}_J$, $\mathbf{W}_K$ the $J-, K-$constituents of $\mathbf{V}_J$). By Composition of two Imputations is simply meant the following. Let an Imputation of Δ attribute to the players in the set J the amounts $\overrightarrow{\beta_J}$ and an Imputation of H attribute to the players in the set K the amounts $\overrightarrow{\gamma_K}$. An Imputation of Γ is composed from these two schemes of distribution, if it attributes to the players in the subset J precisely the amounts $\overrightarrow{\beta_J}$ and to the players in the subset $K(= I - J)$ precisely the amounts $\overrightarrow{\gamma_K}$. Conversely an Imputation of Γ is decomposable, if it distributes to the players in the subset J amounts, forming an Imputation of Δ, and to the players in the subset $K(= I - J)$ amounts, forming an Imputation of H. It is then obvious what Composition (Decomposition) of Solutions should imply: every member of such sets of Imputations must obey the conditions for composability (decomposability) just outlined.

If we confine ourselves to considering ordinary — non-Extended — Imputations for Δ and H, obeying the two familiar conditions

$$\begin{gathered} \alpha_i \geq v((i)) \\ \sum_{i=1}^{n} \alpha_i = v((1, \ldots, n)) = s \end{gathered} \tag{28}$$

composition is always possible but Decomposition is exceptional, as a decomposable ordinary Imputation of Γ must then obey, not only the conditions

$$\begin{gathered} \alpha_i \geq v((i)) \\ \sum_{i=1}^{n} \alpha_i = v(I) = s \end{gathered} \tag{29}$$

but also the conditions

$$\begin{gathered} \sum_{i \in J} \alpha_i = v(J) \\ \sum_{i \in K} \alpha_i = v(K) \end{gathered} \tag{30}$$

i.e., it must always give to the players of J exactly their ≫just dues≫, and the same is true for the players of K. It is clearly a severe restriction for an Imputation of Γ to obey all the conditions (29)—(30), that is to be

decomposable. It follows that decomposability of a Solution of Γ consisting of ordinary Imputations is an exceptional occurrence. Prima facie this is not plausible. ≫Since Γ is the composition of what are for all intents and purposes two entirely separate games, how could any solution of Γ fail to exhibit this composite structure?≪[20] The introduction of Extended Imputations in the sense of (26) for Δ and H removes the conditions (30) and is proved to grant the universal possibility of Decomposition of Solutions of Γ, the game with a decomposable Characteristic Function. Thus the use of Extended Imputations for the constituent games brings the setup in accord with the common-sense view, rejecting the exceptional character of decomposability of Γ-Solutions.

This fairly heuristic line of reasoning supplements the more basic and exact idea of Imbedding of a game, to which important formal analogies are found in the natural sciences. If a game Δ with players J is considered as an ordinary Constant-sum Game it is looked upon as a completely isolated occurrence — compare the study of a closed economy. The first step towards treating it as a non-isolated occurrence amounts to ≫imbedding it without modifying it, into a wider setup≪, which is defined to mean composing it with another game H with players K, arbitrary except for the condition that the two sets of players J and K be disjunct; the Composition of the two games is denoted by Γ and Δ is imbedded into Γ and Γ is an Imbedding of Δ — compare the elementary introduction of international relations into a study of a closed economy. The second step is to investigate all Solutions of Δ, viewing Δ as a non-isolated occurrence, which amounts to listing all Solutions of all possible Imbeddings Γ of Δ, and interpreting them so far as Δ is concerned. The last operation consists in taking the J-constituent[21] of the Γ-Solutions. Consider all imbeddings Γ of Δ, and all Solutions of these. Which are the J-constituents of these Δ-solutions? The answer is as we may expect: The J-constituents of the Γ-solutions are precisely the following sets: all Solutions for Δ with ε between the limits of (27).

Having thus indicated the rationale of the crucial introduction of Extended Imputations, we now turn to the second question about the determination of the limits of the Excess. The lower limit of the Excess ε of Δ, — $|\Delta|_1$, is the point where each participant in Δ is only given the amount that he can secure for himself in isolation, that is $v((i))$, and the totality of players gets $\sum_{i\in I} v((i))$, which is less than or equal to $v(I)$, the equality holding only for Inessential Games. If $\varepsilon < -|\Delta|_1$, ≫a situation would arise where each player would prefer to fall back on the game, even if the worst

[20] von Neumann and Morgenstern [1], p. 362.

[21] Cf. p. 201 above.

should happen, i.e. if he has to play it isolated≫[22], which in an Inessential Game is the only possibility.

The upper limit of the Excess ε of Δ, $|\Delta|_2$, is the point above which no players ≫in any imagined coalition can make such claims as to exhaust the available total. Then the very magnitude of the gift will act as a dissolvent on the existing mechanisms of organizations≫.[23] Thus if $\varepsilon > |\Delta|_2$, then for an Extended Imputation $\overrightarrow{\alpha}$, $\sum_{i\in S} > v(S)$ for every non-empty set $S \subseteq I$, i.e. every coalition is Ineffective for $\overrightarrow{\alpha}$. In an Inessential Game this happens as soon as $\sum_{i\in S} \alpha_i > \sum_{i\in S} v((i))$.

It follows from the last two points that the phenomenon of Excess is absent in Inessential Games, where $-|\Delta|_1 = |\Delta|_2$, but only there. They also indicate that abnormalities in the distribution of income accompany ≫too large≫ Excesses both in the paying and in the receiving community. At the present stage of analysis we do not think that much weight should be attached to the interpretation of the limits of the Excess; the very fact that there is an interval between the lower limit and the upper limit in all games except for the Inessential ones is most interesting in itself and deserves careful consideration.

The discussion of the Excess and of the limits of the Excess is quite general, no restrictions being placed on the games Γ, Δ, and H and their Solutions, except of course the requirement that Γ be decomposable. The detailed discussion of the consequences of an Excess for the distribution of income in a community has only been carried out, however, for the Essential Constant-sum three-person Game. A society experiencing a certain negative Excess has to face a deterioration of the incomes of its units in relation to the total income of the foreign area; the necessary adjustments are discussed in the three person setup, and it is found that the character of the Solutions are not changed through the appearance of a negative Excess — a finite (objective) Solution unique in its class remaining finite and unique, etc. The last proposition would seem trivial, were it not for the fact that it does not apply to the case of a positive Excess. As soon as a certain positive Excess comes into existence additional indeterminacies in the distribution of income arise. In this manner the variability within a family of discriminatory Solutions is strengthened; the character of the unique, objective Solution is even altered. This is turned into a family of very similar Solutions. Each member of this family does not contain three Imputations as before, corresponding to the three alternative coalitions that it is possible to form, but three sets of infinitely many Imputations, corresponding to the three alternative coalitions and letting the ratio between the income of

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22) 23) von Neumann and Morgenstern [1], p. 366.

the coalition and the income of the defeated unit vary continuously within certain limits; this ratio may consequently vary without any changes taking place in the basic power constellation of society, its fixation depending upon ephemeral factors, exactly as the fixation of the ratio between the money incomes of the non-excluded units in a discriminatory three-person Solution. The relation between the money incomes of the coalition partners is fixed within every Solution; it varies, however, continuously in a certain interval between the members of the family of Solutions, all based on exactly the same main qualitative ≫objective≫ principle and distinguished from each other only in less fundamental aspects. As a result all the three relative income shares in a three-person game with a positive Excess show a basic variability, being changeable without any alterations occurring to the grouping of the decision units in coalitions and power blocks and to the main qualitative accepted standard of behavior. In other words: ≫In the presence of a positive excess it may happen that a coalition can obtain beyond its effective maximum also some fraction of the excess. This possibility is then due entirely to the standard of behavior and not to the physical possibilities of the game. The fraction of the excess thus obtained may vary from 0% till 100% and be left undetermined by the standard of behavior. The latter will prescribe, however, uniquely, how the fraction obtained is to be distributed between the members of the coalition. This rule of division will depend on which of the many possible stable standards of behavior is chosen, and if the latter be varied, this rule will vary widely, although not quite unrestrictedly≪.[24)] We emphasize that the variabilities listed correspond to one particular value of the Excess.

Before commenting on the phenomenon of Excess we give the following summary, quoted from the *Theory of Games:* ≫We are considering two games Δ, H played by two disjunct sets of players J and K ...

Let us now find all solutions of the entire arrangement, i.e. of the composite game Γ. Since it is not desired to consider anything outside of Γ, we adhere to the original [constant-sum] theory. Then we have shown that any solution $\mathbf{V}_I$ determines a number $[\varepsilon]$ with the following properties: For every imputation $\overrightarrow{\alpha}$ of $\mathbf{V}_I$ the players of Δ (i.e. in J) obtain together the amount, $[\varepsilon]$, and the players of H (i.e. in K) obtain together the amount $[-\varepsilon]$. Thus the principle of organization embodied in $\mathbf{V}_I$ must stipulate (among other things) that the players of H transfer under all conditions the amount $[\varepsilon]$ to the players of Δ.

[24)] von Neumann and Morgenstern [1], pp. 417–418. It should be observed that we have only discussed the consequences of a positive Excess which is kept within the limits set by (27).

The remainder of the characterization of $\mathbf{V}_I$ — i.e. of the principle of organization, or standard of behavior embodied in it — is this:

First: The players of Δ, in their relationship with each other, must be regulated by a standard of behavior which is stable, provided that the transfer of $[\varepsilon]$ from the other group is placed beyond dispute.

Second: The players of H, in their relationship with each other, must be regulated by a standard of behavior which is stable, provided that the transfer of $[\varepsilon]$ to the other group is placed beyond dispute.

Third: The octroyed transfer $[\varepsilon]$ must lie between the limits

$$\left\{ \begin{array}{c} -|\Delta|_1 \\ -|H|_2 \end{array} \right\} \leq [\varepsilon] \leq \left\{ \begin{array}{c} |\Delta|_2 \\ |H|_1 \end{array} \right\} \tag{27}$$

The meaning of these rules is clearly that any solution, i.e. any stable social order of Γ is based upon payment of a definite tribute by one of the two groups to the other. The amount of this tribute is an integral part of the solution. The possible amounts, i.e. those which can occur in solutions, are strictly determined by [27] above. This condition shows in particular:

First: The tribute zero, i.e the absence of a tribute is always among the possibilities.

Second: The tribute zero is the only possible one if and only if one of the two games Δ, H is inessential...

Third: In all other cases both positive and negative tributes are possible — i.e. both the players of Δ and the players of H may be the tribute-paying group.

The limits [27] are set by both games Δ, H, i.e. by the objective physical possibilities of both groups. But where the actual amount $[\varepsilon]$ lies between these limits, is not determined by those objective data, but by the solution, i.e. the standard of behavior which happens to be generally accepted. These limits express that each group has a minimum below which no form of social organization can depress it: $-|\Delta|_1, -|H|_1$; and, each group has a maximum, above which it cannot raise itself under any form of social organization: $|\Delta|_2, |H|_2$.

Thus, for a particular physical background, i.e. a game, say Δ, the two numbers $|\Delta|_1, |\Delta|_2$ can be interpreted this way: $-|\Delta|_1$ is the worst that will be endured under any conditions and $|\Delta|_2$ is the maximum claim which may find outside acceptance under any conditions. It must be recalled that all this takes the value of the coalition of all players of Δ, $v(J)$, as zero; i.e. we are discussing the losses which are purely attributable to lack of co-operation among the group, and unfavorable general social organizations —

and gains, which are purely attributable to lack of co-operation in outside groups and favorable general social organizations≫.[25)]

We want to focus our comments on three facts. *First:* the most conspicuous and astonishing trait shown by the Decomposition discussion is the variability of the ratio between the total money incomes, e.g. of two separate countries or of one country and the outside world. Thus a ratio of this kind is in general not uniquely determined by factors of technology, taste, natural resources, capital equipment, etc. — the data or the objective possibilities of the situation. If no Excess could develop, as is the case only with Inessential Games, the ratio between the two total incomes would always be

$$r_0 = \frac{v(J)}{v(K)} \qquad \begin{matrix} J = \text{the set of units of area I} \\ K = \text{the set of units of area II} \end{matrix} \tag{31}$$

changing only with changes in the data or ≫the physical background≫; if an Excess can be present — as it always can (but need not) be in Essential Games —, the ratio may, however, vary between

$$r_1 = \frac{v(J) + \varepsilon_{max}}{v(K) - \varepsilon_{max}}$$

and

$$r_2 = \frac{v(J) - \varepsilon_{max}}{v(K) + \varepsilon_{max}}$$

The highest possible value of the Excess — ε_{max} — is determined by the objective possibilities of the situation, but these do not determine the value of r established in the interval $|r_1\, r_2|$; this value depends on other factors.

This result is obtained by logical reasoning on the basis of the fundamental concepts of Characteristic Function, Solution and Imputation, which in their turn are founded on a basic assumption about the aim of the decision units of a society to secure for themselves as large as possible a relative income share. It is not self-evident, i.e. immediately obvious, from the basic assumption, but represents a stage of theory, where the deductions go further than can be guessed by pure common sense; in this methodological respect it resembles the case of discriminatory Solutions in the three-person game — these are found through the rigorous application of the theoretical apparatus, but their existence can hardly be surmised by pure common sense.

[25)] von Neumann and Morgenstern [1], pp. 401–402.

Second: on what factors does the value of r, established at a certain point of time, depend? The variability of r does not always become effective; on the contrary it develops only under special conditions. More explicitly: a zero Excess with $r = r_0$ is always possible, and it represents the most natural state of things taking into account that the decision units of each country, if acting as a collaborating set, are really Effective for the amounts $v(J)$ and $v(K)$ respectively — and for no more; i.e. they can secure these respective amounts and no higher ones through concerted action against equally acting outside sets. Thus the appearance of a non-zero Excess must be somewhat exceptional, due to a relative weakness of one of the areas, prohibiting the full effect of the joint action of its units. The somewhat exceptional character of the Excess is underlined at various stages in the Decomposition discussion of the *Theory of Games*. In the introductory remarks it is suggested that particularly a negative Excess, a withdrawal, may be difficult to visualize. ≫Why should the players who could fall back on a game of constant sum $v(I)$, accept an inferior total? i.e. how can a standard of behavior, a social order, based on such a principle be stable? There is nevertheless an answer: The game is only worth $v(I)$ if all players form a coalition, and act in concert. If they are split into hostile groups, then each group may have to estimate its chances more pessimistically and such a division may stabilize totals that are inferior to $v(I)$≫.[26] The same line of thought is continued in the summary quoted above, where the losses through a negative Excess are termed ≫purely attributable to lack of co-operation among the group, and unfavorable general social organizations≫, and the gains through a positive Excess are called ≫purely attributable to lack of co-operation in outside groups and favorable general social organizations≫.

Fulfilling this line of thought we should expect to observe a persistent zero Excess — $r = r_0$ — so long as no country develops any signs of decisively less favorable general social organizations in relation to the other country, i.e. so long as there is a certain balance of power between the areas. On the other hand it seems plausible that once an Excess has developed — due to unfavorable relative conditions in one country — there will ensue a certain variability in the interval $|r_2\, r_0|$ or $|r_0\, r_1|$. The favored part must be expected to try gradually to enlarge the Excess and push it towards its maximum in order completely to utilize the relative weakness of the disfavored part; this part in its turn is likely to hesitate between accepting the conditions prescribed from abroad and attempting, so soon as a possibility appears, to reverse the course of things and push the Excess back to its more normal zero magnitude.

[26] von Neumann Morgenstern [1], p. 365.

Third: The variability of a non-zero Excess, especially its tendency to be pushed towards its maximum, is, however, much more clearly brought out if the consequences of a positive-Excess for the distribution of income in the relatively strong society are taken into account. As soon as a positive Excess is experienced in a society, certain basic variabilities of relative money incomes occur, no strong factors of power or moral principles completely fixing the ratios between the various incomes. Thus frequent initiating changes of income relations are likely to take place. One group demanding a bigger relative share would require a deterioration of the position of some other group. In a discriminatory Solution an Excluded unit may at least partly take the consequences of an initiating change, as long as it is above its minimum point. Under the existence of Solutions with a positive Excess the consequences of the initiating changes — made likely through the variabilities inherent in such Solutions — may be taken by the foreign area: the trouble arising from the diminution of the relative income share of one or some national units in response to the improvement of the position of some other unit can be avoided by increasing the positive Excess, as long as this is feasible, due to the relative weakness of the foreign area. If this be made, no national income share need deteriorate relative to the total amount $v(J+K)$ or to the national income existing before the increase of the positive Excess $[v(K)+e_1]$. We encounter here the traits of a cumulative process: as soon as a positive Excess comes into existence, this causes a certain indeterminacy of the income distribution to arise; this indeterminacy makes initiating changes likely to occur, the ultimate negative consequences of which no national unit may want to take, and need not, as long as the possibility exists of increasing the positive Excess; such an increase does not, however, remove the indeterminacy of the income distribution in the relatively strong area — on the contrary, it takes on still worse proportions if the Excess be pushed beyond its upper limit.

We also wish here to draw attention to the peculiar position of an economy having to accept a negative Excess, i.e. a withdrawal, gradually increasing from the zero point towards the maximum. Starting from a certain total amount to be nationally distributed — $v(J)$ —, and barring possible changes of the Characteristic Function itself, such a process shows for each new time-interval $t_1, t_2, t_3, \ldots$ a smaller amount to be distributed between the national units: $v(J)-\varepsilon_1, v(J)-\varepsilon_2, v(J)-\varepsilon_3, \ldots$ where $\varepsilon_1 < \varepsilon_2 < \varepsilon_3 \ldots$, or, which is perhaps more adequately expressed, shows for each new time-interval a smaller relative share out of the combined income of the two countries — $v(J+K) = v(J)+v(K)$ — accruing to the own area: $\frac{v(J)-\varepsilon_i}{v(J+K)}$. In such a type of situation, for instance, attempts by some national group to maintain its position relative to that of some foreign groups may be

incompatible with a stable distribution of the national income. The three-person setup with a negative Excess — so far the only example completely discussed — shows the following conditions for an Extended Imputation to belong to a Solution. In the objective Solution the ratio between the incomes of the two winning units must remain the same through a period of increasing negative Excess, and the ratio between their incomes and the income of the losing unit must increase until, at the point of maximum Excess, all units are given solely their minimum amounts. Within a family of discriminatory Solutions the variabilities obviously allow a greater number of alternatives. Maintaining strictly one particular discriminatory Solution, the decline of the Excluded units position due to the deteriorating foreign relations is completely determined, but a larger part of the burden may be imposed on this unit by shifting the standard to another element of the family of Solutions, which treats the Excluded unit less favorably. Certainly, however, the unit cannot be pushed below its minimum point; when this is reached the adjustments to the changed foreign conditions must take place at the cost of the non-excluded units.

β) The distribution of income between countries

In Section 1 of this chapter we distinguished between two types of observations pertinent to the international aspects of inflation, namely an approximate constancy of the relation between two national money incomes during a process of monetary change, implied by an approximate coincidence of the internal and the external values of the currencies, and a changing relation between two national money-incomes during a process of monetary change, implied by a divergence between the internal and the external values of the currencies, and tied, e.g. to price rises taking place faster in one region than in the other, all prices measured in the same currency.

Our way of interpreting the observations involves the assumption that the nominal incomes relevant to our analysis are proportional to price levels. In our preceding discussions we have let the nominal income share going to the savers be represented by an interest rate; this income share we have compared with the rest of nominal shares, represented by a price index. Our procedure in considering the two national price indexes as correlated with the nominal incomes measured in national units is thus at least quite consistent with the earlier approach. By applying the procedure we are disregarding what may be labeled changes in productivity and a number of most important other ≫real≫ changes, but it seems perfectly in agreement with the theory utilized to look upon such changes as changes in the ≫physical background≫, i.e. in $v(J)$ and $v(K)$. Moreover, reservations for ≫real≫ changes that may occur are quite common in the literature on the

purchasing power parity theory. As is well known, this theory is consistent with the first type of observations, showing approximate equality between the external and the internal values of the currencies, i.e. giving evidence of a stability in the ratio between the two money national incomes to be compared.

The first part of our hypothesis is: the observations consistent with the purchasing power parity theory imply the prevalence of a zero Excess between the countries studied. In our comments above on the theory of Excess we supported the opinion that a persistent zero Excess — implying a stable ratio, $r = r_0$, between the two incomes, barring real changes — should be observed so long as no country shows any signs of less favorable general social organizations or of a decisive lack of cooperation between its social groups, i.e. so long as a certain balance of power between countries exists. We now add the almost trivial observation that the period of the gold standard prior to 1914 came close to materializing such a period of international balance with no county showing to a marked degree favorable general social organizations in relation to the other countries and hence taking a dominating position. The purchasing power parity theory did not hold only during the era of the gold standard; also in the years following 1914 up to the 1930's certain important facts comply to the theory; the exceptions, however, begin to appear. This latter period shows many signs of being a period of transition, still characterized by a certain balance of power but with traits developing of less favorable European interior social conditions, e.g. in countries like France, where the lack of cooperation between the decisive units is quite evident in certain time intervals, and in Germany and Central Europe during the early 1920's.

Under the prevalence of a persistent zero Excess and of the purchasing power parity theory, inflation in an international context means only proportional changes of the various national incomes and is no important phenomenon, just as is the case with inflation in a national context under the prevalence of an objective Solution and of the quantity theory of money, implying a relative stability of the national income distribution. In our opinion, however, the purchasing power parity theory and its implications are only time-limited, building implicitly on the political and social premises connected with a zero Excess.

The second part of our hypothesis is: the observations showing successive changes of the relations between the national incomes compared imply the presence of a non-zero Excess; more precisely the observations in the period following the Second World War imply a tendency towards a growing American positive Excess in relation to Europe — partly and temporarily(?) reversed by the economic impact of the Korean War — while the earlier instances of variability in the income ratio imply the appearance and

later disappearance of the same kind of Excess. The observations involved are quite alien to the purchasing power parity theory, and so are the social and political conditions under which they have developed.[27] Also in this case it is almost trivial to add — in addition to what has already been said concerning certain periods of relative weakness of France and Germany — that contemporary Europe in relation to USA is strongly characterized by disfavorable general social organizations and lack of cooperation among its national units and among the social groups within these units. In our preceding comments on the theory of Excess we endeavored to make plausible that once an Excess has developed, due to unfavorable relative conditions in one country or a set of countries, this Excess is likely to grow towards the maximum point — where the ratios $r = r_1$ or $r = r_2$ prevail — or to decline again towards the zero point where the ratio, $r = r_0$ is established —, but not to remain constant. The reason is that a non-zero Excess must be expected to cause a distribution struggle, the strong part trying to utilize its advantages and push the ratio of the incomes towards its extremum, and the weak part attempting to reestablish the zero Excess as soon as a potentiality arises for doing so. The pressure likely to arise towards the maximum point as long as the relative weakness of one region persists is particularly brought out if the following cumulative process be taken into account: the appearance of a positive Excess causes indeterminacies to arise in the distribution of income of the relatively strong region, or strengthens the indeterminacies already existing, e.g. due to the discriminatory type of the national Solution. These indeterminacies constitute the setting for a distribution struggle, which may be most easily solved through an increase of the positive Excess, when this is feasible; the increase does at least not diminish the indeterminacies.

In the French and German cases we observe a period of changing income relations surrounded by periods of stability, which in our interpretation means the following course of events: a zero Excess; a growing negative French (German) Excess; a diminishing negative French (German) Excess; and a zero Excess. Thus the relative weakness experienced by the two economies in one time-interval disappeared in a later time-interval.

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[27] For a somewhat related view of international economic relations see François Perroux, "Esquisse d'une théorie de l'économie dominante," *Économie Appliquée* 1948 pp. 243–300, and François Perroux, "La généralisation de la 'General Theory,' " Conférences faites en Décembre 1949 à la Faculté des Sciences Économiques et Sociales d'Istanbul, Istanbul 1950, esp. pp. 68–71.

The perspective is still too short to admit a judgment whether more than the first two stages of this chain are so far discernible in the recent observations. The pressure towards enlargement of a negative European Excess after the Second World War and the peculiar distribution struggle tied to it is in our opinion inherent in the ≫dollar shortage≫ prevailing.[28] The dollar shortage signifies that ≫at the prevailing exchange rates and price levels it appears that, considering the current items of the balance of payments of the United States, demand for dollars surpasses their supply≫.[29] The remedies for the scarcity that are proposed may be grouped into two categories. The first category comprises measures directly implying the reduction of European money national incomes relative to the money national income of USA: expansion of investment and consumption in the United States, contraction of investment and consumption in Europe, appreciation of the dollar, depreciation of the European currencies, expansion of imports into USA, restriction of exports from USA, expansion of exports from Europe, restriction of imports into Europe (all points e.g. Kindleberger)[30], change of the ≫terms of trade≫ to American favor, i.e. lowering of the ratio p/q, where q = price level of imports into USA and p = home price and export price level in USA (Tinbergen). It follows from our combination of the relevant observations with the theory of Excess that we consider these measures as attempts to establish a certain ratio, $r = r_i$, between the two money total incomes, which is essentially prescribed by non-economic, i.e. by political and social forces. The ratio to which an adjustment must take place is, however, likely to have a tendency to shift in the direction of the extremum ratio so long as the relative political and social weakness of the European countries persists, and thus the measures deemed sufficient at one point of time to cure the dollar shortage, may, when carried out, have to be followed by still further measures. The second category of remedies for the shortage comprises measures which probably should be interpreted as attempts to change the objective possibilities in the favor of Europe: lending to Europe, gifts to Europe (e.g. Kindleberger; Marshall plan), increase of European productivity.

Returning to inflation we think that this phenomenon — in its international aspects — has implications in the cases studied, quite different from those of the purchasing power parity theory inflation. When the

[28] Interesting aspects of this distribution struggle can be found in T. Balogh, *The Dollar Crisis — Causes and Cure,* Oxford 1950.

[29] Jan Tinbergen, "Some Remarks on the Problem of Dollar Scarcity," *Econometrica* 1949, Supplement, p. 73 (Vol. 17).

[30] Charles P. Kindleberger, *The Dollar Shortage,* New York and London 1950, p. 193.

balance of power between two regions is disturbed, a pressure is likely to arise towards a gradual increase of the money national income of the relatively strong region in relation to the money national income of the relatively weak region. Keeping exchange rates constant, this may be achieved through price rises, taking place faster in the dominating area than in the dominated area; certainly it may also be achieved, e.g. through keeping the first-mentioned prices constant and letting the second-mentioned prices fall, but this way of adjustment may be precluded by the interior economic development in the countries involved — for instance, if the economic policy be of the discriminatory type discussed in the previous section, or if the trouble tied to a negative Excess arises, which will be discussed in the following subsection. Taking also exchange rates into consideration the pressure implies: the prices of the dominating area rise faster than those of the dominated area, all prices measured in the same currency — a too heavy price rise in the weak region, as measured by its own currency, may be compensated for by exchange depreciation. When the balance of power between the two regions is restored, a return to the old price ratios will ensue, and if the interior economic development be favorable to inflation, this return may be achieved through further price rises, now taking place faster in the previously dominated area than in the previously dominating area, all prices measured in the same currency.

Thus inflation during the earlier period — under the prevalence of a zero Excess — implies proportional changes of the money national incomes involved, while inflation during the latter period — under the prevalence of a changing negative European Excess — is connected with changes of the relation between the American and the European money national incomes. A historical parallel to the latter case, deserving further comparative study is the rise of Spanish prices in silver, more than proportional to the rise of other prices in silver, taking place during the period of Spanish trade hegemony in the sixteenth century.[31)]

Combining certain of our conclusions about inflation both in its national and in its international aspects we find: In the pre-1914 world, during the reign of objective Solutions with a zero Excess, inflation (or deflation) could be adequately grasped by the quantity theory of money and the purchasing power parity theory of exchange rates, placing the emphasis on the proportional change of all prices during an inflationary period. In the post-1945 world, during the reign of discriminatory Solutions (with the savers as the Excluded unit), and of Solutions with a non-zero Excess (with USA in the relatively strong position and Europe in the relatively

[31)] See Earl J. Hamilton, *American Treasure and the Price Revolution in Spain, 1501–1650,* Cambridge, Mass. 1934.

weak position), inflation is tied to a non-proportional change of various prices and income-shares. During the first period the insignificant type of inflation (or deflation) may be said to depend on ≫economic≫ factors, such as the change of gold stocks; but the basic stability of income relations is due to factors of power and habitual principles, i.e. to political and social elements. During the second period the instability of income relations forms the essence of the type of inflation then prevailing; we also attribute this instability to factors of power and habitual principles, i.e. to political and social elements.

We think, however, that one more aspect of inflation during the second period must necessarily be treated in this context, and turn to this point in the following subsection.

γ) *Inflationary consequences of a negative Excess*

In the remarks which follow we shall connect our analysis with the above discussion of the consequences of a negative Excess for the income distribution within a country, experiencing such a phenomenon. We have attributed the more than proportional rise of American dollar price compared with European ones to the factors causing a growing negative European Excess. There is often, however, a most well-known price movement superimposed on this non-proportional development of international prices, namely violent price rises in the relatively weak countries, accompanied by heavy drops in the values of their currencies in terms of dollars — an outstanding example is the German inflation in the early twenties, superimposed on changes of international price ratios. This type of price rise we attribute to a distribution struggle, being initiated by the appearance and/or growth of a negative Excess. Barring counteracting changes of the objective possibilities, the negative Excess requires a diminution of the national income in relation to the foreign income; if some national groups try to maintain their position relative to, let us say, a price index with foreign elements, this action may be incompatible with the stability of the income distribution and thus cause reactions by other groups; together the actions and reactions may compose a vicious inflationary spiral, which is much more dangerous than the two cumulative inflationary processes analyzed earlier — in this case no safety-valve in the form of an Excluded unit need exist, and by assumption no safety-valve in the form of a relatively weak foreign area is present. To mention only one example: many commentators on the German 1920-inflation stress the fatal importance for the inflationary process of wages — and other national income shares — being tied to import prices and paid on a cost-of-living basis. We quote from Joan Robinson's review of Bresciani-Turroni's *The Economics of Inflation*: ≫The author rejects the view that exchange depreciation caused the inflation. But

the difficulties of his own explanation are just as great. Neither the budget deficit nor the increase in quantity and velocity of circulation of money can produce the effects attributed to them. Clearly in each explanation some essential item is missing.

The missing item is not far to seek. It is the rise in money wages. Neither exchange depreciation, nor a budget deficit can account for inflation by itself. But if the rise in money wages is brought into the story, the part which each plays can be clearly seen. With the collapse of the Mark in 1921, import prices rose abruptly, dragging home prices after them. The sudden rise in the cost of living led to urgent demands for higher wages. Unemployment was low . . . , profits were rising with prices, and the German workers were faced with starvation. Wage rises had to be granted. Rising wages, increasing both home costs and home money incomes, counteracted the effect of exchange depreciation in stimulating exports and restricting imports. Each rise in wages, therefore, precipitated a further fall in the exchange rate, and each fall in the exchange rate called for a further rise in wages. This process became automatic when wages began to be paid on a cost-of-living basis. Thus the authors contention that the collapse of the Mark cannot have caused the inflation, because the exchange rates will always find an equilibrium level, is derived of all force as soon as the rise of money wages is allotted its proper rôle≫.[32)]

Certainly the settling of such a distribution struggle is made somewhat easier if a discriminatory type of Solution prevails in the national economy, but the minimum point of the Excluded unit's relative income share may be reached fairly soon. In case a distribution struggle initiated by an appearing or a growing negative Excess and resulting in heavy price rises, is not accompanied by rising values of the foreign currencies or if some groups tend to make depreciation ineffective through attempts to maintain their positions relative to the foreign area, a serious situation ensues which may require ≫compulsory savings≫ or turnover taxes taken from all groups in the society and ≫sterilized≫. This policy measure seems natural to adopt, from the point of view of an analysis in terms of ≫the inflationary gap≫, a notion which we think is partly connected with economies experiencing the necessity of accepting a deteriorating relation between their incomes and the income of a foreign area. An inflationary gap is often defined to exist when ≫total demand exceeds total supply≫. We think that it may have relevance as regards, for instance, the following type of situation. Two time-intervals are compared, between which a deterioration has occurred to the ratio of the national income and the foreign income. The social groups in the own country act, however, as if no change had taken place,

[32)] *The Economic Journal* 1938, p. 510.

i.e. as if no increase of the negative Excess had happened, — they base, for instance, their requirements for consumption and investment on the conditions existing in the early interval, trying to get their income shares completely compensated for the later rises in the cost of living, which include import price rises coming from abroad, not balanced by corresponding rises of export prices. If there be no Excluded group to be shifted towards a less favorable position, the ensuing situation could be adequately described as an excess of requirements over the means available. Of course the state of things will be still worse if at the same time some social group tries not only to maintain its earlier position but even to improve it; and it will be easier to manage if at the same time counteracting changes of the objective possibilities occur, e.g. increases of ≫productivity≫. We also believe that the notion of inflationary gap and related constructions may have relevance for other situations tied to discriminatory Solutions and to Solutions with a non-zero Excess — the initiating changes may cause requirements to exceed the means available until the consequential changes have been forced to take place — but we consider the theories in question as clearly time-limited, having e.g. nothing essential to say about conditions under objective Solutions with a zero Excess, implying a relative stability in the national and international income distributions.

4) Summary

a) CONCLUSION ON THE RELATION BETWEEN THE FIRST SUMMATION THEOREM AND EMPIRICAL RESEARCH

The discussion of inflation and the rate of interest compares further empirical phenomena, correlated with the prevalence of the orthodox interest rate norm and of the unorthodox interest rate norm respectively, with more extensively discussed theoretical consequences of an objective type of Solution and of a discriminatory type of Solution respectively. On the empirical plane indications of a fairly stable relation between the income shares of the social groups during the first period are contrasted with the frequent ≫initiating inflationary changes≫, followed by the ≫consequential inflationary changes≫, during the second period. The initiating inflationary changes signify an alteration of the wage-profit ratio; the consequential inflationary changes, signifying the deterioration of the relative income share of the savers, permit this alteration to take place without the diminution of the relative income share of one of the two groups laborers and entrepreneurs — as would happen if their combined relative income share should be constant despite the alteration —, or it at least dampens the diminution which would otherwise occur. One of the most important empirical indications of the instability in the wage-profit ratio is constituted by the marked growth, during the second period, of institutions attempting to fix this ratio, viz., arbitration councils and price and wage controls. On the theoretical plane the relative stability of the income relations of an objective Solution are contrasted with the instability of these relations within a family of discriminatory Solutions, putting the savers in an Excluded position. The relative stability is due firstly to the significant, qualitative difference likely to exist between an objective Solution and other possible Solutions; secondly and above all to the limited number of distinct Imputations within such a Solution, each corresponding to a definite power constellation in society. The instability is due firstly to the insignificant difference existing between the various discriminatory Solutions, building all on exactly the same main qualitative principle; secondly to the infinity of Imputations within such a Solution, passing continuously over into each other and being changeable without anything happening to the grouping of decision units in coalitions and power blocks. In the first case an initiating inflationary change is often apt to lack political background (and a consequential inflationary change may lack a moral justification); in the second case a fixation of the wage-profit ratio is in general apt to lack political background and the maintenance of the savers' position is apt to lack a strong moral support.

The discussion of inflation and the rates of exchange also follows the main scheme of analysis: observed phenomena are compared with theoretical consequences of the first summation theorem, and certain elements of change of the theory are identified with the observed phenomena. On the empirical-plane two different types of relations between the social and political conditions in two regions are distinguished. The first type is characterized by a balance of power between a set of countries, substantiated by well-known qualitative observations, and prevailing during the period of the internationally accepted gold standard and in a less pronounced manner and with exceptions in the 1920's and 1930's. The second type is characterized by a marked relative weakness of one subset of countries considered, due to political and social factors: examples are the positions of Germany and France relative to USA during certain years of the 1920's, and the position of Europe relative to USA after the Second World War. On the theoretical plane two different types of national Solutions are distinguished. The first type comprises Solutions with a zero Excess — every Imputation within such a Solution has a zero Excess, i.e. it is not Extended, but gives to the region considered exactly the relative amount which it can expect from the objective possibilities prevailing, say $v(J)$ out of $v(J+K)$. The second type comprises Solutions with a non-zero Excess — every Imputation within such a Solution has the same non-zero Excess, i.e. it is Extended and does not give to the region considered the relative amount which it can expect from the objective possibilities prevailing, but instead a lower relative amount to a relatively weak region — say $[v(J)-\varepsilon]$ out of $v(J+K)$, a negative Excess — and a higher relative amount to a relatively strong region — say $[v(K)+\varepsilon]$ out of $v(J+K)$, a positive Excess. The national Solutions with a zero Excess are by us identified with the conditions prevailing under the first type of time-periods; the national Solutions with a non-zero, negative Excess are identified with the conditions prevailing under the second type of time-periods in the relatively weak region and the national Solutions with a non-zero, positive Excess are identified with the conditions prevailing under the second type of time-periods in the relatively strong region. It should be emphasized that no restrictions are placed on the national Solutions except for the requirement that they show a certain Excess — beyond this they may embody any principle which is feasible, and be, for instance, objective or discriminatory. On the empirical plane the first type of relations between the social and political conditions in two regions is correlated with the approximate equality between the external and the internal values of the currencies, i.e. by a stable ratio between the two money incomes, barring ≫real≫ changes. The second type of relations is correlated with a changing positive difference between the internal and external values of the currency of the relatively weak region, and with a changing negative difference between the internal and external values

of the currency of the relatively strong region; i.e. by an unstable ratio between the two money incomes, the relative income share of the relatively strong region being always greater than it should have been under an approximate equality between the external and the internal values of the currencies. On the theoretical plane the existence of a zero Excess is the most normal state of things; it implies a stable ratio between the money incomes of the two regions. The appearance of a non-zero Excess is tied to the existence of relatively unfavorable general social organizations in one region and lack of cooperation among its decision units. A non-zero Excess may take a whole range of values, i.e. vary between the zero point and the point of maximum Excess. It lies in the interest of the relatively strong region to make its (positive) Excess as big as possible and it lies in the interest of the relatively weak region to push the Excess back to the zero point; this makes the ratio between the two money incomes likely to be unstable under the reign of a non-zero Excess, the relative income share of the relatively strong region being always greater than it should have been under the reign of a zero Excess. The instability of the ratio between the two money incomes on the theoretical plane is, however, also linked to other circumstances. The appearance of a positive Excess has certain consequences for the distribution of income in the region involved: a variability similar to that existing within a discriminatory Solution arises, allowing within certain limits the national Imputations to go over into each other continuously, varying without changes of the power constellation of society. Under the reign of a discriminatory type of Solution, disregarding international relations, an initiating change of the relation between the income shares of two non-excluded participants is likely to be followed by a consequential change, pushing the Excluded participant to a less favorable Solution; in this way the relative income share of no non-excluded unit need deteriorate. Under the existence of a positive Excess, the consequences of an initiating change — which is likely to occur due to the basic variability of Imputations arising when a positive Excess appears — need not be taken by an Excluded unit, if a lowering of a national relative income share out of the total income can be avoided by increasing the positive Excess. Therefore, when a positive Excess has once appeared a constant pressure is likely to be exerted towards its maximum point as long as the relative strength of the region persists, i.e. the relation between the two money incomes will gradually change in favor of the relatively strong region. Thus the empirical phenomena correlated with the two different types of relations between the social and political conditions in two regions fit well into the theoretical consequences of Solutions with a zero Excess and of Solutions with a non-zero Excess respectively.

The phenomenon of inflation we connect with all the variabilities of the relative income shares mentioned. A discriminatory Solution makes initiating changes likely to occur, which signify the rise of a relative income share of a powerful social group; they need not signify, however, a corresponding decline of the relative income share of another powerful social group, since a consequential change may let an Excluded group experience the necessary deterioration as long as this group is above its minimum point. A Solution with a positive Excess can persist only as long as a lack of balance of power exists between the regions involved; it lies in the interest of the favored region to enlarge the Excess, and it lies in the interest of the disfavored region to diminish the Excess when an opportunity arises for achieving this. Moreover, a positive Excess makes initiating changes likely to occur in the relatively strong region, which signify the rise of a relative income share of a national group; they need not signify, however, a corresponding decline of the relative income share of another national group as long as a consequential change can force the relatively weak region to take the necessary deterioration. This might explain why prices in general rise more than the prices tied to an Excluded group, and why prices in a relatively strong region have a tendency to rise more than prices in a relatively weak region (all prices measured in the same currency) — a tendency which should be followed by movements of the prices in the relatively weak region, resulting in the establishment of the original price ratios, as soon as the balance of power is restored.

In addition to this type of inflation with a disproportionate development of various income shares we mention the insignificant type of inflation, likely to be tied to objective Solutions and to Solutions with a zero Excess, implying a proportionate development of all income shares; and the disastrous type of inflation which could happen in an economy experiencing a growing negative Excess, if certain social groups try to avoid adaptation to the deteriorating conditions.

b) CONCLUSIONS ON THE RELATION BETWEEN THE FIRST SUMMATION THEOREM AND OTHER ECONOMIC THEORIES

Certain theories of inflation, such as the quantity theory of money, the purchasing power parity theory of exchange rates, and also various notions of inflationary gaps, are sometimes cramped into Walrasian constructions. Relying on results reached by Patinkin and Marschak we do not consider such an inclusion possible to carry out in a consistent manner. We think, however, that these theories and related constructions can be justified as time-limited theories, valid so long as certain types of elements of change of game theory prevail. Thus the quantity theory and the purchasing power parity theory are approximately consistent with the relative

stability of Imputations likely to exist under objective Solutions and Solutions with a zero Excess; to these conditions also Douglas' theory of wages and Wicksell's theory of the normal interest rate apply. The theories of inflationary gaps and, for instance, various bargaining theories of wages seem on the other hand to fit well into basic features shown by Imputations belonging to discriminatory types of Solutions and to Solutions with a non-zero Excess.

Chapter VII

Business Cycles

1) The Time-series

We want to emphasize two categories of facts, pertinent to business cycles. The first category was discovered by the early pioneers among business cycle investigators: the wave-like fluctuations of many economic time-series, the upswings and downswings being clearly marked although the period be somewhat irregular and much more so the amplitude. In this connection we mention Juglar and his *Des crises commerciales et leur retour, périodique en France, en Angleterre et aux États-Unis* (1860)[1)]; Juglar's name is sometimes explicitly tied to the ≫8–10 year period major cycle≪. The second category belongs to a somewhat later era in the history of business cycle research: the pronounced rise of investment during the upswings, showing itself in a great relative amplitude of the time-series relating to investment activities. In this connection we mention Schumpeter and his *Theory of Economic Development* (1911)[2)]; Schumpeter, more vividly and extensively than others, has underlined investment, and particularly new specific investments, ≫innovations≪ by single firms or groups of firms, as the truly dynamic element in the economic process.

The question will now be posed: in what manner do the various economic theories considered take into account the two categories of facts thus outlined, the fluctuation facts and the investment facts; especially we are interested in their treatment of the investment facts.

2) Economic dynamics I: non-steady-rate flows

In this chapter we return to the disposition employed in Chapter V, discussing first business cycles from the pure production aspect.[1)] There is

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[1)] Juglar [46].

[2)] Schumpeter [47].

[1)] For a most illuminating survey and discussion of traditional analytic business cycle theories see the article by Henri Aujac, "Les modèles mathématiques macrodynamiques et le cycle, *Économie Appliquée* 1949, pp. 496–592, including works by Kalečki, Frisch, Theiss, Haldane, Amoroso,

an important but seldom explicitly recognized point of diverging opinions among business cycle researchers as to how far the development of an analytic theory of business cycles has reached. Many economists, particularly those following traditional econometric and Keynesian lines, hold the view that almost self-evidently the systems of differential and difference equations utilized in ≫economic dynamics≫ and essentially borrowed from classical mechanics, represent a true picture of the observed business cycle phenomena, the crucial matter of investigation being the forms and coefficients and aggregativeness of the relations. In other words: the explicit functions of time, the time-paths or ≫oscillations≫, shown by a variable solved from such a system of equations is considered to be in principle directly comparable with the business cycle time-path of an observed economic magnitude, possibly after the introduction also of shocks, etc. As outstanding representatives of this widely spread opinion we mention Tinbergen and Klein; we further quote the following characteristic words by Metzler about the supremacy of ≫modern≫ business cycle theory developed in close contact with the ≫theory of employment≫: ≫The changes in business cycle theory are of two sorts. First, the modern theory of employment establishes a definition of a norm about which the economic system fluctuates:

Tinbergen, Ezekiel, Samuelson, Radice, Kaldor, Metzler, Tintner, Bennion, Goodwin, Klein, Baumol, Colin Clark and Hicks. Among his remarks the following ones are particularly congenial to our exposition: 1) The significance of the whole type of theory must be doubted if one recalls the fact that it seems little capable of comprising the important and dominating problems of distribution of income between various social classes (p. 574, cf. pp. 572–573); 2) The subunits within the national economy are in all models exclusively chosen from purely economic considerations, leaving out of account all political, sociological and other factors; besides very few subunits are introduced (pp. 575–579); 3) The distinction real *vs.* monetary flows and stocks is often vaguely put, but most recent models seem to favor the real kind, despite the difficulties of measurement (p. 575); 4) The equations of the models studied are without exception linear, but this seemingly extreme restriction is most rudimentarily or even erroneously justified (p. 580); 5) The dynamic elements — mainly time-lags — are introduced, not on the basis of a careful reasoning, but rather for the mere purpose of giving models which show oscillations (pp. 583–587).

The first three points serve to underline our opinion, expressed below in this section and implicit in Chapters I–IV that the researches considered ought to be viewed essentially as special cases of the more general programming analyzes, considered in connection with Leontief's and Hawkins' systems; the fourth point, and to some degree the fifth point, refer to deficiencies which are improved in the programming analyzes.

at any given time, the equilibrium level of income is the level at which intended savings are equal to intended non-induced investment. Business cycles consist simply of fluctuations about this norm or equilibrium ... The second important contribution of the theory of employment to business cycle analysis is in the explanation of turning points of the cycle ... When the concept of a propensity to consume was introduced, the economic system was recognized as an essentially stable system which is subject to more or less regular oscillations about a moving equilibrium. The turning points of these cyclical oscillations were then frequently found to be no more difficult to explain than the cumulative processes of expansion and contraction. In other words, the entire cyclical process was recognized as a process of adjustment to irregular disturbances of the system≫.[2)]

Among the few opponents to the view that business cycles are in principle adequately depicted by analytical theory systems borrowed from classical mechanics we *first* mention Schumpeter. ≫The business cycles with which we are concerned are really not at all what one thinks of when using the terms Wave and Fluctuation. They are the result of a process which, indeed, produces upward and downward movements in our graphs, but these movements are not analogous to the oscillation of an elastic string or membrane — which, once set into motion, would, but for friction, go on indefinitely — because they are due to the intermittent action of the 'force' of innovation, by which the action of the equilibrium 'force' is each time brought into play. But there are other economic fluctuations which answer more nearly to the physical analogy.≫[3)] In what follows Schumpeter continues this line of thought, e.g. by explicitly refuting Tinbergen's attitude that ≫elastic waves [constitute] the only type of 'endogenous' waves and the main object of exact business-cycle analysis≫[4)] and by criticizing two well-known ≫macrodynamic≫ systems by Frisch and Kalečki, as far as these are intended to be analytic theories explaining historical business cycles.[5)] This position of Schumpeter is no doubt often interpreted as implying a misunderstanding on his part of modern business cycle theory, and his own contribution to the study of business cycles is sometimes claimed to be quite reconcilable with the differential and difference equations approach.[6)]

[2)] Lloyd A. Metzler, "Business Cycles and the Modern Theory of Employment," *The American Economic Review* 1946, pp. 279–282. Reproduced by permission of the American Economic Association.

[3)] Reproduced by permission from *Business Cycles,* Vol. I, p. 175, by J. A. Schumpeter. Copyright 1939 McGraw-Hill Book Company, Inc.

[4)] Joseph A. Schumpeter, *Ibid.*, I, p. 179.

[5)] Cf. Wassily Leontief, "Joseph A. Schumpeter (1883–1950, *Econometrica* 1950, p. 106.

[6)] E.g. by R. M. Goodwin.

It is also evident that Schumpeter does not present an analytical theory alternative to the approach rejected by him, but from this it does not follow that his heuristic reasoning based on extensive and careful observations is consistent with the ≫modern≫ line of research.

Second, we mention Johan Åkerman as sharing Schumpeter's critical view. It is significant that, e.g. in discussing Lundberg's ≫model sequences≫ — which in principle are identical to the equation systems employed by Tinbergen and others and which produce the ≫elastic waves≫ indicated by Schumpeter —, Åkerman puts the problem as to whether these are intended to explain a course of real events in a multi-unit-of-decision society or a plan of a central economic agency; he asks: ≫who is planing?≫ in the models, and seems to tend to the opinion that the constructions are most adequately interpreted as plans for a whole society, stressing that ≫the business cycle *is no result of a plan;* its cumulative rise and fall and its turning points are related to the not uniformly regulated activities of the various social groups, and to exogenous impulse phenomena, which cannot be considered as hypothetical changes, acting on the whole mechanism.≫[7] Like Schumpeter, Johan Åkerman, besides criticizing the ordinary approach, develops a heuristic business cycle theory based on extensive and careful observations.

Third, we mention the extremely critical attitude of von Neumann-Morgenstern towards the over-optimistic and careless utilization of physical models in economics, as well in business cycle research as in other fields. They state, e.g. that ≫it is of utmost importance to know how to stabilize employment≫ but ≫we need not concern ourselves with the pretension that there can be scientific answers at present≫.[8]

Fourth, we quote a brief remark by Stigler: ≫Most discussions of economic dynamics, I feel, would be better entitled 'What I Know about Differential and Difference Equations' ... The economics ... is appallingly absent≫.[9]

We shall here attempt to show that the differential and difference equation systems of ≫modern≫ business cycle theories fit quite well into the pure production aspect and should be interpreted as essentially depicting

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[7] Johan Åkerman [14], II, p. 212.

[8] von Neumann and Morgenstern [1], p. 6.

[9] George J. Stigler, "Review of Paul A. Samuelson, *Foundations of Economic Analysis,*" *Journal of the American Statistical Association* 1948, pp. 604–605. Cf. George J. Stigler, "A Survey of Contemporary Economics," *The Journal of Political Economy* 1949, pp. 99–100, and Marschak [27], p. 74.

the planned course of events in a single-unit-of-decision economy; later, this position will be supplemented by attempts to make plausible that the basic observed traits of business cycles in non-centralized economies ought to be explained from the pure distribution aspect. Our discussion in the present section draws much on the reasoning of Chapters I–IV; we start by recollecting some basic points. In our opinion there is a clear line of analytical development of theories:

Keynes ⟶ Leontief ⟶ Dantzig

Dantzig's programming analyzes proper being the wide system, including as special cases the two other constructions, of which the Keynesian one is essentially the most restrictive. The general existence of this line of development is fairly often recognized[10]; we turn here to some propositions about the nature of the relations between the various systems, which so far as we know, are more specific to this thesis. The most crucial issue is: What implications have the restrictive linearity assumptions of the Leontiefian system, stating that the various inputs of each activity are proportional to the output of the respective activity (≫complete complementarity≫, allowing only one fixed method of production for each commodity)? Our answer suggests that a Leontiefian system with as many activities as commodities should be interpreted as the actual choice made from a programming system with more activities than commodities, as the ≫real part≫ of such a system, the ≫virtual part≫ comprising those activities or methods of production among the available ones which were not chosen due to the character of the objectives stated, and the expression for these very objectives. According to this interpretation a system of the Leontiefian type depicts an adopted economic plan, i.e. the planned course of events within a centralized economy during a certain period of time, barring perhaps some disturbances. This much about the relation between Leontief's system and the programming analyzes, proper.

The Keynesian special case distinguishes itself from the Leontiefian case in considering less variables, i.e. less flows, comprising in its simplest form only one commodity — ≫national income≫ —, and two activities — consumption and investment. In some presentations there is the further difference that monetary flows are included in systems of Keynes' type, as contrasted to the real flows of Leontief's system, a procedure which leads into the intricate problems of ≫money illusion≫, etc., but there seems to be a strong tendency in recent literature to prefer deflating the

[10] Concerning the first link in the chain, compare the reference given in footnote 1, p. 19 above; concerning the second link compare the reference in footnote 3, p. 19 above.

aggregative monetary flows into ≫real≫ flows.[11] Let us now superimpose the distinction

statics ⟷ dynamics

on the scheme of development of theories, corresponding to the first type of statics and dynamics mentioned by us on p. 40 above. The ≫modern≫ business cycle theories alluded to by Metzler and utilized by Samuelson, Tinbergen, Klein, etc., include as a special case the static Keynesian ≫theory of employment≫, adding to this more or less comprehensive dynamic elements, mainly in the form of time-lags and almost always comprising some form of an ≫acceleration principle≫: some variable is proportional to a first time-difference (or time-derivative). The dynamic elements being properly introduced, the resulting system of difference or differential equations gives rise to oscillations; i.e., when solved from the system the time-paths of the various variables show oscillatory movements. These movements are identified with observed business cycles, barring sometimes considerations about shocks acting on the system of equations, etc. Our criticism against this identification reads: we regard the core of the ≫modern≫ business cycle theories, the Keynesian system, as a rudimentary planning system, which when somewhat developed from its extreme special-case status, requires the explicit formulation of one set of objectives of the economy; its capacity to explain business cycles in a multi-unit-of-decision economy must be the subject of considerable doubt, particularly when Arrow's theorem is taken into account, proving the impossibility of formulating a consistent set of objectives of an economy on the basis of the value schemes of many individuals.

The motivation for our standpoint lies in the fact that the same relation between the three theory types exists both in the dynamic case as well as in the static case. The programming analyzes proper form the wider set, containing the dynamic versions of Leontief's and Keynes' systems as special cases, the specialization here taking place along three lines: the number of the variables is reduced, the linearity assumptions appear as severe restrictions — when the virtual part of the systems is left out —, and — this is the new trait added — the possibilities for systematically introducing dynamic elements are somewhat restricted. The Leontiefian system is static in the sense that it only considers constant flows of all commodities over time, or as we prefer to say, during the whole planning period; i.e. the time-paths of the variables are straight, horizontal lines, as are the time-paths of a static Keynesian system. A Leontiefian system may be dynamized in the same way as a Keynesian one. We have discussed at some length Hawkins' dynamization of Leontief's system through the

[11] Cf. e.g. the works by Klein [5] and [13].

introduction of a kind of ≫acceleration principle≪, taking into account the creation of new real capital and implying non-steady-rate flows of the commodities during the planning period; the time-paths of the variables included are consequently not horizontal lines. Hawkins' system, including as many activities as commodities, is interpreted in the same manner as Leontief's system: it represents an adopted economic plan, that is the planned course of events in a single-unit-of-decision economy, the plan being of a somewhat greater complexity than a static Leontief plan in considering also some aspects of capital formation.

The general solution of Hawkins' system, that is the time-paths of the variables,[12)]

$$X_i = \sum_{j=1}^{n} C_j D_{ij} e^{\alpha_j t} \qquad (i = 1, \ldots, n) \tag{1}$$

presents the feature that only one particular solution of the system, i.e. only one term in the right-hand sums of (1), has a meaning if taken by itself. This is the unique steady-state solution discussed in Chapter II, the α of which represents the maximal rate of growth of the economy. The other particular solutions are transient states, superimposed on the steady state. The combined solution shows oscillations around the steady-state level. If the system be stable, the oscillations tend to disappear as time goes on, i.e. the transient states tend to vanish. If the system be unstable, the oscillations grow more vigorous with time and the system breaks down ultimately in some kind of a crisis of disproportionate development. The similarity from the point of principle with the ordinary and simpler dynamic economic setups is obvious, as are the limitations of the system in depicting short-run plans. Hawkins discusses stability criteria for the construction presented. We are not interested in any particular details of this discussion, but mention that it obviously must be a concern of the central planning unit to guarantee the stability of the planned course of events and to try smoothing out the fluctuations as far as possible. Hawkins observes that ≫economic devices analogous to electrical or mechanical servo-mechanisms, which would *counteract* the instabilities discussed in this paper, could do so only by temporarily *creating* unproductive storage.≪[13)] This points towards a connection between the field of economics discussed and the theory of servo-mechanisms, an outstanding fundamental treatment of which is Wiener's *Cybernetics, or Control and Communication in the*

[12)] Cf. p. 23 above.

[13)] Hawkins [21], p. 321.

Animal and the Machine (1948).[14] If more activities than commodities be introduced and thus the restrictive linearity assumptions abandoned, Hawkins' system develops into the programming analyzes proper, which can comprise in certain respects much more general dynamic features; besides, they must be supplemented by an Objective Function.

We think that the pure production aspect, and the study of a centralized economy, might profit from a synthesis between the following three elements: the technical relations (and balance identities) of the programming analyzes, which extend further the dynamics of Hawkins' system; the theory of servo-mechanisms, which extends further the stability discussion of Hawkins' paper; and the Objective Function of the programming analyzes, which becomes a necessary explicit link of the analysis as soon as this is pushed beyond the most restrictive features, and which gives to the whole type of analysis its peculiar character.

Our attitude towards the ordinary type of analytical business cycle theories may be summarized as follows. When extended beyond the most rudimentary consideration of one or two ≫commodities≫ and ≫activities≫ and of strictly linear relations the theories in question require logically to be supplemented by an Objective Function, expressing in some way a scale of preferences. The rudimentary forms of the theories being very mechanical in the sense that they are doubtfully derivable from a consistent principle of economic and social action, the necessary supplement gives to the constructions their justification as an instrument of decision for the head of a centralized economy, and opens at the same time grave doubts concerning their capacity of explaining the business cycles of non-centralized economies.

Examining the two categories of time-series observations listed in the first section of this chapter we find, of course, that the theories discussed here can take into account more or less periodic movements; they can even comprise phenomena of investment, that is capital formation, but the character of this investment seems to be quite alien to that of the observed kind — there are no signs that the investments in question are innovations made by many different decision units, i.e. are the results of many economic plans; rather, they form part of one unified plan. A society exhaustively covered by the pure production aspect — a communistic state where no distribution struggle exists — builds on a basic political premise: the complete unity of interests, the assertion of only one economic will. This political fact is expressed in the programming analyzes by the necessary existence of one Objective Function, one value scheme of society, and it implies the actual fulfillment of only one economic plan of society. It would

[14] Wiener [48].

be interesting to explore the possibility of there being joint appearance in an observed centralized economy of capital formation and of certain oscillatory phenomena to be counteracted and damped as far as possible; we must leave the matter with this statement.

3) Economic dynamics II: market adjustments

The previous section dealt with business cycles in relation to the pure production aspect. Following the adopted line of disposition we attempt in this section to give a brief outline of business cycles in relation to the rudimentary Walrasian synthesis between a production aspect and a distribution aspect.

When discussing ≫oscillations≫ Schumpeter mentions market adjustments: ≫The simplest case of Waves of Adaptation or Oscillations may be illustrated by any individual price which happens to be out of equilibrium. Even if no further disturbance occurs, we do not observe that it at once assumes its equilibrium value or that it makes straight for that value and stops there. As a rule, it will miss it, or outrun it, and turn back again. Equilibrium has to be found, as Walras put it, *par tâtonnement.*≫[1)] This kind of oscillation belongs to the second type of dynamics listed by us on p. 40 above: statics denotes a state where the market process has achieved equality between supply and demand; dynamics implies a state where this equilibrium has not (yet) been reached. The general Walrasian problem of dynamics, i.e., the problem of market adjustments when all markets are coupled: ≫the price of any good will fall if its supply exceeds its demand, *these each being regarded as functions of all other prices*≫[2)], has been clearly stated by Samuelson, as remarked on p. 39 above. That which interests us in the special context involved here is the fact that certain authors identify observed business cycle and employment fluctuations with market oscillations shown by the dynamic Walrasian equilibrium conditions. To give an example we refer to the article by Patinkin, "Involuntary Unemployment and the Keynesian Supply Function" (1949)[3)] quoted on p. 118 above. In his opinion the major scientific innovation of Keynes was the argument that the Walrasian market adjustments in modern societies are apt to work in a most unsatisfactory manner, giving rise to unemployment during the long and devastating processes of adaptation.

[1)] Reproduced by permission from *Business Cycles,* Vol. I, p. 183, by J. A. Schumpeter. Copyright 1939 McGraw-Hill Book Company, Inc.

[2)] Samuelson [16], p. 270.

[3)] Patinkin [31].

Of the two categories of time-series facts listed in the introductory section of this chapter, the market constructions can only take into account the least specific one, the more or less periodic movements of the variables studied; any process of investment and new capital creation is hardly touched upon by these theories depicting a society with many quasi-isolated decision units, a society characterized by an extreme compartmentalization.

4) Business cycles and changes in the investment structure

a) NEW INVESTMENTS AND CHANGES IN THE DISTRIBUTION OF INCOMES AND PROFITS

After having indicated the doubtful capacity of the theories within the pure production aspect and within the Walrasian synthesis between a production aspect and a distribution aspect to comprise, besides the fact of oscillations, also the fact of cumulative processes of investment not resulting from one plan of society, we now somewhat revert the line of argument and inquire for some desiderata to be fulfilled by a theory-construction, really capable of embodying the investment facts and at the same time showing some kind of periodic movements.

The investment process in a non-centralized economy seems to be characterized by an outstanding but seldom explicitly recognized feature which we describe thus: ≫in a non-centralized economy it is in general impossible to make a *new specific investment* without inflicting a *potential* gain or loss on other specific investments≪. In order to substantiate this view of ours and to discuss certain phenomena related to the feature just stated we firstly, as an introduction, study a strong statement by Wiener on the investment-potentialities he believes the world will soon have to face; secondly, we turn to two unorthodox investigations of business cycle investments by Domar and Dahmén. We especially classify the last two contributions in line with observations, as they are accounts distilled from a scrutiny of facts; we wish to discuss these somewhat indirect observations.

In his book on ≫Cybernetics,≪ Wiener makes the following vivid prophecy and tendentious statement: ≫Long before Nagasaki and the public awareness of the atomic bomb, it had occurred to me that we were here in the presence of another social potentiality of unheard-of-importance for good and evil. The automatic factory, the assembly-line without human agents, are only so far ahead of us as is limited by our willingness to put such a degree of effort into their engineering as was spent, for example, in the development of the technique of radar in the second world war.

I have said that this new development has unbounded possibilities for good and evil. For one thing, it makes the metaphorical dominance

of the machines, as imagined by Samuel Butler, a most immediate and non-metaphorical problem. It gives the human race a new and most effective collection of mechanical slaves to perform its labor. Such mechanical labor has most of the properties of slave labor, although, unlike slave labor, it does not involve the direct demoralizing effects of human cruelty. However, any labor that accepts the conditions of competition with slave labor, accepts the conditions of slave labor, and is essentially slave labor. The key word of this statement is *competition.* It may very well be a good thing for humanity to have the machine remove from it the need of menial and disagreeable tasks; or it may not. I do not know. It cannot be good for these new potentialities to be assessed in the terms of the market, of the money they save; and it is precisely the terms of the open market, the 'fifth freedom,' that have become the shibboleth of the sector of American opinion represented by the National Association of Manufacturers and the Saturday Evening Post. I say American opinion, for as an American, I know it best, but the hucksters recognize no national boundary.

Perhaps I may clarify the historical background of the present situation if I say that the first industrial revolution, the revolution of the 'dark satanic mills,' was the devaluation of the human arm by the competition of machinery. There is no rate of pay at which a United States pick-and-shovel laborer can live which is low enough to compete with the work of a steam shovel as an excavator. The modern industrial revolution is similarly bound to devalue the human brain at least in its simpler and more routine decisions. Of course, just as the skilled carpenter, the skilled mechanic, the skilled dressmaker, so the skilled scientist and the skilled administrator may survive the second. However, taking the second revolution as accomplished, the average human being of mediocre attainments or less has nothing to sell that it is worth anyone's money to buy.

The answer, of course, is to have a society based on human values other than buying or selling. To arrive at this society we need a good deal of planning and good deal of struggle — which, if the best comes to the best, may be on the plane of ideas — who knows? I thus felt it my duty to pass on my information and understanding of the position to those who have an active interest in the conditions and the future of labor — that is to the labor unions.

I did manage to make contact with one or two persons high up in the C.I.O., and from them I received a very intelligent and sympathetic hearing. Further than these individuals neither I nor any of them was able to go. It was their opinion, as it had been my previous observation and information, both in the United States and in England, that the labor unions and the labor movements are in the hands of a highly limited personnel, thoroughly well trained in the specialized problems of shop-stewardship and disputes

concerning wages and conditions of work; and totally unprepared to enter into the larger political, technical, sociological, and economic questions which concern the very existence of labor.≫[1)]

Obviously, what worries Wiener are the distribution consequences of the anticipated investments. He thinks that the possibilities in the distribution process of the owners of the new automata-investments, and perhaps also of some groups of highly qualified mental labor, will be widely strengthened relative to the possibilities of moderately skilled labor. Thus he fears that this imagined creation of specific investments will change the objective possibilities in the distribution process for very large groups of decision units. Seeing the future possibilities of the laborers diminished to a considerable degree, Wiener turns to the labor union leaders. He finds these very efficient in handling the interest of labor in the short-run distribution struggle, in the negotiations with the employers and in the political disputes, but he encounters no interest of what may happen to labor if the physical background is radically changed in the indicated direction through new specific investments, i.e. changed to such a degree that no effort on the part of labor unions could achieve for labor an adequate share of the national income. The way out Wiener surmises to go over political reversions, but hardly over such of an ordinary type. The modification imagined is so profound as to result closely in a communistic Utopia, where buying and selling values are absent, a complete unity of interests prevails, and no distribution struggle exists.

Prophecies like that of Wiener about the relation between labor and capital — big economic groups commonly referred to also in the political context — have been made earlier in modern history. They have not materialized; the investments made in machinery have been balanced by investments in men, that is in increasing and redirecting the training and education of labor, and possibly also by political reversions favoring labor. But no doubt the relations between various branches of industry have frequently been changed through the creation of new real assets, from the relation between agriculture and manufacturing — two groups that are also often encountered in the purely political sphere — to the relations between smaller branches of manufacturing and between individual enterprises — groups whose action on the political arena is generally inconsiderable.

Thus Wiener indicates the outlines of a process of the following type: new specific investments are made, which imply that the data change, i.e., the objective possibilities of certain groups of decision units in the distribution process are substantially altered; an adjustment of the price

[1)] Wiener [48], pp. 36–38. Reproduced by permission of The Technology Press.

system must take place to fit the new background. The nature of this adjustment process is not specified but it is suggested that the process be not completely automatic as labor union action might counteract the development, if not prevent it.

We continue our consideration of new specific investments as causing changes in the earning possibilities of other real assets by regarding smaller groups than those mentioned by Wiener, and examine an investigation by Domar on *Investment, Losses, and Monopolies*.[2)] Domar observes: ≫For in a private economy it is impossible to make a single investment without inflicting a potential loss on oneself or on someone else!≫ and continues: ≫We may imagine representatives of our big business, perhaps together with those from big labor, sitting around a table. All prospective investments are placed on a platter and passed around. The representative of General Motors might, for example, find all of them splendid, all with the exception of Kaiser-Fraser's new plan for expansion. He takes this one project off and passes the platter on. Pennsylvania Railroad has no objection to Kaiser-Frazer and to all other such projects; but it might decide to veto the St. Lawrence waterway. Its views are seconded by United Mine Workers whose behavior is indistinguishable from that of their business associates at the table. The platter moves on. Real estate interests think that it was a great pity that Kaiser-Frazer and the St. Lawrence were taken off the platter; those were fine projects to achieve prosperity and to raise the American standard of living. The construction of a plant for prefabricated houses, however, is a different matter. That will destroy existing real estate values. And so it goes. By the time the platter completes its trip around the table it is perfectly empty; all projects have been vetoed by one or the other participant. And there goes our prosperity.

If the participants were small and weak, as they are supposed to be in an atomistic society, their vetoes would be just empty talk. On the other hand, if the whole economy were ruled by one closely connected financial group, the latter would not treat the level of income as independent of its own actions, and would find it profitable to invest and expand in many different directions. The gains to the economy as a whole would of course outweigh the losses. Similarly, if the participants at the table could get together and talk things over, each of them would realize his dependence on all the rest, and might agree not to veto the projects, since an empty platter means ruin for everyone. It would even pay the rest to compensate a particularly injured member for his losses in order to remove his opposition. But we may be caught in a situation where the participants are frequently not weak enough to be harmless, and yet not large enough or not sufficiently

[2)] Domar [49].

well organized to perceive the effects of their own action on total income. We should not forget that to lose a few thousands when everyone else is supposedly making money is to lose face and standing. But to lose millions when everyone else is losing millions is perfectly in accordance with sound business practice. No single individual or firm, however large, can be blamed for a depression.≫[3)]

The author concludes: ≫We must recognize the important, though uncomfortable, fact that economic progress in our society is a cruel and destructive process, which on one side can result, and indeed has resulted in a remarkably high and ever-rising standard of living, and on the other — in the destruction of firms and even whole industries. But attempts to tame it so as to achieve safety and security for some, without at the same time introducing some measure of economic planning, may well result in chronic unemployment and insecurity for all.≫[4)]

In illustrating the distribution consequences of new investments, Domar puts the main emphasis on the negative side of the phenomenon: new specific investments as causing losses to certain previously existing real assets; but in his examples is also indicated the positive side of the phenomenon: new specific investments as causing gains to certain other previously existing real assets. The author does not dwell closer, however, on the character of the process of adjustment whereby the distribution of income between different investments is made to fit the changed objective possibilities, implied by the creation of the new specific investments. He further mentions that qualitatively different types of phenomena may arise if society be united in the sense of being ruled by a closely connected group.

The investigations by Dahmén extend logically the considerations made by Wiener and Domar, and build on analyzes by Johan Åkerman and Schumpeter. The investigations are presented in a methodological article of 1942: "Economic Structural Analysis"[5)] and in a treatise of 1950, applying the methodological apparatus to an extensive empirical material: "Entrepreneurial Activity in Swedish Industry in the Period 1919–1939"[6)]; for our purpose it will be sufficient mainly to refer to the first-mentioned piece of research.

Dahmén regards the economic development or ≫transformation process≫ as having two aspects, one positive and one negative. The positive aspect consists in the starting of new lines of production, the negative

[3)] Domar [49], pp. 52–53.
[4)] Domar [49], p. 53.
[5)] Dahmén [50].
[6)] Dahmén [51].

aspect in the cessation or deterioration of old lines of production. We term these two aspects ≫innovation≫ and ≫liquidation≫ respectively. In our opinion the most important concept of Dahmén, built on these general outlines, is that of Block of Development. Let us start from a given point of time with a certain distribution of profits between various existing investments, more particularly from the moment of initial revival after a depression. What factors cause the revival?

It is well known how traditional analysis answers (among other things) this question by resorting to multiplier and accelerator reasoning. Dahmén rejects this type of analysis explicitly on the ground that it is too aggregative, that it deals only with global quantities — such as total consumption, total investment — and not with specific variables, tied to particular subjects, individuals, firms or social groups. Although essentially sharing Dahmén's critical attitude towards aggregative analysis of the kind indicated — compare our remarks on the aggregation problem[7] — we do not think that it is sufficient for an ≫unorthodox≫ approach to criticize traditional analysis for its aggregative character. As indicated at various places above it is perfectly possible to express multiplier, accelerator and kindred reasoning in terms of the non-aggregative Leontiefian and programming systems, including variables tied to particular commodities and productive activities. Our criticism against the application of traditional dynamic theories to observed business cycle phenomena is based on the proposition that these theories essentially depict the course of an economic plan for a unified economy; the statement that this plan is expressed in inarticulate or aggregative terms can from our point of view be only a qualification of the basic criticism.

In trying to explain the revival Dahmén, despite the emphasis of his criticism, does not utilize a non-aggregative form of traditional analysis, belonging more or less rudimentarily to the pure production aspect; he speaks in terms of the changing profitability of various real assets. The initial expansion is ascribed to the realization of the fact that only the carrying out of certain new specific investments may raise the profitability of some of the existing enterprises. Through the change of structure implied by the creation of new specific investments the possibility opens up for a whole group of investments together to earn considerably more than the sum of what could accrue to each, if isolated from the rest, or perhaps even if isolated from only one member of the group. Such a set of complementary investments Dahmén labels a Block of Development. The initial business cycle expansion and more widely the positive side of the

[7] pp. 56–63 above.

economic process of development or transformation is due, in this explanation, to the existence of Incompleted Blocks of Development, which means that it is in general possible to increase the profitability of *some* existing investments by changing the structure and creating certain new real assets. ≫One might say that there is a deficient structural balance, or vacua in the (production) structure during a depression, for certain enterprises or branches show unsatisfactory profitability because of the fact that certain other specific new investments have not been made. This deficiency of balance in the production structure amounts to a force of expansion: a structural vacuum will be filled out if certain *specific* new investments are made, and thus conditions created for removing the unprofitability [of some old investments]≫.[8] Further: ≫It is possible to differentiate in many ways between various kinds of structural vacua. It is important to realize, however, that we deal with a repetitive phenomenon: in principle the same type of vacua appears under various disguises; the issue consists in all cases of a deficiency of balance in the production structure.≫[9] As simple examples of Blocks of Development Dahmén mentions that ≫the operation of a new railroad often will not be profitable until industries have grown up in the surrounding territory, and a railroad may not be satisfactorily profitable until certain other railroads have been completed.≫[10] In connection with the positive side of the transformation process and its innovations, Dahmén further stresses that the existence of the particular type of complementarity — ≫complement-investments≫ — embodied in the Blocks of Development creates a strong tendency towards trustification, ≫big capitalism≫, coalitions among enterprises, etc. ≫The interest spheres of some entrepreneurs and capitalists are widened, when their enterprises show relative unprofitability due to structural vacua.≫[11]

This much about the positive side of the economic transformation process with its peculiar kind of complementarity between investments. The negative side of the process consists in the partial or total liquidation — i.e. the more or less complete writing down of capital values of existing investments, which turn unprofitable: Dahmén, following Schumpeter, speaks essentially of the liquidation of *old* investments, old firms, etc. Thus a kind of substitutability enters the picture: profits of new investments are partly substituted for profits of some of the old investments.

In our opinion Dahmén gives a remarkable contribution to our understanding of the phenomenon of economic progress or expansion in a

[8] Dahmén [50], p. 182.
[9] Dahmén [50], p. 182, footnote 2.
[10] Dahmén [51], p. 425.
[11] Dahmén [50], p. 183.

capitalist economy, an expansion which is not due to the carrying out of a plan of society in fulfillment of certain objectives, stated in terms of desired commodities and services, but an expansion which is due to the fulfillment of a pure profit motive on the part of individual entrepreneurs or perhaps groups of entrepreneurs and intimately connected with the Block of Development kind of complementarity. It may, however, be asked: can this analysis of the economic transformation process give some clue towards the understanding of business cycles with crises and depressions? Is it not conceivable that the expansion implying completion of various Blocks of Developments and the retreat of old investments take place in a moderately smooth way without such heavy interruptions as crises and depressions? How can the step be taken from the study of individual enterprises or branches of enterprises and their mutual relations to the consideration of business cycle fluctuations permeating the entire national economy, and particularly of breakdowns occurring simultaneously in all or almost all branches of industry and showing themselves in severe dips of statistically measured global quantities such as real national income, total real investment, total real consumption? In the discussion following Dahmén's second-mentioned work related questions were put, for instance, by Erik Lundberg and Johan Åkerman. The former asked for a synthesis between Dahmén's transformation analysis and traditional business cycle analysis dealing with global quantities; the latter posed the problem about the relation between Dahmén's ≫micro-structural≫ analysis of specific firms and branches and their mutual relations and a ≫macro-cyclical≫ analysis of the changes of national income, total investment, total savings, etc.

It should be clear from our previous discussion that we do not believe in the possibility of a straightforward synthesis between traditional business cycle analysis belonging to the production aspect and an analysis centered around changes in the money-profitability of specific investments. Nevertheless, we find a serious problem to lie in the relation between Dahmén's picture of the transformation process with innovations in expanding branches and Blocks of Development and liquidations in declining branches, and the explanation of total crises and depressions. In his second-mentioned work Dahmén takes up the position that his analysis, in conformity with that of Schumpeter, is essentially applicable to ≫the economic transformation process≫, and not so much to the phenomenon of business cycle fluctuations. In his earlier article he regards business cycle fluctuations as due to ≫a lack of synchronism between the positive and the negative side of the economic development.≫[12)] This idea he elaborates by saying that, despite the rather intermittent appearance of liquidations

[12)] Dahmén [50], p. 189.

during fairly limited periods of crises and ensuing depressions, unprofitability presents itself much more continuously. Due to certain ≫factors of inertia≫ the explicit recognition of the unpleasant fact, however, and the taking of its consequences by various economic subjects, tends to cluster in limited periods. Among ≫factors of inertia≫ he lists mere unwillingness to depreciate capital values; investments which may temporarily retard an impending decline: ≫throwing good money after bad≫; all sorts of trusts and cartels; social and political factors; and so forth. In some cases the factors of inertia are said to take on an absolute character in the sense that they may completely prevent the unprofitability which otherwise should have been caused by the change of the objective possibilities implied by the appearance of new investments and their substitution effect: an example of this is the agricultural policy followed in Sweden and elsewhere after 1932, maintaining or raising the relative income share of agriculture despite the technico-economic development, unfavorable to agriculture.

We stressed above that Dahmén mainly regards liquidation as occurring to old investments. In discussing the business cycle problem partly from the same viewpoint as Dahmén, Johan Åkerman puts emphasis on the liquidation of new investments: ≫The real capital formed during the upswing is to a greater or smaller degree diminished during the downswing through the liquidation of 'malinvestments,' which have not been able to settle themselves in the production structure and the monetary structure.≫[13] Dahmén touches the same subject in holding, as it seems, the view that the existence of Incompleted Blocks of Development may, when a crisis has occurred, severely aggravate the depression in rendering fatal unprofitabilities, which would have the chance of disappearing when the Blocks were completed — a potentiality that the pessimism of a crisis may obliterate.

Let us now summarize what is to be found about the nature of the investment acts in the researches discussed: we distinguish four points.

1) The new investments have consequences for the distribution of income in the society, sometimes very pronounced consequences. This is the basic theme of all the investigations; moreover, the new investments are regarded almost solely in their rôle of upsetting the previously existing income or profit distribution, between big social groups in the case of Wiener, and between smaller groups within the entrepreneurs in the two other cases — if not essentially between individual enterprises, then between various branches of industry. However, particularly Wiener, and to some extent also Domar indicate that new investments may be viewed from a different angle if society be united: in the first case, if it employs a common value scheme and is not based on monetary values in the second and less pronounced

[13] Johan Åkerman [14], II, p. 43.

case, if it be ruled by a closely connected group. Thus we find even here rudiments of the fundamental distinction between the production aspect and the distribution aspect.

2) The consequences of the new investments are explicitly differentiated into positive ones and negative ones by Domar and Dahmén. The positive consequences, consist in the potentiality of new investments to raise the profits from some previously existing real assets; the negative consequences are embodied in the potentially of new investments to decrease the profits from certain older investments.

3) Both types of consequences are further studied by Dahmén, who particularly elaborates the positive side. The only way to increase the profitability of some investments may be to wait for certain other investments to be made, all investments concerned jointly forming a Block of Development. A main incentive to making new investments and hence to expansion in a free group society is the realization of the profit potentialities that appear if the objective possibilities are changed through new investments which complete Blocks of Developments. Dahmén also elaborates the negative side: the profits of new investments are often partly substituted for those of older ones, which must be liquidated; in Johan Åkerman's opinion, however, new investments also frequently fail to achieve a satisfactory profitability and must be written down fairly soon.

4) Only Dahmén dwells at any length on the nature of the process of adjustment which forces the income and profit distribution to adapt itself to the change of background, implied by the creation of new investments. This process of adjustment displays itself in the observed features of crises and depressions. In discussing the topic Dahmén explicitly notes one fact of discontinuity: the losses and liquidations are concentrated to discrete periods of time and not evenly spread over the cycle; more implicitly there seems to be another fact of discontinuity in his exposition: the potential gains resulting from innovations should not have any possibility of materializing until the relevant Blocks of Development are to a considerable degree completed. Despite these two facts of discontinuity, Dahmén reasons as though the economic transformation process were in both its positive and its negative aspect essentially continuous, certain ≫factors of inertia≫ accounting for the marked clustering of liquidations, notwithstanding the asserted continuous appearance of unprofitability.

We give a more condensed summary of these observations: New specific investments cause changes in the existing distribution of incomes and profits in the society; the process whereby the income distribution adapts itself to the changed conditions is of a highly troublesome character, consisting of a crisis and an ensuing depression; to the process is tied

a certain kind of complementarity between different investments embodied in the concept of Block of Development. From a somewhat different viewpoint the observations may be expressed in still another way: New specific investments are made by certain units of the society, be it separate enterprises or branches or groups of enterprises; these investments, tied to particular units, cause consequences for the whole society in starting an adjustment process, displaying itself in a crisis and an ensuing depression with a marked lowering of the economic activity of all or most sectors of the society, with nonproportionate but overall price and income declines, i.e. with a breakdown of the pricing system, and so forth; some sort of a connecting link between the initiating (micro)changes, tied to particular units, i.e. the new investments, and the resulting (macro)consequences, tied to the whole society, may possibly be constituted by the complementarity relation between different specific investments indicated by the concept of Block of Development.

In introducing the terms micro and macro we wish to stress the similarity between our discussion and summary of the nature of the investment facts and the exposition given in Johan Åkerman's ≫Economic Theory≫ (1944)[14)] of the business cycle phenomenon utilizing these terms. They should not be confused with the equally named concepts which are sometimes connected with the problem of aggregation within a Walrasian universe.

The observations discussed and summarized by us and made by Johan Åkerman and Schumpeter, by Dahmén and Domar, and by Wiener, give in our opinion very important desiderata, which should be fulfilled by a theory capable of explaining business cycles in a free group-society. We now turn to the direct inquiry for such a theory.

b) BUSINESS CYCLES AS ADJUSTMENTS OF IMPUTATIONS TO CHANGES OF CHARACTERISTIC FUNCTIONS

α) *Variations between Strategically Equivalent Characteristic Functions*

A Characteristic Function describes the physical background of a society or the objective earning possibilities of the various decision units in the distribution process — i.e. how big a share each decision unit acting alone and every conceivable set of decision units acting in cooperation can obtain out of the national income. A new investment changes the structure of the real capital of a society and it gives to its owner new

[14)] Johan Åkerman [14], II, pp. 38–52 and [15], pp. 505–511; cf. p. 71, above.

possibilities of earning money. It would seem quite natural to view new investments as causing changes in the objective earning possibilities, i.e. the Characteristic Function of a society. We adopt this proposition; it can never be proved in the same way as a mathematical theorem, but it can be made plausible through the establishment of marked similarities between theoretical consequences, following from variations of Characteristic Functions, and empirical phenomena, correlated with new investments. In other words: theoretical implications, tied to variations between different Characteristic Functions, have here to be compared with the observations on investment facts and business cycles, discussed and summarized above.

There are two qualitatively very different types of variations between Characteristic Functions. In discussing the first type of variations we shall fail in establishing any similarities between theoretical implications of the variations and empirical observations tied to new investments: this is the case of variations between Strategically Equivalent Games.[15)] We start by considering Inessential Characteristic Functions, which as we know, all belong to one and the same class of Strategically Equivalent Games, thus being only able to comprise changes of the insignificant kind involved.

In these setups certainly nothing contradicts the circumstance that new investments have consequences for the distribution of incomes and profits in the society. A change in an or some of the $v((i))$ — the constituent parts of an Inessential Characteristic Function — is reflected in changes of the Imputation $\overrightarrow{\alpha}$ prevailing, as $a_i = v((i)), a_j = v((j))$, etc. Thus a new investment may tentatively be viewed as causing an increase in $v((j))$ with a resulting higher relative income-share, i.e. a resulting higher relative money-income, accruing to decision unit no. j. Before the new investments are carried out, a certain Characteristic Function prevails: $v((i))(i = 1, \ldots, n)$, with the corresponding imputation $\overrightarrow{\alpha}$; when the new investments are completed, a new Characteristic Function comes into existence: $v((i)(i = 1, \ldots, n)$, with the corresponding Imputation $\overrightarrow{\beta}$; What is then the nature of the process of adjustments of Imputations — $\overrightarrow{\alpha} \to \overrightarrow{\beta}$ — to changes of Inessential Characteristic Functions?

There are no signs that such a process could incur any considerable trouble. It leads from one set of values $v((i))$ with the corresponding *unique* Imputation $\overrightarrow{\alpha}$, to another set of values $\overline{v}((i))$, on the background of which the new Imputation $\overrightarrow{\beta}$ is to be established. No agreement of any kind whatsoever is needed to establish this new Imputation, when the data have once changed; no set of many alternative new Imputations is available to the society: the new Imputation is uniquely implied by the changed data and is no result of a struggle between various group-interests as no other

[15)] Cf. pp. 55–56.

and better alternative can be achieved by *any* unit, if all units treat their own affairs in an intelligent manner — the adjustment is apt to take place in a smooth way, automatically and harmoniously. Thus the theoretical consequences of a variation between Inessential Characteristic Functions do not mirror the pronounced empirical phenomenon, sometimes correlated with a process of new investments: a crisis and the ensuing depression.

Also the other empirical phenomenon listed, which is sometimes correlated with a process of new investments: the complementarity relations of Blocks of Development, seems alien to Inessential Characteristic Functions. No doubt a certain set of relative income-shares may experience a correlated increase as a result of the variation from one Inessential Characteristic Function to another, but this positive connection is no significant case of complementarity. It is either of a fortuitous character, or depends systematically on factors that the theory cannot take into account — which, of course, would be quite detrimental to the theory. We shall have opportunity further to discuss this topic below.

We may summarize our arguments as follows: The Inessential Games can comprise investment microchanges, i.e. new specific investments made by particular independent decision units, causing changes in the various $v((i))$; these micro-changes can have no macro-consequences, i.e. no consequences for any pricing or distribution agreements between several or all units in the society, as all adjustments of the distribution of income to the changed data take place automatically and harmoniously.

The situation is almost the same if we turn to a class of Essential Characteristic Functions which are Strategically Equivalent. An Imputation, established on the background of an Essential Characteristic Function is no longer uniquely conditioned by this background — it depends on the standard of behavior accepted, on the power constellation of the society and the agreements which have been formed to suit this constellation, and possibly on the bargaining abilities existing — but a change of this Function to another member of the class of Strategically Equivalent Games causes no important consequences for the Imputation: the bargaining abilities, the strategical possibilities of the decision units to form agreements, etc., are materially unaffected by such a change of Characteristic Function, and hence the whole superstructure of possible standards of behavior and potentialities for groupings into power blocks and coalitions is not modified to any significant degree. If no other events occur, the income adjustments are apt to take place in a smooth way, the investment micro-changes causing no macro-consequences for the pricing and distribution system, i.e. no necessity of changing any pricing or distribution agreements in a way which may be the matter of dispute; moreover, no significant complementarity phenomena intervene.

In our opinion many theoretical analyzes in economics, dealing particularly with problems of business enterprise, could be adequately considered as valid within the universe of a class of Strategically Equivalent Characteristic Functions. These analyzes take into account certain repercussions of the actions — e.g. the carrying out of new investments — of one decision unit on the profits of other units, but they assume the ≫structure≫ of the society, i.e., the prevailing social norms, the existing ≫institutional≫ power constellation, the distribution agreements, the bargaining abilities, etc., as constant. We have repeatedly emphasized that a variation between Strategically Equivalent Games shows close similarity to a change of a Walrasian structural coefficient, causing an adjustment of the relative prices; we now add that the consequences for the distribution of income of a variation from one Characteristic Function to another within the same class of Strategically Equivalent Functions should be in principle uniquely predictable, knowing all data completely, assuming the most intelligent behavior of all decision units, and barring spontaneous changes of Solutions and Imputations to occur.

β) Variations between Reduced Essential Characteristic Functions

As we know, however, variations between Strategically Equivalent Games are rather unimportant at the game level of analysis. There is another qualitatively different type of variation, which may take place between Reduced Characteristic Functions — in the three-person setup not only between the (unique) Reduced Inessential Characteristic Function and the (unique) Reduced Essential Characteristic Function, but in the n-person setups with $n > 4$ also between the multitude of feasible Reduced Essential Characteristic Functions. Such a change in some or all of the $v(S)$ alters definitely the strategic possibilities of the units to form agreements, enter into coalitions, etc., *in upsetting the relations of Domination.*

After the change the Effectivity of some sets of units may diminish — if a $v(S)$ falls this obviously has consequences for relations of the type

$$\sum_{i \in S} \alpha_i \leq v(S)$$

i.e., the set becomes Effective only for Imputations giving to it less than the previous maximum amounts —; the Effectivity of some other sets of units may increase. As we know, the conditions for one Imputation $\overrightarrow{\alpha}$ to dominate another Imputation $\overrightarrow{\beta}$ with respect to a set S are[16)]

a) S is not empty

[16)] Cf. p. 65 above.

(2) b) S is Effective for $\vec{\alpha}$

c) $\alpha_i > \beta_i$ for all $i \in S$

A change in the relations of Effectivity and Domination thus is sure to impact on the sets of Imputations which are undominated by each other, that is on the Solutions.

Little is as yet known about the complicated and variegated process: change of a Reduced Essential Characteristic Function $\longrightarrow$ change of the relations of Effectivity and Domination $\longrightarrow$ change of Solutions and their Imputations.

The evidence found in the investigations of Zero-sum four-person Games and Simple Games suggest that in some cases a change of a Reduced Essential Characteristic Function implies only a minor modification in the structure of the Solutions and their Imputations[17]; in other cases a slight change in a Reduced Characteristic Function may upset profoundly this structure.[18]

Suppose that a particular Imputation out of a feasible set, i.e. belonging to a Solution, is established on the background of a Characteristic Function. This Imputation is conditioned by certain agreements between the decision units of the society, or perhaps the sector of society involved, behind which agreements stands a certain power constellation of units. This power constellation may be of a relatively simple structure, one set of units having all the advantage, the remaining units being pushed to their minimum amounts; in other cases a whole hierarchy from most favored sets to least favored sets must be said to exist; and in some cases the advantage may be fairly evenly distributed. If now a significant change happens to the physical background a situation arises which makes the established Imputation outmoded and calls for new distribution agreements: it may be that the previously most favored units face a necessary retreat and completely new possibilities open up for previously less favored units, whose Effectivity as a set has increased. However, and this is very important: as we speak about significant changes between *Reduced, Essential* Characteristic Functions, the new Imputation to be established, i.e. the new pricing and distribution agreements and the corresponding grouping of the decision units, is not uniquely given; even if a similar *type* of Solution, i.e. accepted standard of behavior, be maintained, a number of alternative Imputations

[17] Compare the Zero-sum four-person Games, e.g. the movement from Corner VIII to the point $x_i = -1/5$, referred to on p. 151 above; see von Neumann and Morgenstern [1], Figure 4, p. 313.

[18] The limiting points in the figure of the preceding footnote exemplify to some extent this phenomenon.

are feasible and a distribution struggle is likely to ensue, the outcome of which can impossibly be exactly foreseen. Agreements between a set of units with satisfactory power of Domination being formed, this ensures the putting up of a feasible Imputation, which holds until new significant changes of data occur — or until spontaneous changes of the established Imputation or the prevailing Solution take place.

Thus certain theoretical consequences of significant variations between Reduced Essential Characteristic Functions, namely the non-automatic and non-harmonious character of the adjustments of an Imputation to the changed data, seem to fit quite well into one important empirical phenomenon, often correlated with a boom of new investments, namely the breakdown of the pricing system in the crisis and the consequential diminution of the volume of most economic transactions, the liquidations, etc., taking place during the depression. It appears plausible that the changes caused by investments fulfilled in the first part of the business cycle upswing, and by the work on incompleted investments during this phase, may be of an insignificant kind, from the distribution point of view, similar to variations between Strategically Equivalent Characteristic Functions, and implying a fairly smooth process of transformation with no major conflicts of distribution to be settled. As time lapses and further new investments become completed, the crisis arrives — this could be, explained with resort to the hypothesis that the changes in objective possibilities caused by the new investments, which are fulfilled during the last parts of the business cycle upswing, belong to the category of significant variations between Reduced Characteristic Functions. In that case: the new data do not uniquely condition the distribution of profits; room is left for alternatives which favor different sets of units; hence a problem of distribution open within certain limits presents itself, and the problem can only be settled through a struggle between different group interests; until this problem is solved the transactions between the various units are likely to be severely reduced, no stable pricing agreements having been arrived at. Outstanding signs of the struggle are also the liquidations occurring during crises and ensuing depressions, to old firms negatively influenced by the change of data (Schumpeter-Dahmén), but also to some of the progressing branches with newly created investments (≫malinvestments≫: Johan Åkerman). When a new acceptable Imputation is established the transactions are apt to take on a more normal magnitude; besides new plans may be devised for shaping investments, which, when completed, once again may upset the data.

The theoretical consequences of significant changes in Characteristic Functions just discussed — the non-automatic and non-harmonious character of the adjustment of Imputations to new data — are conditioned by the basic non-additivity characteristic of Essential Games, implying both

the multiplicity of Reduced Essential $\geq$ 4-person Characteristic Functions and the non-uniqueness of the Imputations possible to establish on the background of one Essential Characteristic Function. This non-additivity characteristic should display itself in certain complementarity relations appearing between the profits of various investments. As we remarked above a variation between non-reduced Inessential Games — and between Strategically Equivalent Games at large — most characteristically shows no systematic connection between, e.g. the increase of income of one unit and the possible increase of income of some other units. The non-additive Characteristic Functions embody possibilities of the most pronounced systematic connections of this complementarity kind. After a change having occurred to such a Characteristic Function, various sets of units whose Effectivity is increased may face completely new possibilities of forming agreements and entering into collaboration, with raised relative income shares as the outcome for all members, if an agreement is really formed. This raise is made possible through the change of Characteristic Function, but it does not come about automatically, because it needs agreements in order to be established; nor does it develop harmoniously, as certain other alternative agreements are always feasible, involving other units of the society and showing inferior conditions for some members of the coalition experiencing the potential rise of profits.

The empirical observations behind the important concept of Block of Development seem strongly to suggest a systematic connection of a complementarity kind between the profits of let us say one branch of industry and the profits of some other branches — the new investments completed in the latter change the data of the situation and make not only possible their own profitability but increase also the profits of the former. The observations do not immediately tell us, however, if the connection is established automatically and harmoniously or if it requires agreements and collaboration, and if other alternative profit distributions would have been feasible, making e.g. some of the new investments in the Block ≫malinvestments≫, and letting the profits mainly go to other units. No doubt a historical investigation of new investments causing changes in the profit distribution has an inherent bias to let the changes look automatic — only what is realized is directly observed, only the ≫real part≫ of a theoretical system can be directly observed, not the ≫virtual part≫ — in this connection we mean by ≫real part≫ the Imputation that is in fact established and by ≫virtual part≫ the possible alternative Imputations which might have been established, the power constellation and the bargaining abilities being different from those actually arising. Certainly intensive empirical investigations of particular crises and depressions would be necessary in order satisfactorily to ascertain the possible connection

between the Block-of-Development observations on complementarity and certain complementarity phenomena tied to Essential Games. There are, however, already some indications that the complementarity relations of the Blocks of Development do not come into existence in an automatic and a harmonious way.

Firstly, it is likely that part of the relevant agreements are made before the completion of the new investments, i.e., certain units may decide to collaborate in the situation that will arise when the new investments are carried out. This point is quite in conformity with Dahmén's emphasis on the fact that the existence of Blocks of Development creates strong centralizing tendencies within industry. The change of data and part of the agreements made to fit the new situation thus form a certain kind of unity from the observational point of view. Secondly, also this in conformity with Dahmén's remarks, there are examples that a pronounced change in what may be labeled the objective possibilities does not have the effect expected in a first approximation — an unmistakable technological change disfavoring one unit is not necessarily accompanied by the relative decline of the profit share of this unit. This circumstance Dahmén attributes to ≫absolute factors of inertia≫, being connected with phenomena of a political, social or cartelization kind, and exemplified by the development of agriculture in the Western countries during the recent decades; it could imply that the resulting income distribution is not uniquely conditioned by changes in data.

We summarize our arguments as follows: the Essential Games can comprise investment micro-changes, i.e., new specific investments made by individual units, causing changes in the various $v(S)$; these microchanges have, if they are significant enough, pronounced macro-consequences i.e., they upset the basis for some or all pricing or distribution agreements between the units within society or a sector of society, requiring new agreements to be found through negotiation and struggle. The new investments are carried out during revival and prosperity; at least some important ones are completed towards the end of prosperity, causing the significant micro-changes; the macro-consequences are felt during the crisis and the ensuing depression, when the diverging interests have to fight for the shape of the new agreements necessitated and when therefore the volume of economic transactions greatly diminishes and liquidations, i.e. the writing down of capital values, take place; in the time-interval when such agreements are reached the path is opened for revival to come, accompanied and strengthened by new investments which may result in further significant micro-changes at the time of their completion. The fact that the micro-changes can have macro-consequences is due to the non-additivity or complementarity characteristic of Essential Games; there are indications

that this characteristic displays itself in the observations inherent in the Blocks of Development, showing a pronounced systematic positive connection between the profits of various specific investments. Finally, we re-stress the circumstance that the consequences for the income distribution of a significant variation from one Reduced Essential Characteristic Function to another could never be uniquely predictable, even knowing all data completely and assuming the most intelligent behavior of all decision units. This circumstance is obviously implied by the non-uniqueness of Imputations in Essential Games; it is probably of relevance to the discussions of ≫malinvestments≫[19]: due to factors of power and negotiation, the units owning certain new investments may get a disfavored position within the Imputation that is established, far below the maximum possible amounts — this despite the fact that they act ≫rationally≫.

γ) *Economic progress in a free group-society*

In a centrally planned economy, exhaustively covered by the pure production aspect, economic progress is achieved through new investments, made according to a plan, which is conditioned by the value scheme of the society, i.e. which is the outcome of attempts to maximize the social utility through the exercise of choice-activities. Within the Walrasian universe the problem of economic progress was never clearly posed, but the possibility at least exists that progress may be achieved relative to the various consumer preferences, harmonized through the market mechanism — progress thus being related to the choice-activities depicted by the system. How can any form of economic progress be grasped by a theoretical system within the pure distribution aspect, which views the aim of an economic subject to consist in the maximization of its income-share out of a given total amount, i.e. of its relative money income, no meaning being attached to the maximization of the total money-income of the (closed) society?[20] In such a system — the Zero or Constant-sum Games — the profit-motive has a pure form and plays an independent rôle, while in a simple Walrasian construction the profit motive is only subordinate or auxiliary to the final aim of satisfying so far as possible the consumer preferences, involving goods and services.

Given a Characteristic Function, i.e., a set of objective possibilities for the various decision units in the distribution process, and a Solution, i.e. a habitual principle, the negotiation activities, guided by the pure profit motive, fix, under both conflict and collaboration, the Imputation. These activities ensure a pricing system, necessary for the performance

[19] Cf. p. 240 above.

[20] Compare our discussion in Chapter III.

of the economic transactions in the society, i.e., for the utilization of the existing assets, but they can hardly be said to create any net value to the society implying some kind of economic progress. In our above discussion we have attributed economic progress in a free group-society — i.e., the new investments carried out in such a society — to activities, which change the physical background or the Characteristic Function of the society and in that way try to achieve a better profit position for the unit involved. As long as these changes imply variations only between Strategically Equivalent Games the activities are of a rather unproblematical character; when the changes take on the character of significant variations between Reduced Essential Characteristic Functions the activities firstly become somewhat more strongly motivated, as their effect may be greatly increased through the existence of new potentialities for the owning units of entering into favorable agreements — a complementarity effect; secondly, they become more risky, as the result for the income distribution of a variation between Reduced Essential Characteristic Functions depends on ≫institutional≫ factors of power, negotiation and struggle, and does not come about automatically, granted the rationality of behavior of the owning units; thirdly, in adding to the nation's amount of real assets they open certain distribution problems, which can only be solved through a struggle between diverging interests — a distribution struggle which we believe to be the essence of crises and depressions.

Particularly the third and last point underlines the important fact that economic progress and economic crises and depressions are most intimately connected in our discussion, as they are, e.g. in Johan Åkerman's and Schumpeter's elaborations of the business cycle phenomena, but not in the linear econometric models, superimposing oscillations on a straight trend and letting the oscillations develop notwithstanding the slope of the trend.

δ) *Business cycles, the hierarchy of distribution problems and the variability of Imputations*

In discussing business cycles from the pure distribution aspect we have only been able to utilize the most general and basic traits of the theory involved, namely, the multiplicity of Imputations on the background of one Essential Characteristic Function, and the multiplicity of Reduced Essential n-person Characteristic Functions with $n \geq 4$; we have not been able to specialize the problem, e.g. in the sense of connecting one restricted type of games with business cycle fluctuations. This is quite in accord with our opinion that business cycle fluctuations constitute a very general phenomenon which should appear at the very basis of a significant economic and social theory. Our discussion above all focuses attention on the specific investments carried out, and on the interest group constellation and its

changes. It must appeal to empirical research for further specialization, which is apt to be rather tied to particular regions and periods. This is quite in accord with the ≫empirical≫ character of the theory of games, in the sense that this theory contains a variety of elements of change or ≫empty boxes≫, the particular shape of which could in principle be determined only through empirical investigations, even if the theory were perfectly completed.

We think, however, that it may be possible here to make two minor specializations of some interest. On carrying out the first specialization we refer to our above remarks about the hierarchy of distribution problems, the big social groups including labor, agriculture, industry, etc., competing on the highest national level for their shares of the national income; various sectors within industry competing on a lower level for their shares of the total industrial amount; various branches competing within each sector; etc. Our suggestion is now that the severity of crises and depressions may be less if the distribution problems made crucial through the change of Characteristic Function be found only within the industrial group, the relations between the big social groups not being questioned; and it may be worse if the open problems simultaneously occur on various levels, not only within industry but also between the big social groups. In fulfilling this line, of thought we contrast the picture of business cycle fluctuations given above to Veblen's opinion expressed in 1923 that crises and depressions were no longer possible. ≫There is in the current situation an element of sobriety and a factor of salutary reserve that were lacking in the nineteenth century. Prosperity ran somewhat headlong in those days, and came in the ordinary course, to a headlong liquidation, ending in panic, crisis and depression. Under the surveillance of the Federal Reserve and the One Big Union of the [Vested] Interests in the twentieth century the prosperity of business, that is to say the inflation of capital and prices — does not run riot. It is an orderly advance, in the course of which the progressive creations of credit and capital are duly stabilized and 'digested,' distributed and consolidated under the aegis of the Interests, with such effect as to make them a secure ground on which to hypothecate further creations of the same character, in indefinite progression. And no undesigned liquidation need be apprehended in this case, since the major debits and credits are pooled, in effect, by being drawn in under the custody of these major Interests that informally make up the One Big Union≫.[21)]

This prophecy of 1923 stands in glaring contradiction to the events of 1929 and the early thirties. We would criticize to a first approximation

[21)] Thorstein Veblen, *Absentee Ownership and Business Enterprise in Recent Times,* B. W. Huebsch, New York 1923, p. 391, footnote.

Veblen's statement thus: Veblen overlooks the possibilities of new investments to upset the income-relations between the various subunits that may exist within the One Big Union of American industry, and/or he overestimates the degree of unification of the Big Union, forgetting the divergent interests that may be found within it, between which a distribution struggle may appear as the outcome of an investment boom, changing the objective possibilities of the various units. A further factor should, however, also be mentioned: it is possible that the extreme severity of the depression following the crisis of 1929 was due to the fact that the income-relations between the big social groups were questioned to a pronounced degree simultaneously with some intra-industrial income relations; in other words: the existing income-relations between the One Big Union of industry and, the other big social groups were also upset. The rôle of investment facts in this reversion may be important, but it seems that other factors apt to change the objective possibilities of the big social groups in the distribution process should also be taken into account, notably political facts — decisive changes in the composition of the national Parliament are likely to make feasible a number of completely new political groupings and coalitions with corresponding consequences for the income distribution of the society when a stable coalition is actually established.[22)]

In our discussion in Chapters V and VI of the changes occurring around 1932 we spoke about the shift from one type of Solutions to another with

[22)] Compare in this context Johan Åkerman's investigations about the influence of economic conditions on political elections, presented in *Ekonomskt skeende och politiska förändringar,* Lund 1946, and in "Political Economic cycles," *Kyklos* 1947. According to Åkerman, elections held during prosperities are likely to result in essentially unchanged political conditions, but elections held during depressions have often led to sharp changes in the composition of the national Parliament — the course of events in the depression after 1929 constituting a typical example. Looking at political reversions as being capable of changing in a significant way a Characteristic Function involving particularly the big social groups of society, we think that the type of political behavior during depressions just indicated may be a factor aggravating the depression. More particularly: suppose that a change of investment facts has put distribution problems open within the industrial group and initiated a depression; an election held during this depression and leading to a political reversion may also bring the relations between the industrial-group and the other big social groups into dispute and thus create another open distribution problem on a higher level in the hierarchy. Compare also Myrdal's discussion of desiderata for an economic technology in Myrdal [29], pp. 276–285.

corresponding implications for the variability of Imputations, comprising the big social groups. Probably this shift should in a closer approximation be viewed as the consequence of changes in the objective possibilities tied to the crisis of 1929 and caused by the investment boom of the 1920's and the political reversions taking place after the crisis; the Imputation within a new type of Solution established around 1932 should thus be considered as the outcome of the distribution struggle made necessary by the changes in Characteristic Functions happening around 1929.

In speaking about business cycles and the distribution of income between the big social groups the character of two of the most well-known verbal business cycle theories should be observed: we think of the over-investment theory and the under-consumption theory. Certainly these theories are presented in many variants, but it seems fair to conclude that they form the bases of two distinct types of business cycle policy, which are exactly opposite to each other. The first theory recommends every measure favoring savings and disfavoring ≫spending≫, i.e. consumption and investment — e.g. a rise of the rate of interest in relation to wages and profits; the second theory sponsors every action disfavoring savings and favoring spending — e.g. a fall of the rate of interest in relation to other prices.[23)] Thus both theories are related to a distribution struggle, but they propose two widely different solutions of this struggle; they should be viewed in connection with the two interest rate norms, the orthodox norm prevailing before 1930 and the unorthodox norm prevailing during the recent decades.

We now turn to the second specialization of our discussion of business cycles. In this thesis important features of the economic course of events after 1932 have been attributed to the variability of Imputations which is a consequence of discriminatory Solutions — each discriminatory Solution containing infinitely many Imputations and belonging to a family of infinitely many members, all based on exactly the same qualitative principle — and in some degree also of Solutions with a non-zero Excess. How is this variability of Imputations — in our discussion tied to inflation — related to significant changes of Characteristic Functions — in our discussion tied to business cycles?

This is a question of the nature of the process: change of a Reduced Essential Characteristic Function $\longrightarrow$ change of the relations of Domination $\longrightarrow$ change of the Solutions and their Imputations
under the reign of, above all, a discriminatory type of Solution. Now it is a characteristic of discriminatory Solutions — and of Solutions with a

[23)] Cf. Tinbergen [4], II, pp. 188–190.

positive Excess — that the relations of Effectivity, upon which the relations of Domination build, are to a varying extent set aside: the relations $\sum_{i\in S} \alpha_i \leq v(S)$ involving the Excluded unit as a coalition partner are of no real importance as this unit is excluded from all agreements by the accepted standard of behavior; some or all relations $\sum_{i\in S} \alpha_i \leq v(S)$ have no actual importance under a positive Excess as some or all feasible sets of units obtain a higher income than the relations admit.[24)] Hence it seems fairly plausible — although the matter has not been systematically explored so far — that changes occurring to the relations of Effectivity and Domination, i.e. changes from one Reduced Essential Characteristic Function to another, are much less likely to necessitate a significant adjustment of the existing Imputation under the reign of a discriminatory type of Solution and a Solution with a positive Excess than under the prevalence of a type of Solution which strictly maintains the relations of Effectivity and Domination. In other words: the more indeterminacy of Imputation which the prevailing type of Solution admits, i.e. the bigger the set of Imputations which are consistent with one and the same power constellation, the greater seems the possibility to be that an Imputation established on one Characteristic Function may continue to be feasible essentially unchanged on another Characteristic Function as long as the same type of Solution is valid.

Thus changes of Characteristic Functions should not be so likely to necessitate a troublesome distribution adjustment between the big social groups when a discriminatory type of Solution applies to the relations between these groups as when an objective type of Solution is accepted.

5) Summary

a) CONCLUSIONS ON THE RELATION BETWEEN THE TWO SUMMATION THEOREMS

First, a type of oscillations is discussed, which is found in many economic theory-constructions, utilizing differential and difference equations, and which is often intended to explain actual business cycle movements. If logically completed these systems, however, turn out to be planning models, requiring the establishment of one social value scheme; hence their relevance for free group-economies must be severely doubted. These logically completed systems can take into account a process of new investments, but view the investments as carried out in fulfillment of a plan of society. The systems raise the problem of dampening their oscillations or of counteracting possible instabilities; this leads into the theory of servo-mechanisms.

[24)] Compare; e.g. the quotation from von Neumann and Morgenstern [1] on p. 205 above.

Second, business cycles are discussed in connection with the distribution process of a multi-unit-of-decision society. New investments are viewed in their rôle of upsetting the existing income distribution of a society. The process of income-distribution adjustment made necessary is investigated, utilizing concepts of the Zero or Constant-sum Games; it is found that this process can be of a highly troublesome nature, giving rise to pronounced breaks in the economic development.

Thus the general dualism between a production aspect and a distribution aspect — in our opinion implied by the apparent contradiction between the two summation theorems — is exemplified in a special context: the field of economic fluctuations, or, as we might also say, the field of new investments, which cause various types of economic fluctuations. Two widely different rôles of new investments are distinguished: the rôle of new investments in a planned economy, being parts of the economic plan of society and, being capable of giving rise to certain oscillations, which are to be counteracted and damped; and the rôle of new investments in a free-group economy, giving to their owners (chances of) increased profits, and, if significant enough, necessitating serious processes of adjustments involving struggle between divergent economic Interests.

b) CONCLUSIONS ON THE RELATION BETWEEN THE FIRST SUMMATION THEOREM AND EMPIRICAL RESEARCH

Our examination of the two summation theorems in relation to each other leads to the acceptance of the dualism between the production aspect and the distribution aspect. The general dualism is exemplified in the special field of the analysis of economic fluctuations or of processes of new investments. The next logical step should be the investigation of empirical fluctuation and investment-phenomena, utilizing the production aspect — essentially for studying features of a centrally planned economy —, and the distribution aspect — essentially for studying features of a free group-economy. Only the second task is attempted here.

Starting from observations pertinent to business cycles, we tentatively try to identify certain types of elements of change of Essential Games with the observed phenomena. On the empirical plane, new specific investments made by single decision units, e.g. certain groups or branches of enterprises, are known to have consequences for the distribution of income in the society; booms of new investments are often followed by severe crises and depressions, characterized by heavy decreases of the volume of economic transactions in all sectors of the society and by pronounced liquidations of economic capital values, both of old and of new investments; moreover, these investment booms are accompanied by certain phenomena of

complementarity, displaying themselves in a marked positive connection between the profits of certain new investments and of certain previously existing assets, the profitability of the old assets being unsatisfactory until the new assets are created. We identify new investments with changes of Characteristic Functions, i.e. of the objective possibilities of the decision units in the distribution process. On the theoretical plane changes of Characteristic Functions certainly have consequences for the distribution of income in the society, but as long as the variations only take place between Strategically Equivalent Characteristic Functions the income-distribution adjustments are apt to take on a smooth character: no conflict of interest is opened by such variations since the new Imputation to be established is uniquely conditioned, granted the fact that no spontaneous changes happen, e.g. to the power constellation of the society; moreover, no pronounced phenomena of complementarity display themselves in connection with such variations. Significant variations between Reduced Essential Characteristic Functions — tied to the first summation theorem —, i.e. variations changing the relations of Effectivity and Domination, have, however, much more serious implications: conflicts of interest occur, as the material basis for the existing power constellation is disrupted and as the new Imputation to be established is not uniquely given but must be found through a struggle between different group-interests; moreover, pronounced phenomena of complementarity arise because of the fact that certain sets of decision units face completely new possibilities of increasing all profits involved through agreement and collaboration.

Significant variations between Reduced Essential Characteristic Functions illustrate the fact that in a free group-society economic progress — i.e. new investments — is intimately tied to economic crises and depressions — i.e. distribution struggles with a break-down of the existing price system, a heavy decrease of the volume of economic transactions and numerous liquidations of capital values. The process of adjustment made necessary by an investment boom is likely to be more troublesome if the objective possibilities are disrupted, not only as regards certain groups or branches within the entrepreneurs, but also as regards the entrepreneurs in relation to other big social groups.

c) CONCLUSIONS ON THE RELATION BETWEEN THE FIRST SUMMATION THEOREM AND OTHER ECONOMIC THEORIES

If variations occur between Inessential Characteristic Functions — which are all Strategically Equivalent — and between Essential Characteristic Functions, which are Strategically Equivalent, the implications for the distribution of income, i.e. for prices, profits, etc., are in principle uniquely predictable — in the last-mentioned case granted the fact that no spontaneous changes occur to the existing accepted standard of behavior or to the existing power constellation and bargaining abilities. At least if significant variations occur between Reduced Essential Characteristic Functions, the implications for the distribution of income are in principle not uniquely predictable, as a number of alternative new Imputations may be established — the actual choice must be made through a struggle between different group-interests; moreover, the nature of the accepted standard of behavior may change in a non-unique way.

The first type of variations thus take place within a universe which we may label closed or determinate; the second type of variations develop within a universe which we may label open or to a certain degree indeterminate. We have concentrated our interest on one type of action likely to change the existing Characteristic Function: the carrying out of new investments. It seems plausible, however, that many more types of economic and social action should be grasped as changes in Characteristic Functions. The Characteristic Functions of the simplest case of the Inessential Games, the Zero-sum two-person Games, consists essentially of one number: $v((1)) = \alpha_1$ [as $v((2)) = \alpha_2 = 0 - v((1))$]; the theory discusses the fixation of this number, given a certain set of ways of action or Strategies of one decision unit and his adversary. Starting from a certain existing value of $v((1)) = \alpha_1$, and carrying out a project involving the choice between various Strategies, the value of $v((1))$ changes to a new position β_1. This situation is similar to, although more general than, the situation arising in the ordinary theory of the firm, when firstly a certain value of profits exists, say α_1, and secondly, various ways of action are discussed, the optimal choice between which changes the profits to β_1. As far as we can see, in one respect no important qualitatively new phenomena meet a generalization of this kind of analysis, if it be extended to Inessential Games with more than two decision units: the consequences for prices, profits, etc. of the actions changing the Characteristic Function are still in principle uniquely predictable; or, if it be extended to a class of Strategically Equivalent Games: the consequences, for prices, profits, etc. of the actions changing the Characteristic Function are also then in principle uniquely predictable, granted the condition that the superstructure of social norms and institutional power constellation and bargaining abilities remain unchanged.

Completely new qualitative phenomena, however, meet the analysis if the actions carried out cause significant changes from one Reduced Essential Characteristic Function to another, because in that case the superstructure cannot possibly be regarded as constant — it must change decisively to fit the new objective possibilities, and it changes in a non-unique way.

In our opinion many analyzes of modern economics, particularly those related to theoretical business economics, should in principle be valid within a closed or determinate universe as long as no spontaneous changes — i.e. changes not caused by variations of Characteristic Functions — happen to the superstructure of social norms and institutional power elements. The investigation of spontaneous changes of Solutions and Imputations and of variations within an open or to a certain degree indeterminate universe require other methods of analyzes, taking into account sociological, political and similar factors. In Chapters V–VII we have tried to outline an investigation of the last-mentioned type.

Conclusion

Our thesis is based on two important methodological *postulates.* The first postulate states that economic theories should be derived from explicit assumptions on human economic action. The second postulate states that a contact should be sought between the elements of change of economic theories and empirical observations in order to achieve a mutual corroboration between theory and observation. Both postulates are also explicitly accepted by, for instance, traditional econometrics. In our opinion, however, econometrics does not go far enough in pursuing the implications of the first postulate. This is due to its almost complete neglect of the economic problem of summation; as a consequence, its attitude towards the second postulate becomes very special.

On examining, from the point of the view of the *first postulate,* a competitive Walrasian construction — the chief economico-theoretical background of traditional econometrics — we find that the behavior equations, i.e. the supply and demand equations, may be derived from assumptions on individual utility maximization, granted that prices be taken as given. When equations are added, however, with the purpose of making prices also variable within the system — equilibrium conditions — the question immediately arises: How are these equations related to the underlying principle of human economic action? Certainly they are not directly derived from this principle. Through the functioning of the price mechanism which they depict, the various individual value schemes can in harmony with each other influence the production of goods and services in the society — the relevance of this aspect of the construction looks a priori uncertain if Arrow's summation theorem be taken into account; moreover, the price mechanism or distribution process which they describe always proceeds in an automatic and ultra-harmonic manner without any distribution agreements or any conflicts of interest, or even any possibility for one economic subject directly to influence prices more strongly than another economic subject — the relevance of this aspect of the construction looks a priori uncertain, if von Neumann's and Morgenstern's summation theorem be taken into account.

Realizing the doubtful character of the equilibrium conditions from the point of view of the first postulate we may proceed along two lines.

Firstly, it may be investigated what becomes of a Walrasian construction if the equilibrium conditions be deleted. In this case the prices should not appear as variables in the system; hence we substitute the proposition that total supply of a commodity is identical to total demand of this commodity for the proposition that the equality of total supply and total demand of a commodity is achieved in the long run through the functioning of the pricing process, depicted by the equilibrium conditions. This step is taken by Leontief, who at the same time introduces severe linearity restrictions. The prices in Leontief's system do not play an independent rôle; they are of a ≫shadow≫ character, do not vary so as to achieve equality between supply and demand, and may be completely omitted from the analysis. The assumption about identity between supply and demand of every commodity gives to Leontief's system its only teleological trait: the full-utilization-of-resources assumption. Relaxing the linearity restrictions we arrive at a programming system, and must at the same time introduce a most explicit teleological element: the best-utilization-of-resources assumption, embodied in the Objective Function, i.e. the preference function of society which is to be maximized. Arrow's summation theorem prevents in general a social value scheme from being based on many individual value schemes. The pure production aspect is thus the end result of an analysis, trying to investigate what becomes of a Walrasian construction, if the dubious equilibrium conditions be deleted.

Secondly, it may be attempted to generalize the rudimentary pricing or distribution process described by the equilibrium conditions, and thereby to connect the distribution process with assumptions on human economic action. A first step in this direction is taken by Marschak, but his distribution process is still automatic and harmonious; for, although the various decision units may have an unequal influence on the prices to be established, neither any explicit distribution agreements, nor any open conflicts of interest are embodied in the theory. The theory of games achieves the fundamental generalization of the equilibrium conditions: von Neumann's and Morgenstern's summation theorem, and the concepts combined with it, describe a process embodying explicit distribution agreements and open conflicts of interest. The limiting case of Inessential Games closely resembles the traditional automatic and harmonious distribution analyzes; the general case of Zero or Constant-sum Essential Games comprises the basic characteristic of non-additive monetary economic values. The aim of each participant in a Zero or Constant-sum Game is to maximize his relative money income, i.e. his income-share out of a given total amount, the magnitude of which is of no real importance. Severe difficulties, especially due to Arrow's summation theorem, meet an analysis trying to make the total income and the total income shares of importance, thereby introducing into the analysis features

other than purely monetary ones, i.e. utility derived from the production of goods and services. The pure distribution aspect is thus the end result of an analysis, trying — when generalizing the equilibrium conditions — to connect the distribution process with assumptions on human economic action.

In our opinion the provisional acceptance of *the dualism between the pure production aspect and the pure distribution aspect* must be at the present stage of scientific development in economics the outcome of the strict following of the first postulate — that economic theories should be derived from explicit assumptions on human economic action — if the economic problem of summation be taken into account. We now turn to the *second postulate* — that a contact should be sought between the elements of change of economic theories and empirical observations in order to achieve a mutual corroboration between theory and observation. The structural coefficients are the elements of change of a Walrasian construction; the particular nature of these elements is a consequence of the rudimentary treatment within a Walrasian construction of the economic problem of summation. Guided by this theoretical background, traditional econometrics tries essentially to determine by statistical methods economic behavior equations, i.e. to estimate the values of certain structural coefficients from empirical observations. Starting from the two dual aspects, the preliminary step in our following of the second postulate is tentatively to assume that the pure production aspect be mainly relevant for explaining the economic course of events in centrally planned societies; while the pure distribution aspect essentially covers the course of economic events in free group-societies. Although certain reasons may be listed which immediately make this hypothesis plausible, it can only be finally tested by a repeated success in establishing a contact between the elements of change of the two categories of theory and empirical observations from the corresponding type of economy.

In our thesis we only treat very rudimentarily the task of combining theories within the pure production aspect and observations from *planned economies.* We emphasize the distinction between the real and the virtual parts of a programming system, the real part consisting of the methods of production actually employed, the virtual part comprising the alternative methods of production which were not chosen, and the Objective Function. Recalling this distinction it may be added that the elements of change of a programming system are the structural coefficients of the real part of the system; these structural coefficients are, however, dependent on the virtual part of the system, and are likely to be changed at time-points when a new Objective Function is adopted by the society, and perhaps at the same time new methods of production are taken into account — i.e.

in the interval between two economic plans of society or in other words between two planning periods. We further indicate how certain phenomena of rates of interest = rates of growth fit exactly into the pure production aspect, and how phenomena of oscillation, linked to capital formation and of a kind treated by many modern dynamic economic theories, are quite congenial to this aspect; we do not, however, attempt to discuss these findings in relation to observations from planned economies.

In a centrally planned society economic progress is achieved through capital formation, carried out according to a plan which is conditioned by the objectives of the economy. On examining the pure distribution aspect the following question is apt soon to arise: How can any economic progress be achieved, if the decision units be only concerned with their relative money incomes, no meaning being attached to the maximization of the total income, i.e. in this case, of the total amount of money in a closed society? Our answer to this question runs: Economic progress — i.e. capital formation and new investments — in a free group-economy is tied to changes of the data, i.e. of the objective possibilities of the various decision units in the distribution process.

At this point we begin the summary of our attempts to discuss the elements of change of the theories within the pure distribution aspect in relation to observations from free group-economies, the nature of these elements of change being conditioned by the fundamental non-additivity characteristic of von Neumann's and Morgenstern's summation theorem.

The objective possibilities in the distribution process of a *free group-economy* are expressed in a Characteristic Function. Identifying changes of Characteristic Functions with new specific investments, we compare theoretical implications of such changes for the distribution of income in the society, i.e. for the established Imputation, with empirical observations tied to booms of new investments: In the case of variations between Strategically Equivalent Characteristic Functions the process of income-distribution is apt to be smooth with no complementarity phenomena arising; in the case of significant variations between Reduced Essential Characteristic Functions, however, the process of income-distribution adjustment is apt to necessitate a struggle between various group-interests and to show pronounced complementarity phenomena. |——| Booms of new investments are often followed by crises and depressions, correlated with complementarity relations of the Block-of-Development kind.

In the case of significant variations between Reduced Essential Characteristic Functions, economic progress, and economic crises and depressions are thus intimately related.

Each Essential Characteristic Function probably admits several (types of) accepted standards of behavior, i.e. Solutions, each Solution being a set of Imputations. Thus, irrespective of changes of Characteristic Functions, changes of Solutions and changes of Imputations may be considered.

Identifying objective Solutions which have a zero Excess with the national economic norms characterized by the treatment of interest on a par with other distributive shares and the international economic conditions characterized by a balance of power in the world economy; and discriminatory Solutions which have a non-zero Excess with the national economic norms characterized by the exclusion of the savers from all distribution negotiations and the international economic conditions characterized by the dominating position in the world economy of USA relative to European countries, we compare theoretical implications of each type of Solutions with empirical observations tied to the corresponding category of national economic norms and international economic conditions: An Imputation established within an objective Solution with a zero Excess is likely to be fairly stable, because it corresponds to a definite power-constellation of the society and varies only together with changes of this constellation and because the zero-value of the Excess is the most normal state of things, indicating a balance of power between the regions involved and implying a constant ratio between the national money-incomes — hence no frequent initiating changes are likely to occur; moreover there may exist a strong moral unwillingness to admit consequential changes of the very norm established, due to the qualitative differences between this norm and other feasible norms. |——| The period of the orthodox interest-rate norm and of the balance of power in the world economy shows a maintained position of the savers' income share relative to the income shares of the other big social groups and an approximately constant ratio between the American money-income and the money-incomes of European countries, all incomes measured in the same currency; these quantitative facts are correlated among other things with observations indicating the stability of various important income-relations, e.g. Douglas' theory of wages and Wicksell's theory of the normal rate of interest and also the quantity theory of money and the purchasing power parity theory of exchange rates, all devised and taught during the period. I The exclusion of one decision unit within a discriminatory Solution introduces a basic indeterminacy into the national income distribution, making changes of the income-relations between some of the non-excluded units or some sets of such units likely to occur frequently; these initiating changes take place without modifying the power constellation of the society and need not imply the deterioration of the relative income-position of any non-excluded unit, as long as a consequential change may let the Excluded unit face the necessary deterioration, a

way of resolving the problem which can be utilized without the sacrifice of any main principles, due to the qualitative similarity between the various norms within a family of discriminatory norms. A positive Excess is likely not to remain constant, due to the facts that the Excess may vary in a certain interval and that once a positive Excess has appeared, a basic indeterminacy is introduced into the income distribution in the country experiencing the Excess, making initiating changes likely to occur, the negative consequences of which need not be taken by any national social group as long as the magnitude of the positive Excess may be increased — thus the money-income of the country with the positive Excess should have a tendency gradually to increase relative to the money-income of the region with the negative Excess as long as the balance of power between the two regions is disturbed; a gradually increasing negative Excess should cause very serious problems of income-distribution adjustments in the countries involved. |——| The period of the unorthodox interest-rate norm and of the American dominance relative to Europe in the world economy shows a gradual deterioration of the savers' relative income share and a pressure towards an increase of the American money-income relative to the money-incomes of European countries, all incomes measured in the same currency; these quantitative facts are correlated with observations showing the increased attention paid to the instability notably of the income-relations between laborers and entrepreneurs, to the European dollar shortage, etc.

The main variations of Imputations discussed are those caused by changes of Characteristic Functions — tied to our explanation of business cycles — and those occurring within infinite types of Solutions — tied to our explanation of the kind of inflation, which implies a non-proportional development of various prices and income-shares.

Thus our way of analyzing cycles, inflation and interest in free group-societies indicates that these seemingly diversified phenomena may be viewed under a common heading: the distribution of income within and between countries.

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